Optimal Control of the Bi-Rotor SeeSaw System

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I. Introduction

Control Theory deals with the development of system models and designing of control algorithms. Based on the mathematical system design, control algorithms are built and tune to get the desired response, thereby improving system performance, robustness, and efficiency. Complex systems are primarily based on the "Sense-Plan-Act" methodology, wherein sensor data is accumulated, fused, and perceived to be further exploited as a tool for tackling motional, behavioral, and path-planning challenges.

Practically observed systems are usually linear or non-linear in nature, wherein the former adheres to the superposition principle and the later does not. The superposition principle states that the net response of a linear system is simply the sum of individual responses added together. Similarly, no observed system is purely Single-Input-Single-Output (SISO), but has multiple inputs and outputs bifurcated as SIMO, MISO and MIMO systems.

The system under consideration here is a Multi-Input-Multi-Output (MIMO) Bi-Rotor Seesaw system. As the name implies, the system consists of two rotors balanced on either side of the seesaw beam pivoted at the center and possesses a base of rollers capable of linear motion along the beam axis. The lift force from the propellers affect the angular offset (θ) of the beam about the pivot and a thrust source that allows linear offset (x or d) of the system. Fig. 1 shows the free body diagram of the system. The non-linearity of the system is justified by the presence of trigonometric quantities in the dynamic equations. This system forms the base for the controlled maneuver of the "Parrot Rolling Spider Minidrone". This special drone is capable of flights as well as crawling maneuvers. A similar system is replicated in the bi-rotor seesaw system. The project aims at development and implementation of optimal control techniques such as Linear Quadratic Regulator (LQR) and Model Predictive Control (MPC), followed by subsequent system response behavior analysis and tuning of control parameters to improve system performance and robustness.

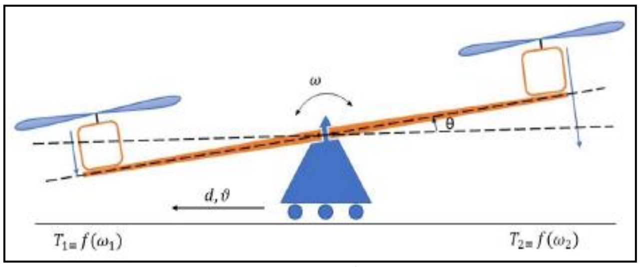

Figure 1: Force body diagram of bi-rotor seesaw system

II. Solution

A. Definition

The system under consideration comprises of a beam with one propeller on either side. The lift and the thrust forces from the propellers govern the angular and linear offset of the system. A system needs to be modelled in terms of a controls problem to develop and implement control algorithms on the same. This system is in equilibrium at zero-displacement position and horizontal orientation with the horizon. The objective backing the algorithm development is to achieve equilibrium state with optimal efficiency.

B. Mathematical Modelling

The inputs to the system are the thrusts at each of the propellers – T1 and T2. These thrust inputs are functions of angular velocity. If the beam were to be perfectly balanced, then the thrust force would be purely lift force causing only angular offset. With the increase in the inclination of the beam, the horizontal component of the thrust also allows for a linear offset. This behavior maintains that the beam will undergo a see-saw motion, before attaining a stable equilibrium. The states of the system are linear displacement (x or d), linear velocity ($\dot{x}$), angular displacement (θ) and angular velocity ($\dot{\theta}$). Additionally, the output of the system is the linear and angular displacement, based on which the equilibrium achievement is determined.

As observed, there are two inputs and outputs and four states for the system. Hence, the state vector is a (4*1) matrix, input vector is a (2*1) matrix and the output vector are (2*1) matrix. These dimensions define the state space model of the system.

Based on the free-body-diagram, the system is mathematically modelled based on the dynamic equations:

$$States : x = [x_1, x_2, x_3, x_4] = [x, \dot{x}, \theta, \dot{\theta}] \quad (1)$$

