L & R method for Frequency Synchronization

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Actual approximation part of L and R technique

Our primary aim was to implement L and R technique of frequency offset estimation.

There are 2 parts in it, one using estimate and other as exact.

Estimated part:

$\Delta\hat{f} \simeq \frac{1}{\pi T_s(M+1)} arg(\sum_{k=1}^{M} R(k))$

This is true for small frequency offset , as what we observed , it is true until 800 Hz frequency offset.

Condition on $\Delta\hat{f}$: T_s must be grater than or equal to $1/2*\Delta f$

Condition on M: $M*\Delta f*T_s << 1$.

Calculation of R(k)

R(k) is the auto correlation of our received signal r(k),which is complex oscillation with added gaussian noise.

$$R(k) \triangleq \frac{1}{N-k} \sum_{i=k+1}^{N} r_i r_{i-k}^{*} \ , \ 1 \leq k \leq N-1$$

N = length of signal r(k) i.e. received signal.

The above expression is the auto correlation of received digital signal.

But because of the drawback that the frequency offset should be small (as what we observed , should be upto 800 Hz), we implemented the exact part for frequency offset estimation.

Exact part

For large frequency offset , above estimation goes wrong.So we use the following.

Exact part:

$$\Delta\hat{f} \simeq \frac{1}{2\pi T_s} \frac{\sum_{k=1}^{M} Im(R(k))}{\sum_{k=1}^{M} kRe(R(k))}$$

Constraints on M is same as previous.

This yields a better approximation even for frequency offset as large as 40 MHz.

Practical values of the parameters used

Signal Length N $= 36$

frequency offset $\Delta f = 5MHz$

Ts $=$ 1e-9 .

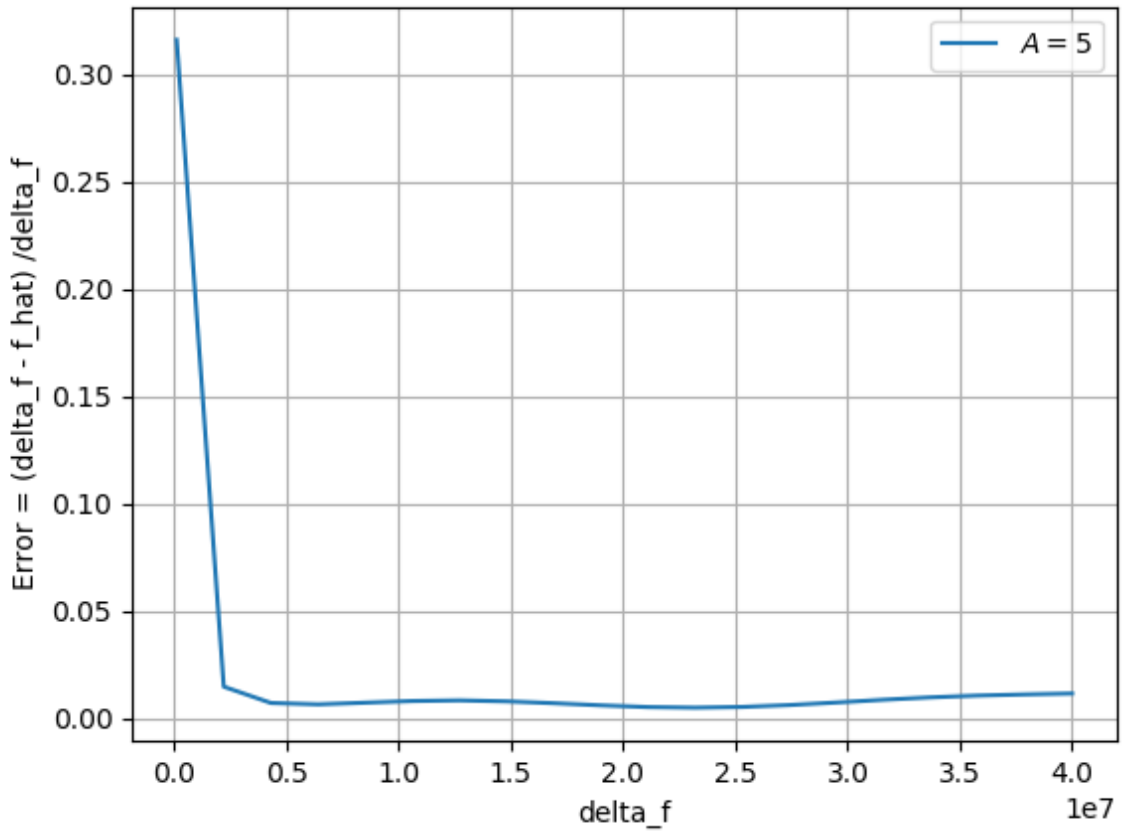
Fc $=$ 25 GHz. (central frequency)

