Stochastic Volatility

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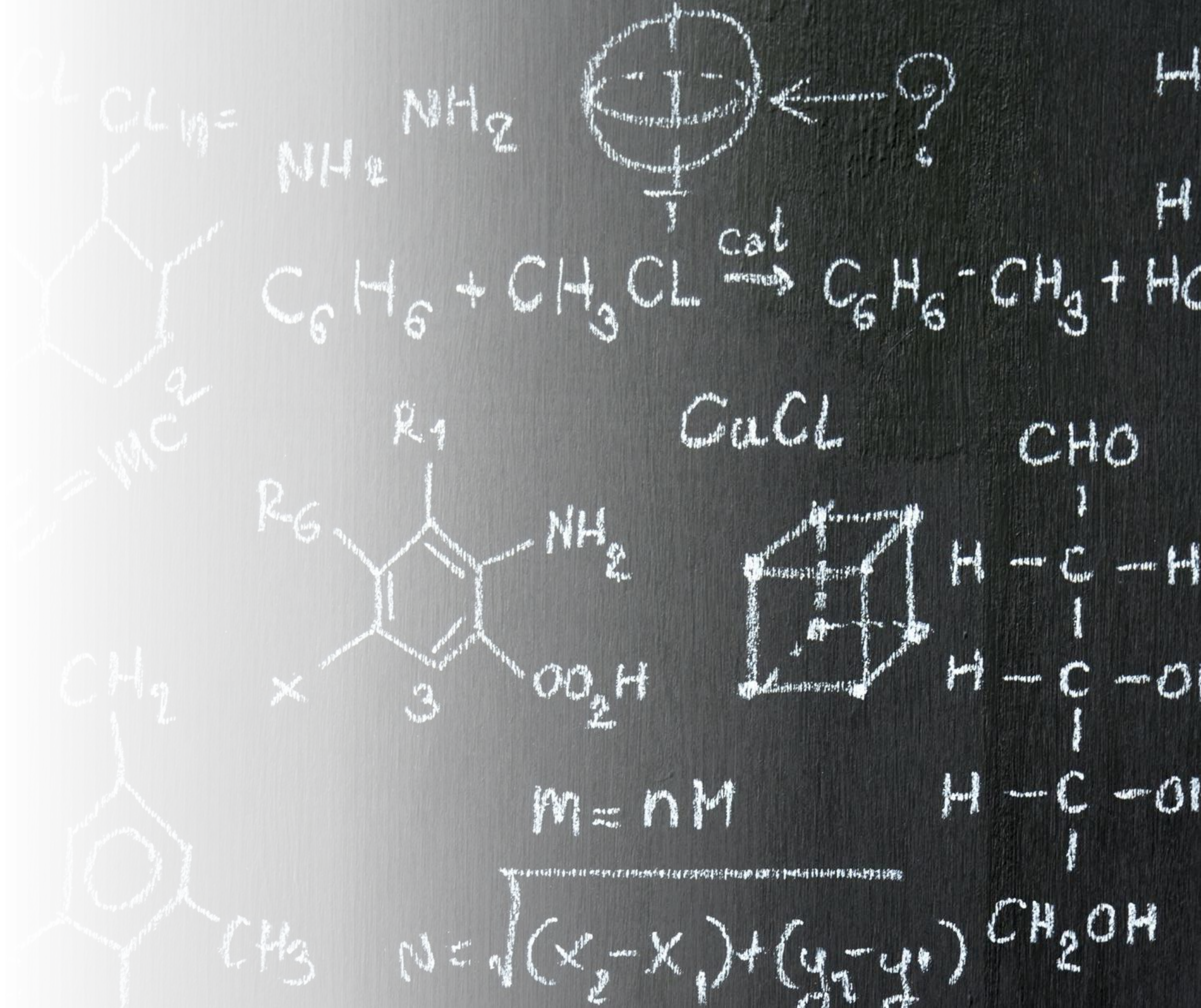
Agenda

- Model Introduction
- Foreign Exchange Rates Introduction
- Simulated Example
- Fitting The Model To Data
- Conclusion
- Q/A

Model Introduction

- State-space Model
  - Observation
  - State-Space Equation
- Bayesian Inference
  - Draws From Posterior Distribution of Parameters and Latent Variables
- Used in Finance To Predict Future Volatility of Time-Series