$$Inputs : u = [u_1, u_2] = [T_1, T_2] \quad (2)$$

$$Outputs : y = [y_1, y_2] = [\theta, d] \quad (3)$$

$$J * \left(\frac{d^2}{dt^2}(\theta)\right) = (T_1 - T_2) * r \quad (4)$$

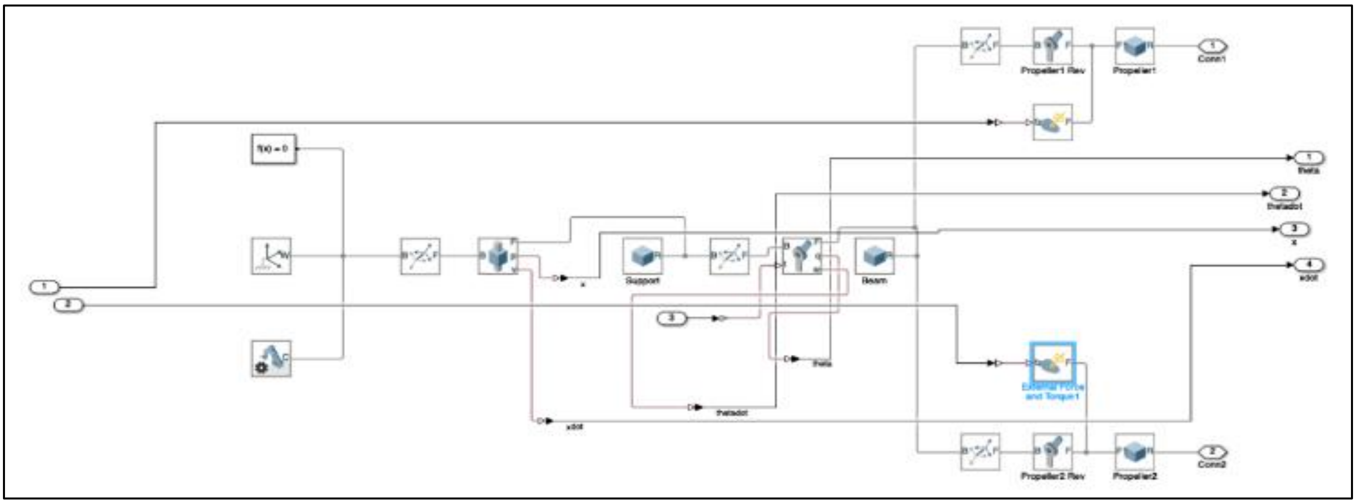

Figure 2 : Simscape model block diagram

$$m*\left(\frac{d^2}{dt^2}(d)\right)=(T_1+T_2)*\sin\theta \tag{5}$$

$$\dot{x}=\left[x_2,\ \frac{(T_1-T_2)*r}{J},\ x_4,\ \frac{(T_1+T_2)*\sin\theta}{r}\right] \tag{6}$$

where, J : Polar Inertia of the beam and m : total mass of the system (600 grams)

Simscape is a MathWorks tool that allows modelling the dynamics of the plant conveniently. As seen in Fig. 2 , initially world link, gravity link and solver block were connected to the base link, which further undergoes a frame transformation with prismatic and revolute joints to represent the links in bi-rotor system. Thrust inputs were externally provided to model the propellers, based on which four state outputs $x, \dot{x}, \theta, \dot{\theta}$ were obtained.

C. Control Objectives

A controller is designed for a system in order to get the desired output and performance eventually exploiting resources optimally. Controller takes into account the error between the desired response and the current system response and implements appropriate gain to mitigate the error and output the desired result. Numerous optimal control techniques have been developed such as – Proportional Integral Derivative (PID), Linear Quadratic regulator (LQR), Linear Quadratic Gaussian (LQG), Model Predictive Control (MPC)[1]. In order to analyze the performance and behavior of the system, certain control objectives were set. The initial condition of the system is considered as a stationary slider at $x = 0$, with a vertical orientation of $\theta = 0$. The control objectives and constraints defined were as follows:

- Slider can be moved to a new position between a linear offset-range of [-10,10] units, with a step setpoint change.
- For the setpoint change being traced, the rise time should be less than 4 seconds with an overshoot of less than 5 percent.
- For an impulse input disturbance of 2 units, the slider should attain equilibrium state with a maximum linear offset of 1 unit. Similarly, the slider should attain the original orientation with a maximum angular offset of 0.26 radian.

Here, the vertical orientation of the beam is an unstable equilibrium, attaining and maintaining of which is challenging.

III. RESULTS

A. Linear Quadratic Regulator

The system needs to linearize in order to implement any of the aforementioned control techniques. A system generally behaves non-linearly over a range of states but is capable of exhibiting a linear behavior about the equilibrium states. Control behavior analysis is primarily carried out for this region. The set of linear differential equations defined in the "Mathematical Modelling" section and a quadratic cost function defined form the LQ problem, which can be solved through Linear Quadratic Regulator technique. Based on the functions, the states and inputs are penalized through Q and R penalty matrices, which penalizes states and inputs respectively. Fig. 3 shows the LQR block diagram for the system. The state estimator forms the closed-loop feedback system, wherein the estimated states are compared with the step input resulting in the error, based on which the controller parameters Q, R is iterated to mitigate the same. The system is linearized about the equilibrium states $x = 0$ and $\theta = 0$, through piecewise linearization technique.

