GVV Assignment(Ques-36)

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Question

Find the equation of the tangent to the circle,at the point

$$\mathbf{P}= \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

whose centre is the intersection of the straight lines

$\begin{pmatrix} 2 & 1 \end{pmatrix}\mathbf{x} = 3$ .....1st Equation

$\begin{pmatrix} 1 & -1 \end{pmatrix}\mathbf{x} = 1$ .....2nd Equation

Solution

1.Finding intersection of lines:

$\begin{pmatrix}2 & 1\end{pmatrix}\mathbf{x} = 3$ .....1st Equation

$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 1$ .....2nd Equation

Both equation can also write as:-

$$\begin{pmatrix}2 & 1\\ 1 & -1\end{pmatrix}\mathbf{x} = \begin{pmatrix}3\\ 1\end{pmatrix}$$

$$\implies (N^T)\mathbf{x} = \begin{pmatrix}3\\ 1\end{pmatrix} \text{.....}(N^T \textit{ means transpose of } N)$$

$$\implies \mathbf{x} = \begin{pmatrix}3\\ 1\end{pmatrix}(N^T)^{-1} \text{.....}(N^{-1} \textit{ means inverse of } N)$$

$$(N^T)^{-1} = -(\tfrac{1}{3}) \begin{pmatrix} -1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$x = (-1/3) \begin{pmatrix} 3 \\ 1 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$x = \begin{pmatrix} 4/3 \\ 1/3 \end{pmatrix}$$

x = intersection of two given lines

x= Center of circle

2.Finding normal to the tangent

$\mathbf{P}= \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

P = point on circle(Given)

$\mathbf{C}= \begin{pmatrix} 4/3 \\ 1/3 \end{pmatrix}$

C = Centre of circle

$$\mathrm{T}_{PC} = \begin{pmatrix} C & P \end{pmatrix}$$

$$\mathrm{T}_{PC} = \begin{pmatrix} 4/3 & 1 \\ 1/3 & -1 \end{pmatrix}$$

$$\mathbf{m} = \mathrm{T}_{PC} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\mathbf{m} = \begin{pmatrix} -1/3 \\ -4/3 \end{pmatrix}$$

m = direction vector of PC

m= normal to the tangent

3.Tangent to the circle

Now normal vector of tangent is direction vector of PC

Therefore, N for Tangent $=$ $\mathbf{m}$

4.Equation of tangent

Equation of any line

$$\implies n^T(x - P) = 0$$

$$\implies n^T x = n^T P$$

$$\begin{pmatrix} -1/3 & -4/3 \end{pmatrix} x = (1)$$

Hence

Equation of tangent

$$\begin{pmatrix} 1 & 4 \end{pmatrix} x = (-3)$$

Plotting Parameters

For circle Radius P= $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

P = point on circle(Given)