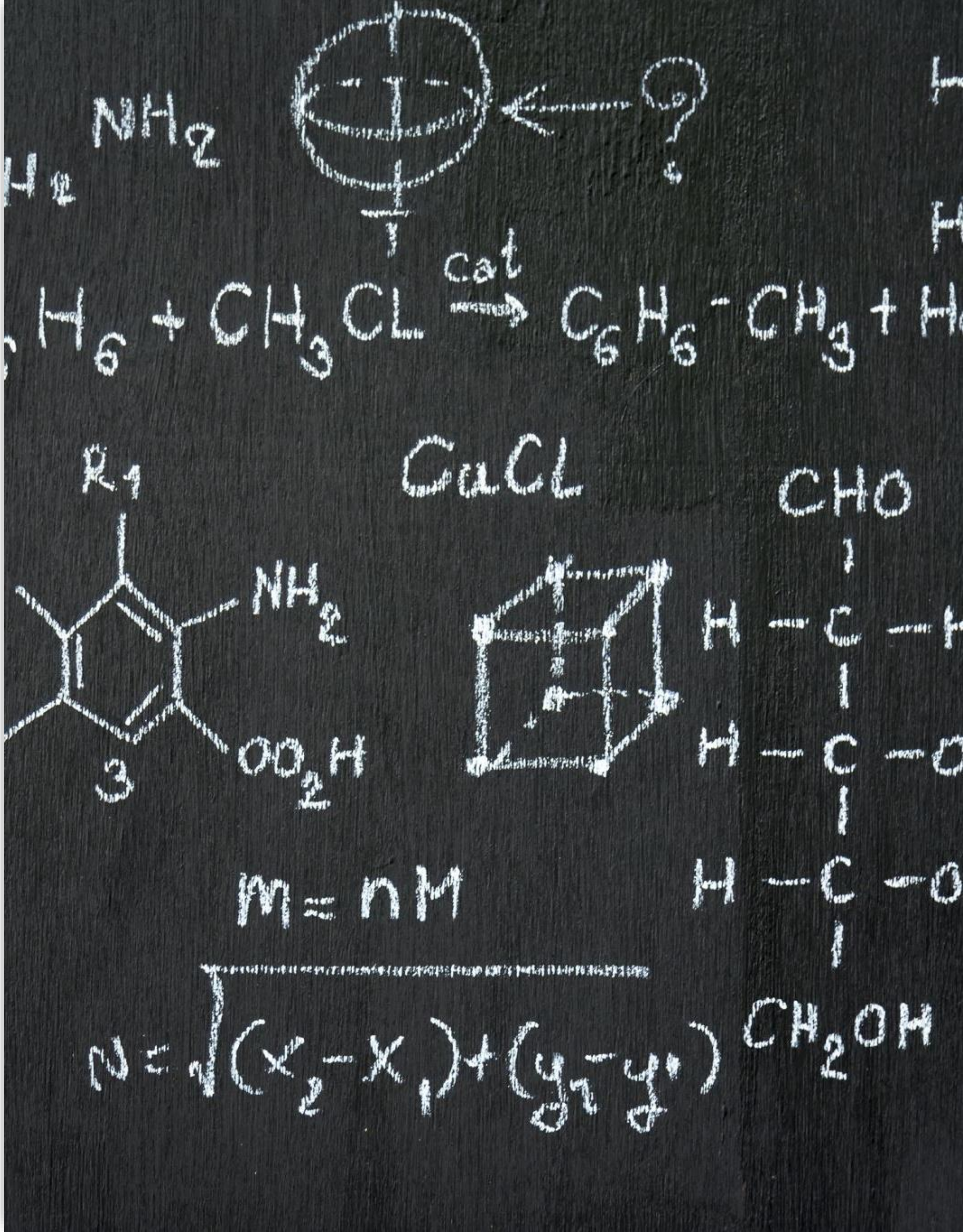
$$
\begin{aligned}
y_t | h_t &\sim \mathcal{N}(0, \exp h_t), \\
h_t | h_{t-1}, \mu, \phi, \sigma_\eta &\sim \mathcal{N}\left(\mu + \phi(h_{t-1} - \mu), \sigma_\eta^2\right), \\
h_0 | \mu, \phi, \sigma_\eta &\sim \mathcal{N}\left(\mu, \sigma_\eta^2/(1 - \phi^2)\right),
\end{aligned}
$$

Model Introduction: Equation

- Random Vector of Returns with Mean Zero: y = (y1, y2, . . . , yn) ^T
- Each Observation Has Its Own Variance: e^ht
- Centered Parameterization:
  - N (μ, σ2 η) denotes the normal distribution with mean μ and variance σ2 η.
  - Theta is refered to as the vector of parameters (θ = (μ, ϕ, ση) ^T)
  - The process h = (h0, h1, . . . , hn) is an unobserved and interpreted as the log-variance process or time-varying volatility process.