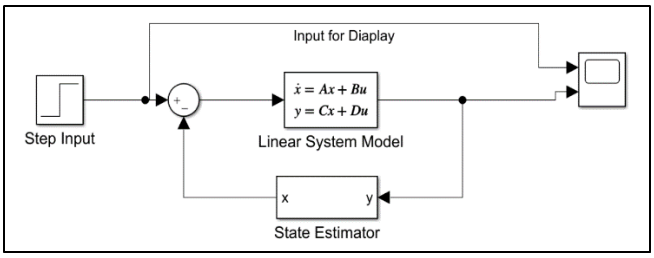

Figure 3 : Linear Quadratic Regulator block diagram

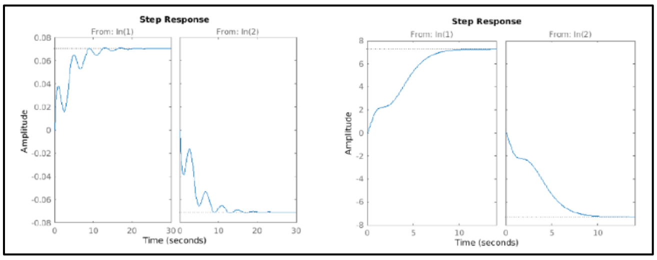

Figure 4 : System Response for Linear Quadratic Regulator Controller

With appropriate penalties on states and inputs, the system response can be observed in Fig. 4. Before tuning Q and R, the settling time of the system was about 20 seconds, which was reduced to about 8 seconds. A trade-off between the transient response and overshoot is prevalent and hence with minimal overshoot, a rise time of 8 seconds is optimal with smooth response.

B. *Model Predictive Control*

Model Predictive Control is an optimal online control technique, which controls and optimizes the system behavior based on the time horizon. After the model is linearized, the controller optimizes the current horizon and takes into account subsequent horizon. Based on the current state estimation and system dynamics modelled, state alterations in subsequent horizons are calculated while considering saturation constraints on inputs and outputs. Fig. 5 shows the MPC Block diagram for the system. The input to the MPC controller is the current and reference state, which outputs a calculated thrust estimate to the model. The derived outputs were represented through the scope. MPC designer is a tool provided my MathWorks, to model, linearize and study system response with a MPC controller.

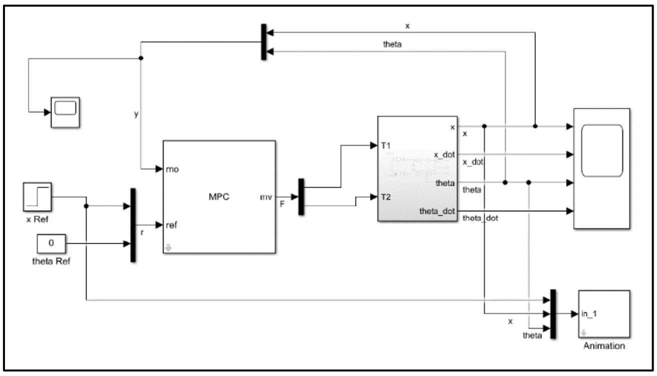

Figure 5 : Model Predictive Controller system block diagram

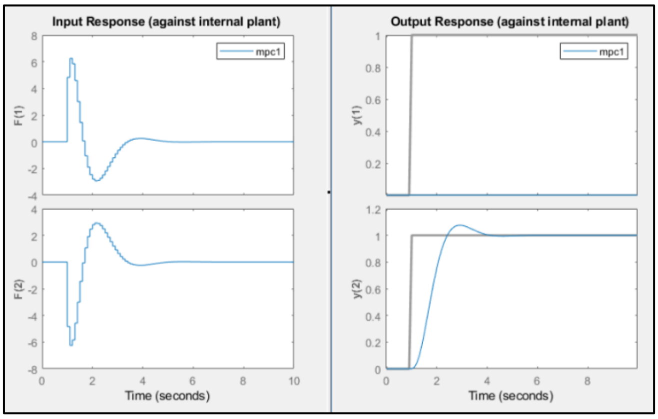

Figure 6 : System Response for Model Predictive Controller

As observed in Fig. 6, the system overshoot is less than 5 % and the rise time is about seconds, which justifies that the control objectives were achieved based on the reference impulse input.

IV. CONTRIBUTIONS

My contributions to the project span over a few developments. I was primarily involved in the system identification, mathematical modelling and designing the dynamic equations of the system. I was responsible for FBD schematic diagram. Along with this, I contributed to the Linear Quadratic Regulator controller implementation on the seesaw system – block diagram and response plot in Matlab Simulink. With a linearized system as base, I was able to iterate and interpolate the penalty matrices (Q & R) for optimal response, tracing predetermined control objectives. Finally, I was involved in the project report and presentation drafting.

My team members collaborated on the project development. Karan Shah contributed strongly on implementation of MPC controller and tailoring the algorithm to our system. Adding to this, he helped with system modelling, system identification and Simscape model development as well. Dhaivat Dholakiya and I worked jointly on the LQR implementation. He worked on the physical model for free body diagram including a foundation of planned project execution. He also worked on PID analysis for system behavior

V. SKILLS AND KNOWLEDGE GAINED

Through this project, I was personally able to learn and hone skillset mentioned below:

• I was able to learn about system identification execution for a real-world system. This helped me understand underlying dynamics fundamentally.

• I was introduced to novel tools per se Simscape, PID auto-tuner, Simulink MPC Toolbox, etc., and how it could be exploited for project development.

• Developed an understanding of controllability, observability, penalty matrices, etc., on real-world dynamic systems.

• Learnt how to consider a physical hardware and tune the designed controller through MathWorks. Learnt how to extend the theoretical concepts to physical systems.

• Learnt how to linearize a non-linear practical system about its equilibrium state and perform pole-placement to manipulate the unstable system behavior.

VI. REFERENCES

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