C= $\begin{pmatrix} 4/3 \\ 1/3 \end{pmatrix}$

C = Centre of circle

Radius= $(\text{C-P})^T.(C - P)$

Radius= $\sqrt{(17/9)}$

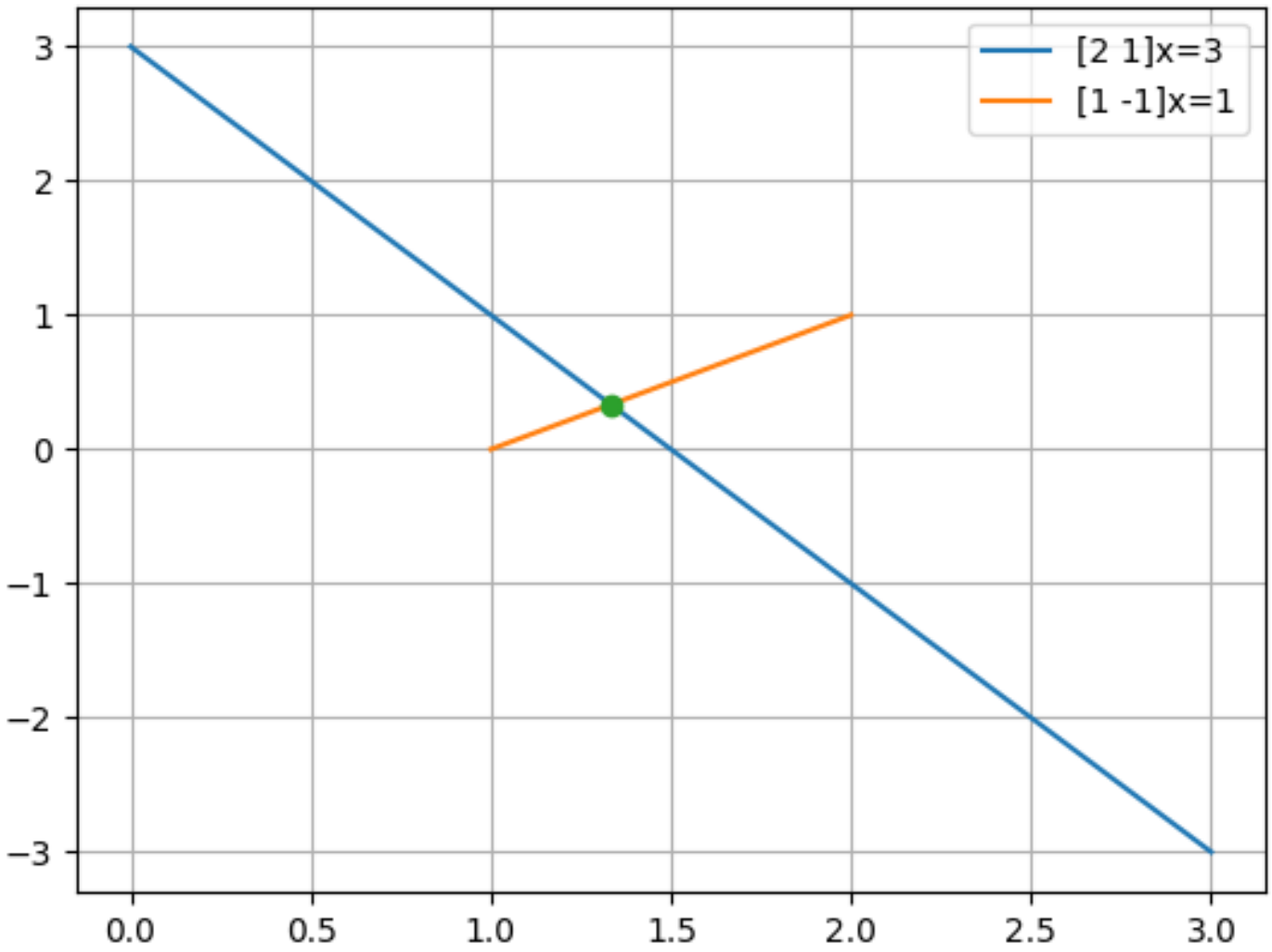