Model Introduction: Prior Distribution

- The model specifies a prior distribution for the parameter vector θ where each component can be independent and follow a different distribution such that $p(\theta) = p(\mu)p(\phi)p(\sigma\eta)$.
  - $p(\mu)$ = level of log-variance
  - $p(\phi)$ = persistence of log-variance
  - $p(\sigma\eta)$ = volatility of log-variance
- Each parameter has it's set of hyperparameters:
  - $\mu \sim N(b\mu, B\mu)$
  - $(\phi + 1)/2 \sim B(a0, b0)$
  - $\sigma 2\ \eta \sim B\sigma\eta \times \chi\ 1^2 = G(½, 1/2B\sigma\eta)$



Model Introduction: stochvol

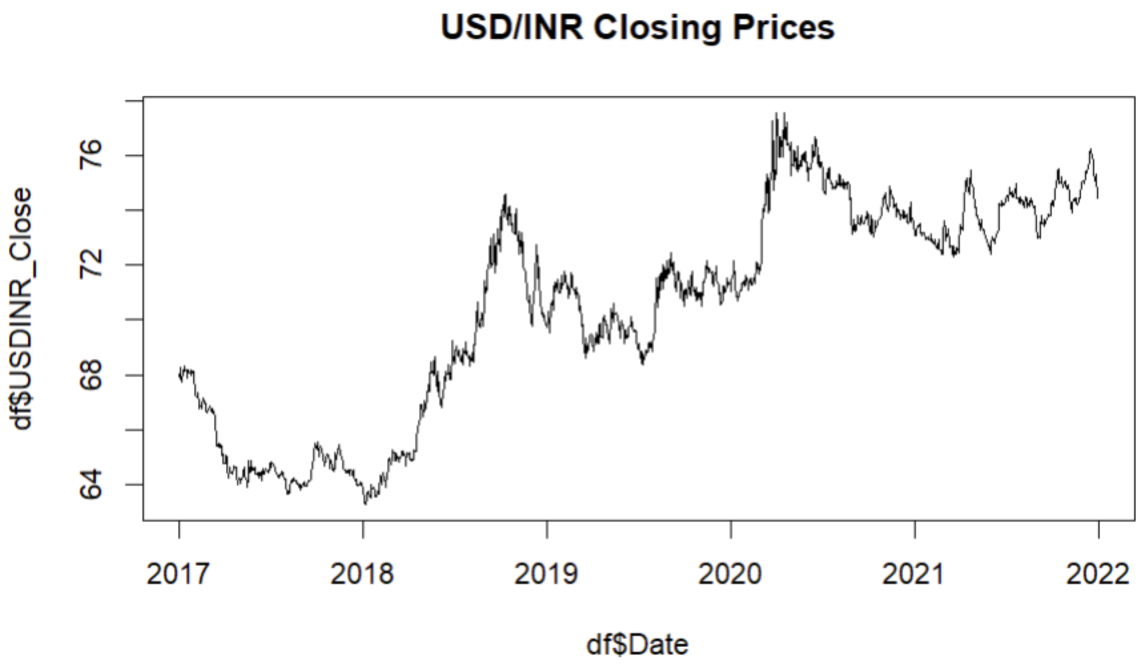
- stochvol is the R package used to forecast future volatilities.
- Key Functions:
  - svsim(len, mu, phi, sigma)
  - svsample(y, priormu, priorphi, priorsigma)
  - volplot(x, forecast, dates)

Foreign Exchange Rates (daily updates) – Yahoo!

