23-12-20

Wednesday, December 23, 2020 4:27 PM

1) If $\vec{A} = (3x^2+6y)\,i - 14yz\,j + 20xz^2\,k$ evaluate the line integral $\oint \vec{A}\cdot d\vec{r}$ from $(0,0,0)$ to $(1,1,1)$ along the curve $x=t,\ y=t^2,\ z=t^3$

2) Evaluate $\int F\cdot N\,ds$ where $F = 4x\,i - 2y^2\,j + z^2\,k$ over the region bounded by $x^2+y^2=4$, $z=0$ and $z=3$

3) Evaluate $\int_S F\cdot N\,ds$ where $F = y^2z^2\,i + z^2x^2\,j + x^2y^2\,k$ and S is the surface $x^2+y^2+z^2=1$ above xy plane

4) Evaluate $\int_V F\,dV$ when $F = x\,i + y\,j + z\,k$ and V is the region bounded by $x=0$, $y=0$, $y=6$, $z=4$ and $z=x^2$

5) Evaluate the surface integral $I = \int_S a\cdot ds$, where $a = x\,i$ and S is the surface of the hemisphere $x^2+y^2+z^2=a^2$ with $z \geq 0$

6) $\int_{x=0}^{1}\int_{y=0}^{2} (x^2+3xy^2)\,dx\,dy$

7) $\iint_A (5-2x-y)\,dx\,dy$ where A is given by $y=0$, $x+2y=3$, $x=y^2$

8) $\iint_R x^2\,dx\,dy$ where R is the two-dimensional region bounded by the curves $y=x$ & $y=x^2$

9) Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$ $\qquad \frac{\sqrt{\pi}}{2}$

10) Evaluate $\iint_R \sqrt{x^2+y^2}\,dx\,dy$, where R is the region in the xy-plane bounded by the circle $x^2+y^2=4$ $\qquad \frac{38\pi}{3}$

11) change the order of the integration and evaluate.

$$\int_0^a \int_0^y \frac{dx\,dy}{\sqrt{(a^2+x^2)(a-y)(y-x)}}$$ $\pi \log(1+\sqrt{2})$

12) change the order of integration and evaluate

$$\int_0^a \int_0^x \frac{\sin y \, dy\, dx}{(4-5\cos y)^2 \sqrt{[(a-x)(x-y)]}}$$ $\frac{\pi}{5}\log(5\cos a-4)$

13) change the order of the integration $\int_0^a \int_{\sqrt{a^2-x^2}}^{x+3a} f(x,y)\,dx\,dy$

14) Evaluate $\int_0^a \int^x \frac{f'(y)}{[(a-x)(x-y)]^{1/2}}\,dy\,dx$ $\pi[f(a)-f(0)]$

15) Find the area bounded by the curves $y^2=x$ and $y=x^3$

16) Find the area lying inside a cardioid $r=1+\cos\theta$ and outside the parabola $r(1+\cos\theta)=1$ $\frac{3\pi}{4}+\frac{4}{3}$

17) Find the area inside the circle $r=2a\cos\theta$ and outside the circle $r=a$

$2a^2\left(\frac{\pi}{3}+\frac{\sqrt{3}}{4}\right)$

18) Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+y} e^{x+y+z}\,dx\,dy\,dz$ $\frac{5}{8}$

19) Evaluate $\iiint x^2yz\,dx\,dy\,dz$ throughout the volume bounded by the planes $x=0,\ y=0,\ z=0,\ \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

$\frac{a^3b^2c^2}{2520}$

20) Evaluate $\iiint \frac{z^2\,dx\,dy\,dz}{x^2+y^2+z^2}$ over the volume of the sphere $x^2+y^2+z^2=2$ $\frac{8\pi\sqrt{2}}{9}$

21) A triangular prism is formed by the planes whose equations are ... the prism between

$ay = bx$, $y = 0$ and $x = a$ Find the volume of the

the planes $z = 0$ and the surface $z = c + xy$ $\frac{ab}{8}(4c + ab)$

1. Show that the two variable function $u(x,t) = e^{2(2t+x)}$ is a solution of the Diffusion Equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$
2. If $f(x,y) = \tan^{-1}(xy)$ find the approximate value of *f (1.1, 0.8)* using the Taylor's series (1.) Linear approximation (II) Quadratic approximation if their initial point is *(1,1)*.
3. If $u = r^m, r = \sqrt{x^2 + y^2 + z^2}$ find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$
4. Verify that $\phi(x,y,z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$, satisfies the three-dimensional Laplace's equation $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$.
5. A function f is given by $f(x,y) = x^3 + x^2 y + y^4$
(a) State the second-order Taylor polynomial generated by *f* about *(1, 1)*.
(b) Use the polynomial to estimate *f (1.2, 0.9)*. Compare this value with the true value.
(c) Verify that the second partial derivatives of the function and the Taylor polynomial are identical at *(1, 1)*.
6. For the function $f(x,y) = xy^2 + \exp(x^2 y)$, show $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.
7. Show that $u(x,y) = log(x^2 + y^2)$ satisfies the **partial differential equation:**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

8. Find the extreme value(s) of the following function:

i) $f(x, y) = y^2 + 4xy + 3x^2 + x^3$ **Ans.** $\left(\frac{2}{3}, -\frac{4}{3}\right)$, Minimum

ii) $f(x, y) = x^3 y^2 (1 - x - y)$ **Ans.** $\left(\frac{1}{2}, \frac{1}{3}\right)$, Maximum

9. If the function is given by $f(x,y,z) = 2x + 2y + 2z$ then use Lagrange's multiplier method to find the greatest value if it is related as $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 6$.

10. If $f(x, y) = x^2y + cosy + ysiny$ then find all second order partial derivatives.

11 Find the extreme values of $f(x, y, z) = 10x^2 + 8yz - 32z + 1200$ takes on the ellipsoid

$g(x, y, z) = 5x^2 + y^2 + 4z^2 - 15 = 0$.

12 : Find the shortest distance from the origin to the surface $xyz^2 = 2$.

13: If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$.

14: Expand $e^y cosx$ in ascending powers of $(x - \frac{\pi}{2})$ and y up to term of third degree.

15: Expand $x^2 + xy + y^2$ in ascending powers of $(x - 1)$ and $(y - 2)$ up to third degree.

16. : If $u = (x^2+y^2 + z^2)^{-1/2}$ the prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x} + z\frac{\partial u}{\partial x} = -u$.

17.: Expand $e^y\log(1 + x)$ about origin up to term of third degree.

18. To compute the partial derivatives of the functions at the specified points.

$f(x, y) = 1 - x + y - 3x^2y,\quad \frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at $(1, 2)$

$f(x, y) = 4 + 2x - 3y - xy^2,\quad \frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at $(-2, 1)$

19. find the value of dy/dx at the given point.

$$x^2 + xy + y^2 - 7 = 0, \quad (1, 2)$$
$$xe^y + \sin xy + y - \ln 2 = 0, \quad (0, \ln 2)$$

20. A flat circular plate has the shape of the region $x^2 + y^2 \leq 1$. The plate, including the bouodary where $x^2 + y^2 = 1$,is heated so that the temperature at the point (x, y) is $T(x, y) = x^2 + 2y^2 - x$. Find the temperatures at the hottest and coldest points on the plate.