Grace Under Pressure

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Abstract

Vague terms like "tall" and "bald" raise difficult questions in the philosophy of language. Attempts to understand such questions have focused mostly on ordinary language, with some recent focus on specialized languages such as legal language. I argue that it's worth looking farther afield to physics—specifically, modeling practices in physics. Physics may seem an unlikely place to find vagueness given its reputation for mathematical precision, but I show that it is shot through with vagueness. Grasping how vagueness arises in physics helps us better grasp an important function served by vagueness in general. I argue that vagueness endows our descriptive practices with *fault-tolerance*. Fault-tolerance is a concept from engineering and refers to systems being able to continue functioning despite faults. I argue that the characteristic features of vagueness—namely, sorites-susceptibility, borderline cases, and higher-order vagueness—ground the characteristic features of fault-tolerance—namely, robustness, graceful degradability, and error-correctability. Fault-tolerance is a significant benefit that accrues to vague terms and this may help explain the prevalence of vagueness, even in mathematical disciplines like physics.

1 Introduction

Much of ordinary language is vague. Perhaps this is no great surprise; we have little need for extreme precision in everyday discourse. But what about discourses where extreme precision is sought after and, indeed, often achieved? Physics is one such discourse and, I will argue, it contains much vagueness. This suggests that vagueness might be in service of something valuable in our descriptive practices because it is retained even when we are seeking extreme precision. I will argue *fault-tolerance* is plausibly one such valuable feature.

By 'vagueness', I mean the phenomenon that many of our terms—such as "heap", "tall", and "bald"—seem to lack sharp boundaries and that such sharp boundaries seem impossible to specify without at least some arbitrariness (I will characterize vagueness more carefully in Sec. 2). In philosophy of language, vagueness is often thought of as a property of linguistic items such as predicates or sentences. But my focus isn't going to be specifically on vague *language* in physics. Rather, I will focus on the primary descriptive technology of physics—*models*.[1] This shift in focus is justified because the language used in physics is secondary to the models employed in physics. One signature of this: theoretical terms in physics are typically defined, at least partially, via the role they play in models. E.g., we understand the term 'viscosity' via the role it plays in the equations governing fluid flow.[2]

In what sense do models in physics have boundaries? Models in physics typically come with regimes of validity. We can describe the behavior of a pendulum using a simple harmonic oscillator model—i.e., as a system with a definite oscillation frequency—only when the pendulum's amplitude is small.[3] Thus, the *boundaries* of the simple harmonic oscillator model for a pendulum are at those amplitudes at which this model *breaks down*, i.e., where it fails its descriptive-explanatory task. In general, the boundaries of a physics model are marked by the values of the parameters of the model at which the model fails its descriptive-explanatory task. These boundaries, I claim, are unsharp: physics doesn't offer principled ways to mark precise boundaries between where a model

1. There is a large and complex philosophy-of-science literature on the nature and function of models in science; see, e.g., Frigg and Hartmann (2020) and references therein. To keep things tractable I will largely avoid engaging with this literature by not taking on any substantive view of models, except to note that they play a central role in descriptive and explanatory practices in physics. The notion of 'models' relevant for my argument should become clearer further into this paper.

2. A useful recent framework to think about such theory-dependent understandings of terms is the 'constitutive role functionalism' of Knox and Wallace (2023).

3. See, e.g., (Nelson and Olsson 1986).

is successful and where it isn't.

To talk about where a model breaks down, it is necessary to compare the model to the behavior of a target system. That means we need some way of talking about the behavior of the system, which would be what we compare our model to. But this other way of talking—which supplies the basis for comparison—is yet another way of describing the system, and hence yet another model. In saying this, I am assuming that models can be very thin: a model could simply be what might be considered *data*, i.e., an assignment of numerical values to certain measurable quantities.[4] The standard way in which we compare models to the world is by comparing models to data. (This doesn't mean our models can't be about the world. It simply means that the way the world becomes available to us for comparison is via data.)

Beyond data, often in physics, other, 'less thin' models might also serve as the basis for comparison. This might be because we have reason to believe that another model is more accurate or in some sense closer to the phenomena. For instance, the second model might derive from a more fundamental theory or might be a so-called phenomenological model.[5]

Hence, model-model relations subsume model-world/model-system relations. Focusing on such relations allows us to precisely specify the set of situations where one model is applicable or inapplicable in terms of another model. Consequently, in this paper such relations will be my focus.