- Daily time series of FX rates from Kaggle for Stochvol
- Focused on USDINR_Close for univariate time series forecasting

EDA

- No Null Values
- Duplicates handling by imputing with mean
- Filtered for 2017-2021 data

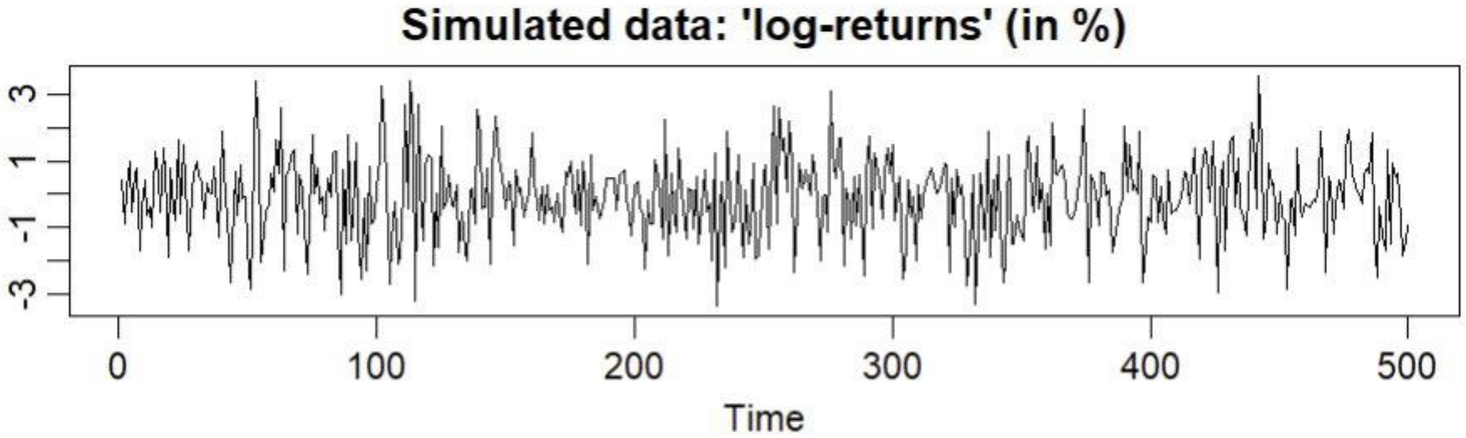


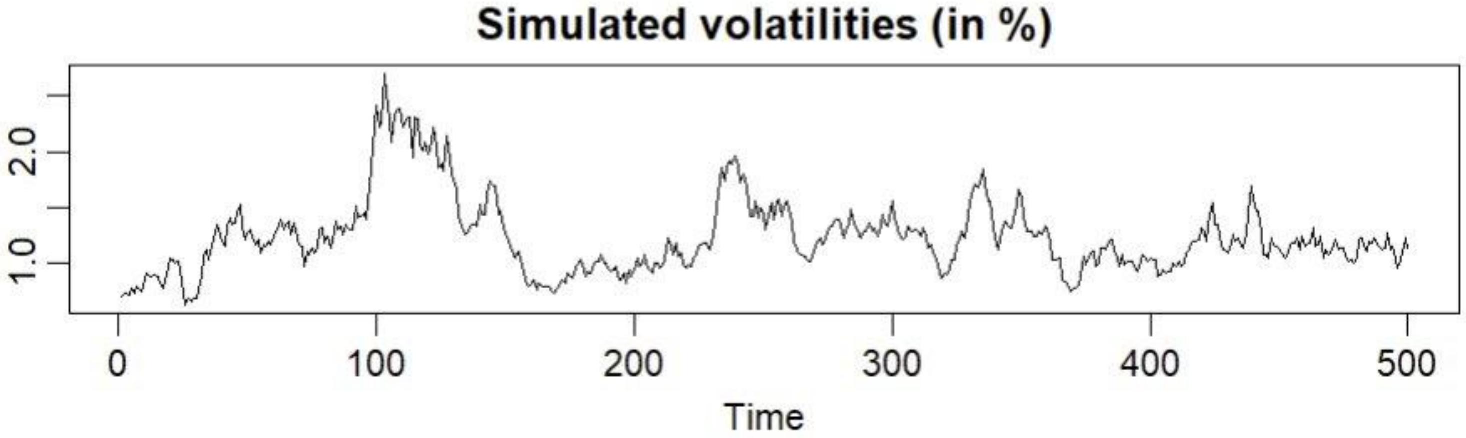
Simulated Example

- First, we have simulated a time series data with 500 observations and arbitrary values of mean, phi and sigma using svsim() function from stochvol package in R.

````
```{r}
sim <- svsim(500, mu = -9, phi = 0.95, sigma = 0.15)
par(mfrow = c(2, 1))
plot(sim)
```
````