Figure: Intersecting lines

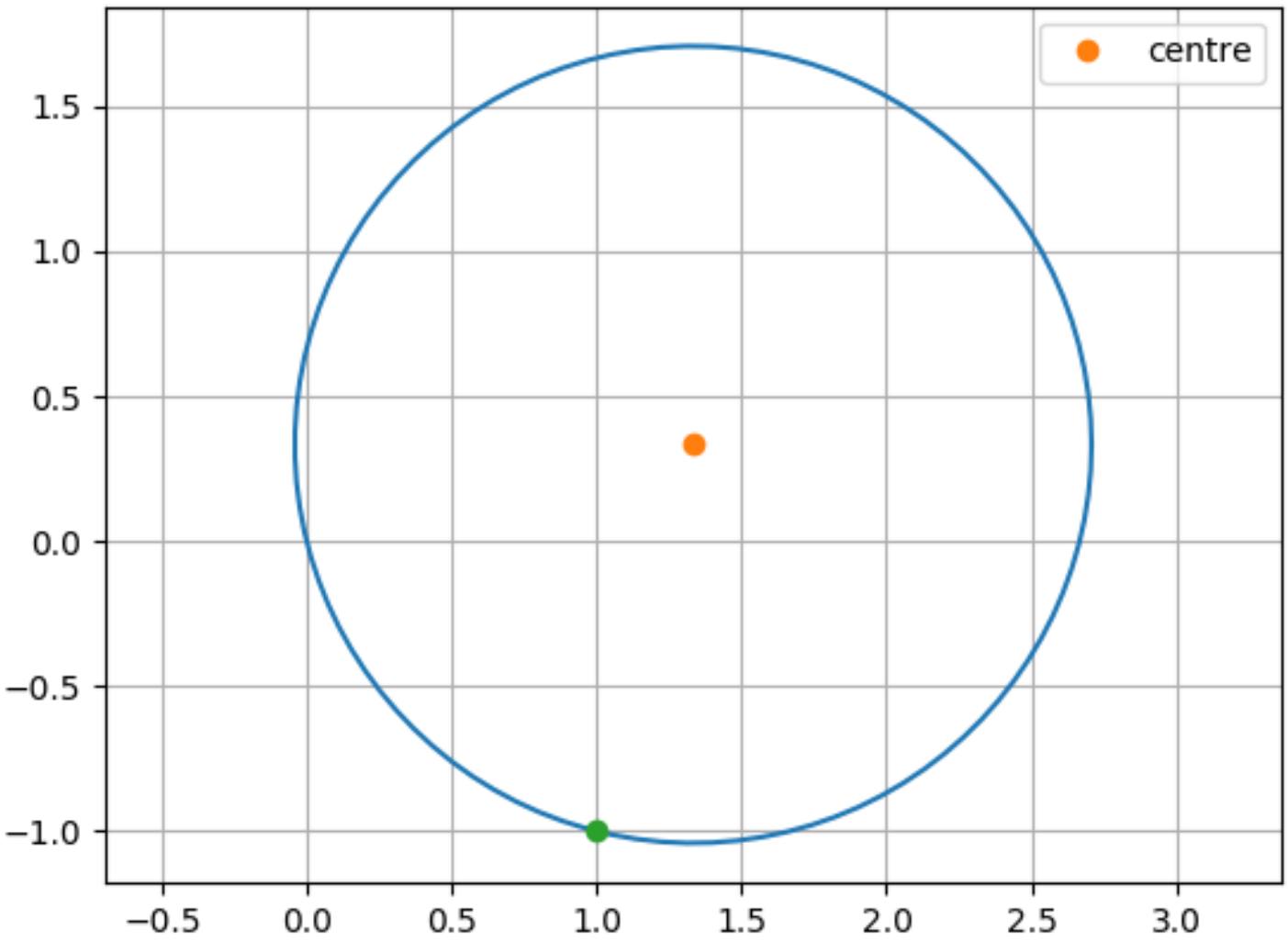

Figure: Circle

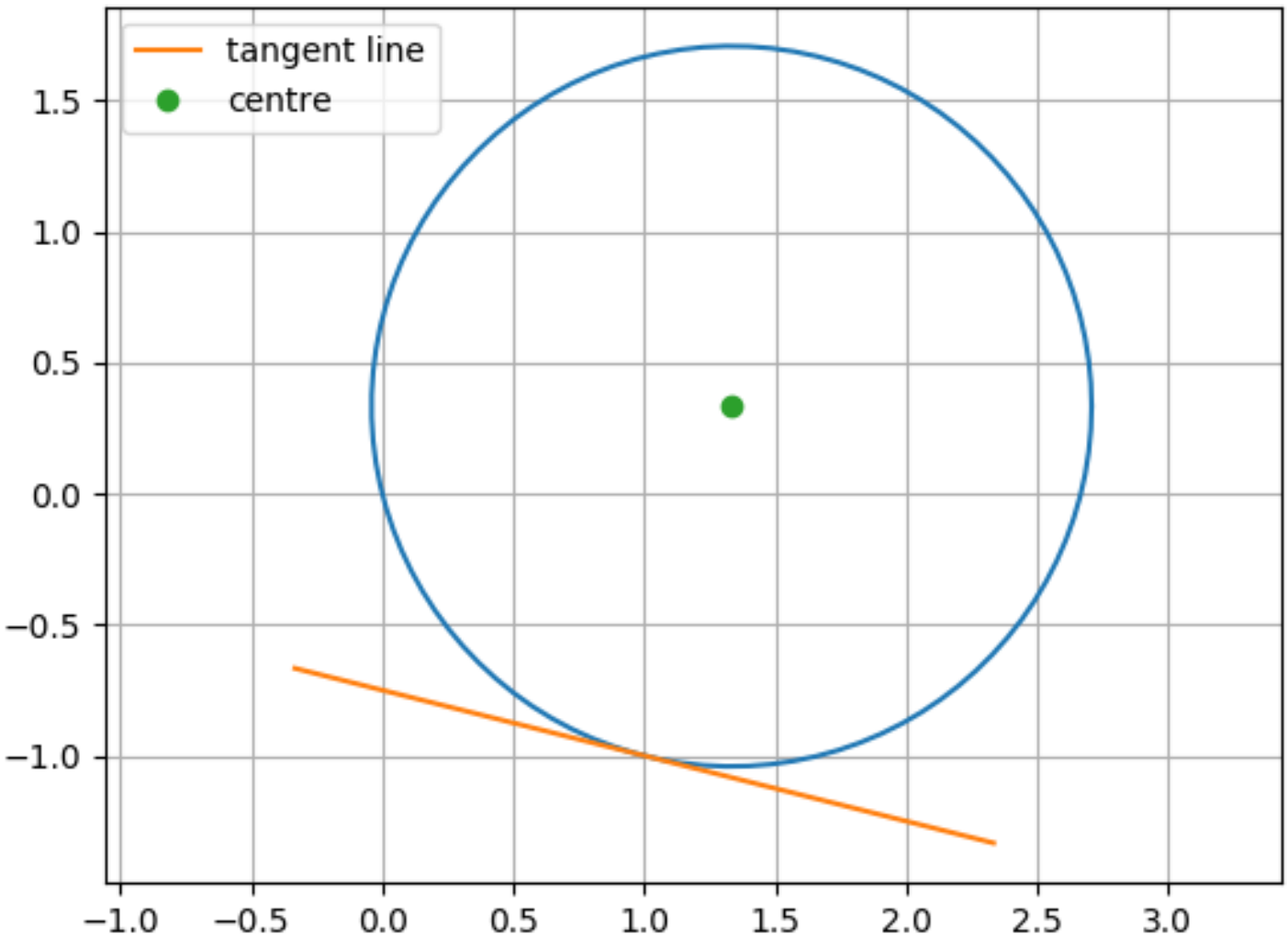

Figure: Tangent to the circle