I will focus on an example from celestial mechanics, where we model a solar system consisting of a large central star around which two planets orbit—one small inner planet and one large outer planet (akin, respectively, to Earth and Jupiter). We can construct different models for the inner planet's orbit: in particular, a two-body model that only represents a central star and an inner planet, and a three-body model that also represents an outer planet. There will then be an unsharp boundary between those three-body models that agree with a given two-body model and those three-body models that disagree. There is nothing special about this example; we can see this kind of vagueness throughout physics. (The physics examples will be set up and explicated in Sec. 3.)

Despite the focus on modeling, my conclusions aren't only restricted to

4. Compare with Van Fraassen (2008)'s notion of *data model*.

5. Paradigmatic examples of phenomenological models in physics are Kepler's laws of planetary motion (which were an encapsulation of Tycho Brahe's data, lacking a deeper explanation prior to Newton's gravity) and Hubble's law in cosmology (a simple model encapsulating Hubble's painstaking observations and that was only later explained by general relativity).

physics. When we use ordinary descriptive language, we model the world in a way not dissimilar to how physics models the world. In particular, our descriptive terms carve the world into categories that carry with them implicit or explicit 'theoretical' commitments—e.g., if we call an item 'solid', we are committed to it resisting some attempt at deformation. These networks of commitments are similar to the way models work in physics. Hence, ordinary language use encodes physics-like models. (This will be spelled out in Sec. 4.)

Prima facie, we have every reason to believe that physics would work very hard to eliminate vagueness. After all, physics is known for its extremely precise successful predictions, such as predicting the values of some particle-physics quantities down to 12 significant digits.[6] But this penchant for precision doesn't seem to reach to drawing sharp boundaries for its models, which gives us reason to believe that vague boundaries are playing a valuable role in physics. What might that valuable role be?

I will argue that one very plausible candidate for the value conferred by vagueness—in both physics and in ordinary language—is *fault-tolerance*. Fault-tolerance is a central normative requirement in engineering: roughly, it is the requirement that a system continue functioning despite encountering faults.[7] Note that the focus on fault-tolerance is suggested by *when* we see vagueness arise in physics models: namely, as models *break down*. I will argue that the standard characteristic features of vagueness—susceptibility to sorites paradoxes, the presence of borderline cases, and higher-order vagueness—can respectively be seen as grounding three central sub-components of fault-tolerance—robustness, graceful degradability, and iterated error-correctability.

Robustness is the requirement that our models continue functioning despite variability. We will see that the principle of tolerance (Wright 1975)—which is the basis of the sorites paradox—confers robustness. Graceful degradability is the requirement that our models degrade in proportion to the amount of errors they encounter, and so avoid failing entirely and suddenly. We will see that the presence of borderline cases confers graceful degradation. Iterated error-correctability is the requirement that we be able to introduce new models to handle the breakdown regime, and moreover that those new models can in turn be made fault-tolerant. We will see that higher-order vagueness confers iterated error-correctability. (All this will be spelled out in detail in Sec. 5.)

What sort of project am I undertaking in this paper? Let me first clarify

6. I'm thinking here of the magnetic moment of the electron; see, e.g., (Gabrielse 2013).
7. See, e.g., the entry "fault tolerance" in IEEE Std 610.12-1990 (1990).

what I am not doing. My goal here isn't to offer an account of vagueness, of the kind that leading accounts like epistemicism (e.g., Williamson (1994)), supervaluationism (e.g., Fine (1975)), contextualism (e.g., Shapiro (2006)) and others aim to be. In particular, I do not aim to offer a systematic or formal semantics for vague terms. Nor do I offer an answer to the question of what the *nature* of vagueness is (a question articulated by Eklund (2005)), which would be answers such as *vagueness is incompleteness of meaning* (Fine 1975), *vagueness is semantic indecision* (MacFarlane 2016), *vagueness is boundarylessness* (Sainsbury 1996), *vagueness is ignorance* (Williamson 1994), and "the vagueness of an expression consists in it being part of semantic competence to accept a tolerance principle for the expression" (Eklund 2005). Neither am I attempting to show how vagueness might emerge from the likes of signaling and representation games.[8]