Simulated Data log returns and volatilities



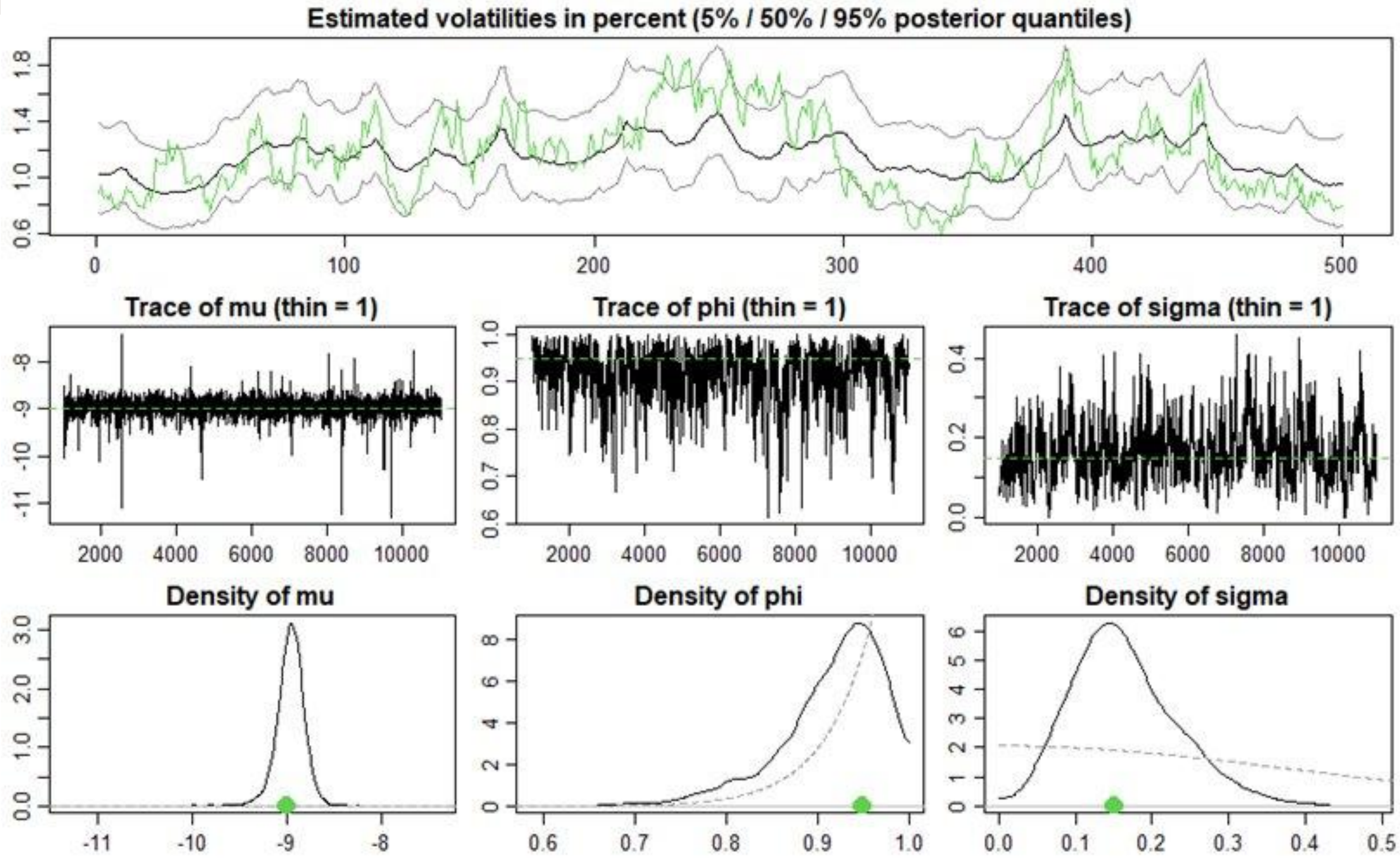


Simulated Example Contd..