But Fc isn't required anywhere in code.

$M = \frac{N}{2} = 18$

Error for SNR = 5 units

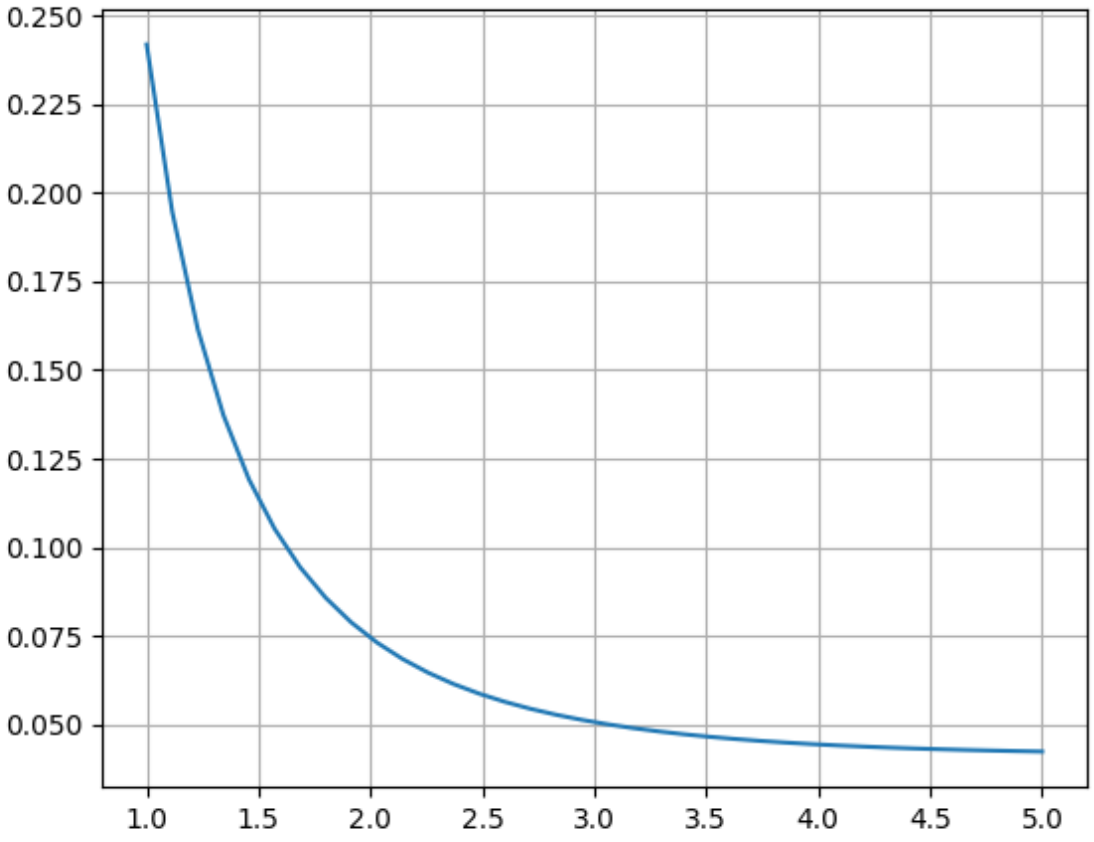
Here, the frequency offset, $delta_f$, is varying from 0.1 to 40 MHz.



Clearly, the error for our destined frequency offset, 5 MHz, is very low.

Error vs SNR for high frequency offsets/ practical offset

Here, frequency offset is fixed to be 5 MHz.



We can clearly see that the error is very very low for high SNR.