What I am saying is that fault-tolerance is an important function, use, or value of vagueness. Identifying the value of vagueness is the sort of project that has been carried out in the context of law by Endicott (2011). One might think that law, given that its function is to specify norms of behavior, would be entirely disadvantaged in being vague. But Endicott argues that leaving the law vague protects against arbitrary precision. In particular, it allows judges to exercise appropriate judgment in varied contexts, and it allows subjects to conform to laws in varied ways.[9]

My project is in this spirit. One might think that physics, given that its function is to supply precise predictions and quantitative explanations, would be entirely disadvantaged in being vague. But I argue that leaving the relations between models vague provides tolerance to faults—it allows for a model to be useful despite the situation it's intended to model veering away from its paradigm domain of validity; when it does fail, for the model to fail in graceful ways; and for the model to be "patchable" in the failing regime.

When one identifies a value of vagueness, then that goes at least some way in explaining why vagueness is so prevalent. Despite the fact that some may take it to be—and in some cases it may well be—a bug, my arguments suggest that it is also frequently a feature. This claim is significantly bolstered by the fact that vagueness is prevalent in physics, a field both with both the mathematical capability and the desire for precision.

I should note that I am not insisting that fault-tolerance is the only, or

8. See O'Connor (2020, p. 24) and references therein.

9. In this context, see also recent monographs by Lanius (2019) and Asgeirsson (2020).

sole, value of vagueness nor that it is the sole reason why vagueness is so prevalent in our descriptive practices. For instance, one plausible reason why vagueness is so prevalent in our descriptive practices is that it takes far too many social resources to set up and police sharp boundaries. Think of the amount of resources required to enforce even a simple sharp boundary such as a driving age cutoff—the complexity of the legal system, the construction and maintenance of the institutions which constitute and legitimate it, the breadth of the cultural enforcement of such norms.[10] Or perhaps, and even more simply, we typically lack sharp boundaries for many of our terms simply because "no one has been fool enough" (to use Lewis (1986)'s turn of phrase) to set boundaries for most of our terms.

While I am not going to evaluate such explanations here, I do want to note that such explanations for the prevalence of vagueness do not attach any particular *positive* value that accrues to users of vague terms. On such explanations, vagueness is a mere artifact of our lack of resources or desire to fix a sharp boundary. On this picture, one might well imagine a supremely advanced civilization with unimaginable resources deciding to pin down sharp boundaries for all their terms. However, on the sort of explanation that I'm offering here, such civilizations would do no such thing, because if they did they would be jettisoning fault-tolerance, a valuable feature of their descriptive practices. My focus on physics supplies evidence for this. While physicists certainly don't constitute an exceptionally advanced civilization, but they do constitute a highly motivated and fairly unified community with both the resources and the penchant for precision. Nevertheless, vagueness persists in their modeling practices.

2 What is vagueness?

There's no univocal definition of vagueness since part of what's at stake in disputes about vagueness is what is definitive of vagueness. We can, however, state some characteristic features of vagueness and display them in examples. This is enough to get a working handle.

To see these characteristic features, consider the following sentence.

(B) Jack is bald.

There could be situations in which it is clear that Jack is bald, and hence clear that (B) is true. And there could be situations in which it is clear that Jack is

10. I thank [acknowledgment redacted for blind submission] for pointing this out.

not bald, and hence clear that (B) is false. But, crucially, there could also be situations in which there seems to be no principled way to decide whether or not Jack is bald—even if we know all there seems to be to know about Jack's hair, including how many hairs he has on his head, their length, and their arrangement. These are *borderline cases* of baldness. This is the first characteristic feature of vagueness: a predicate is vague only if there are borderline cases—cases in which it is neither clear that the predicate applies nor clear that it doesn't, and moreover, there seems to be no principled way in which we can decide whether it applies or not.[11]

The second characteristic feature of vagueness is *sorites susceptibility*, i.e., susceptibility to sorites paradoxes. We cannot simultaneously endorse all three of the following statements though they're individually attractive: (i) A man with zero hairs is bald; (ii) A man with a hundred thousand hairs is not bald; (iii) If a man with k hairs is not bald, then so is a man with $k-1$ hairs.