- And then with simulated data, we proceeded to infer the model parameters with the svsample() function. We applied a Bayesian approach, incorporating prior beliefs to get posterior draws.

```{r}
res1 <- svsample(sim, priormu = c(0, 100), priorphi = c(39, 1),
                 priorsigma = .15)
plot(res1, showobs = FALSE)
```

Trace Plot of Simulated Data

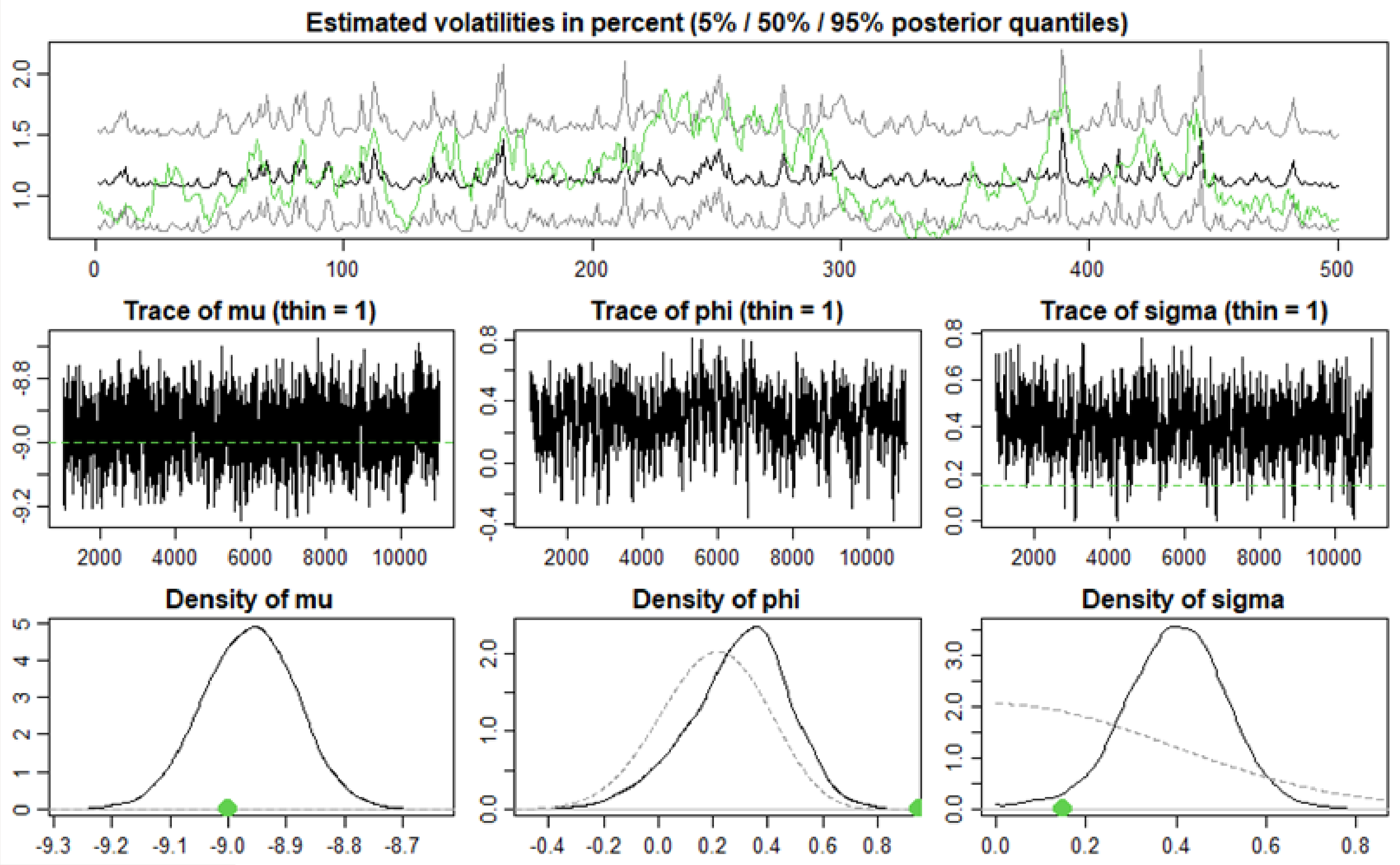


```{r}
res2 <- svsample(sim, priormu = c(0, 100), priorphi = c(15, 10),
                 priorsigma = .15)
```

Simulated Example Contd..

- So, we repeated the same plot with the different priorphi now.

