Savitribai Phule Pune University
Department of Scientific Computing
M.Sc I (Semester I) 2019-2020
Subject: SC 104 Foundations Scientific Computing II
Final Exam

Marks: 50   Calculators are allowed.   Time 3 hours

Attempt all

1. Draw the graph of function $|x + y| = 1$ on $\mathbb{R}$ axes.   2 marks

2. Find $\lim\limits_{x \to \pi/3} \frac{\sqrt{3} - \tan x}{\pi - 3x}$ OR $\lim\limits_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$ 2 marks

3. Show that the function defined by $f(x) = |x|$ is continuous on $\mathbb{R}$.   2 marks

4. Find dy/dx for $x = 2(\theta - \sin \theta)$ and $y = 2(1 - \cos \theta)$ at $\theta = \pi/2$ 2 marks

5. Find dy/dx if $y = 2 \cot(5x^2 + 3)$ or $y = \sec^2(4x + 7)$ 2 marks

6. Find dy/dx if $2y + 4xy - x^2 y - y^2 x + 2x = 0$ 3 marks

7. Find the equation of tangent and normal to the equation $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$ at $(1, 1)$.
   OR
   Find the intervals in which the function $f(x) = 4x^3 - 6x^2 - 72x + 30$ is
   (a) strictly increasing (b) strictly decreasing.   3 marks

8. Find maximum and minimum values of the function $y = x^2 - 32\sqrt{x}$ 4 marks

9. Evaluate $\int \frac{(1 + \log x)^2}{x}$ or $\int \frac{\sin x}{(1 + \cos x)^2}$ 2 marks

10. Evaluate $\int \frac{5x - 2}{1 + 2x + 3x^2} \, dx$ or $\int e^x \cdot \sin x \, dx$ 4 marks

11. Evaluate $\int \frac{5x}{(x+1)(x^2 - 4)} \, dx$ 3 marks

12. Find the area of the region in the first quadrant enclosed by the $x - axis$,
    the line $y = x$, and the circle $x^2 + y^2 = 32$.
    OR
    Find the area of the region in the first quadrant enclosed by the $x - axis$,
    $y - axis$ and the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.   3 marks

13. Using integration find the area of region bounded by the triangle whose
    vertices are $(1, 0), (1, 3)$ and $(3, 2)$.   4 marks

14. Solve the differential equation
    $x \frac{dy}{dx} - y = \sqrt{x^2 + y^2}$ or $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$ 3 marks

15. Solve the differential equation and find the particular solution at $x = \pi/3$
    $\frac{dy}{dx} + 2y \tan x = \sin x$ 3 marks

16. Examine the convergence of any one series $\sum \frac{(n+1)}{2^n}$ or $\sum \frac{(n!)^2}{(2n)!}$ 2 marks

17. Check for absolute/conditional convergence $\sum (-1)^{(n+1)} \frac{\tan^{-1} n}{n^2 + 1}$ 3 marks

18. Calculate the fourier series for the function $y = \frac{x^2}{4}$ in $-\pi \leq x \leq \pi$ 3 marks