Borderline cases tempt us to cordon them off into a category of their own, and to then argue that vague terms such as *bald* don't just have an extension and an anti-extension, but also a borderline extension, elements of which we might call *borderline bald*. But we can't dispense with borderline cases or the sorites paradox just with this move because if we do so, there will still be cases which we can't categorize, in any principled way, as *bald* or *borderline-bald* and cases which we can't categorize, in any principled way, as *borderline-bald* or *not bald*. So we will have new borderline cases between the intermediate category and the old categories. And we can raise sorites paradoxes here as well, by transitioning, small step by small step, from *bald* to *borderline bald* and from *borderline bald* to *not bald*. This is second-order vagueness. This can be iterated to third-order, to fourth-order, and to higher orders. This is *higher-order vagueness*, the third characteristic feature of vagueness.[12]

I will take these three features of vague terms—the presence of borderline cases, sorites susceptibility, and higher-order vagueness—to both be identifying

11. The term "borderline" can be misleading because "line" suggests that the border is sharp. But that's precisely what isn't the case for vague terms. A better phrase might be "border-zone" or "border-region", since there might be quite a large region of cases within which it is unclear whether or not a predicate applies. Indeed, this is important to keep in mind when discussing higher-order vagueness, for we need to be able to subdivide border-zones into further subcategories. However, since "borderline" is standard terminology, I will stick with it.

12. Keefe and Smith (1996) characterize vagueness using the following features: the possibility of borderline cases, the presence of fuzzy boundaries, and sorites susceptibility. My characterization swaps out fuzzy boundaries for higher-order vagueness. But these are interchangeable; this is one of the imports of Sainsbury (1996, p. 257).

features of vagueness and (I will argue) the bases on which the fault-tolerance of vague terms rests.

It's important to note here that although there are three characteristics laid out here, and those characteristics are conceptually separable, that doesn't mean that it is possible for them to occur separately from each other. Typically, they are co-extensive. Outside of some artificially constructed cases[13] they always go together. This is important for us to note because, later, when I'll argue that vague terms enjoy various features of fault-tolerance by virtue of various features of vagueness, I'm separating these various features only conceptually, to help in elucidating my point, and not as suggesting that those features can be realized independently of each other.

3 Vagueness in physics

The primary example we'll be working with in this paper concerns different models for the orbit of a planet. Say we are modeling the following sort of solar system: a central star around which two planets stably orbit. (You might imagine a system with just the Sun, the Earth, and Jupiter.) And say we are interested in the orbit of the inner, Earth-like, planet. We can construct many models for the inner planet's orbit, at increasing levels of detail. At the simplest level, one can model the inner planet as being on a circular orbit. At a more sophisticated level, we can construct a model that ignores the outer planet and treats the star as stationary. This yields a standard *two-body model* in Newtonian gravitation.[14] Going even further, we can consider *an intermediate model* which includes the outer planet as a perturbation to the two-body model. An even more detailed model is the full *three-body model* of the star and the two planets.[15] One can construct even more detailed models: models with general-relativistic corrections, full general-relativistic models, models that include the size and rotation of the bodies, and so on.

In what follows, we'll largely focus on the *two-body* model and the *three-body* model and later bring in the *intermediate* model. These models are systems of equations that take as input the masses of the bodies and their positions and

13. See, e.g., Sainsbury (1996)'s example of *child** for a term that may be argued has borderline cases but is neither sorites-susceptible nor higher-order vague.

14. Such a model is commonly analyzed by mapping onto a *central-force* model. See, e.g., Kleppner and Kolenkow (2014, Ch. 10) for more details.

15. While such a model resists a closed-form solution—the famed three-body problem (Barrow-Green 2010)—it can be numerically solved on computers.

velocities at some point in time and output a solution for the trajectory of the inner planet (this trajectory is what we've chosen to focus on). For some of these inputs, the two-body model will say the inner planet is on an elliptical orbit. Meanwhile, on the same inputs, the full three-body model will say the inner planet is on a more complicated, fluctuating orbit that's not a perfect ellipse. (Note that the inputs that enter into the two-body model will be a strict subset of the inputs that enter into the three-body model, since the three-body model represents the star, the inner planet, and also, the outer planet.)