Trace Plot of Simulated Data

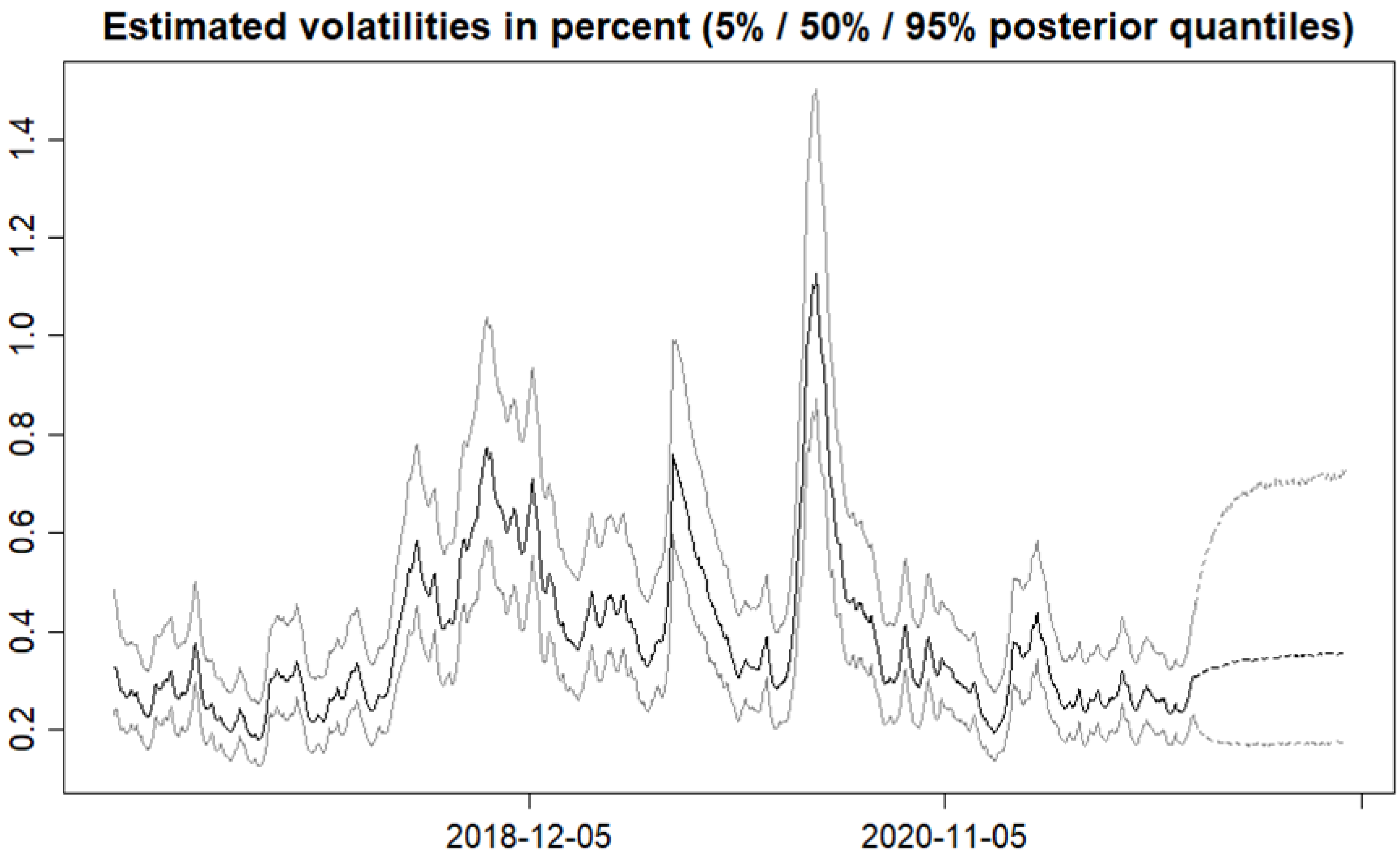


````
```{r}
ret = logret(na.omit(df$USDINR_Close), demean = T)
res2 <- svsample(ret, priormu = c(0, 100), priorphi = c(10, 2), priorsigma = .1)
volplot(res2, forecast = 180, dates = df$Date[-1])
```
````

Fitting the model to data

- So after careful observation from Bayesian Inference with simulated data, we fitted the model with our original foreign exchange data with high priorPhi and tuning other prior beliefs.

Volatility Forecast



PROS

- Capacity to build model that captures randomness.
- Bayesian Inference for posterior draws using prior knowledge.
- UniVariate and MultiVariate

CONS

- Requires domain knowledge on priors and model specifications for modelling expertise.
- High significance of hyperparameters on the model.

Conclusion

Thanks

Any Questions