When do these two models *agree* about the inner planet's orbit? The natural criterion for agreement is some scheme of *approximation*. If a three-body model (with appropriate input parameters) produces an orbit for its inner planet that is *approximately* the same as the orbit produced by the two-planet model for its inner planet, then we can take the two models as agreeing about the motion of the inner planet.[16]

Vagueness enters when we ask *which* three-body models agree with a given two-body model. To see this, take a specific two-body model (i.e., a model with its parameters fixed at some values); this model will make a claim about the orbit of the inner planet—say, that it follows a particular elliptical orbit. Now note that there are many different three-body models whose parameter settings are consistent with the specified parameters of the two-body model: i.e., there are many different three-body models which have the same mass, initial velocity, and initial position for the inner planet and central star as the two-body model, but all these three-body models differ amongst each other on the mass or initial velocity or initial position of the outer-planet. Each such three-body model will make different predictions about the inner planet's orbit. Some of these three-body models can be taken as *agreeing* with the two-body model because they approximate the two-body model's inner planet orbit; contrariwise, some three-body models will *disagree*. What is vague here is that there is no way to draw a sharp boundary, in a principled way, between those three-body models that agree with the given two-body model and those that don't.

When I say we can't draw a sharp boundary in a principled way, I mean that neither the two-body model, nor the three-body model, nor anything in the

16. Why is approximation and not consistency (two models agree just if they make exactly the same claims about a system) the relevant criterion of agreement? Because consistency is poorly motivated by scientific practice. For instance, the two-body model would almost never be strictly consistent with the three-body model. Nevertheless, the two-body model is enormously useful in physics, and it is so because it's a good *approximation* to the three-body model.

methods for deriving one from the other within the framework of Newtonian physics, provides a precise quantitative cutoff at which one can take the two-body model to have failed in its task of approximating the three-body model.

However, this doesn't mean one cannot provide any quantitative characterization of where the disagreement occurs. For instance, the two-body model agrees well with the three-body model when the outer planet isn't too massive relative to the inner planet and is sufficiently far away from the inner planet, and disagrees otherwise. But clearly this characterization of when the two-body model fails, while quantitative, isn't *precise*.

Straightforwardly, we can exhibit in this example the characteristic features of vagueness (specified in Sec. 2 above).

(i) Borderline cases: There will be three-body models such that it is neither clear that they agree with the given two-body model nor clear that they disagree, and moreover, there are no principled reasons that allow us to decide whether or not they agree.

(ii) Sorites-susceptibility: There will be sequences of three-body models—adjacent models differing ever so slightly (say, the adjacent models' outer planets' mass differs by grams)—that start out clearly agreeing with the two-body model but end up eventually disagreeing.

(iii) Higher-order vagueness: Separating the three-body models that are in borderline agreement with the given two-body model into a new, distinct category will result in new borderline cases between the new and the old categories; and this will continue at higher-orders if we carve out further intermediate categories.

The example I've given above isn't by any means cherry-picked. It is easy to generate many examples in physics where one constructs two different models for a system or a class of phenomena, and where the relation between the two models is vague. Let me list a few further examples (one can generate many more):

• *Temperature of a gas.* Consider a box of gas. We can model it using statistical mechanics, and attribute a certain temperature T to it. We can also model it as a swarm of particles, with definite positions and momenta at any given time, obeying Newtonian equations of motion. There are uncountably many Newtonian models—where a Newtonian model is an assignment of particle positions and a

Newtonian dynamical time-evolution of that configuration of particles—that are compatible with the same statistical-mechanical description, and hence the same temperature attribution. (It might help to imagine ever so slightly disturbing the position of just one particle in a box with $\sim 10^{23}$ particles and seeing how that won't make a difference to the overall temperature attributed to the box.) There is no principled way we can draw a sharp boundary between those Newtonian models that count as agreeing with the statistical-mechanical model and those that don't, leading to vagueness.

This example can be extended: Any macrostate defined by some macroproperty (or collection of macroproperties) will bear a vague relation to microphysics.[17] More precisely, there is no principled way we can draw a precise boundary between those microstates that count as realizing a particular macrostate and those that don't.[18]

• *Viscosity of a fluid.* We can model a fluid using the Navier-Stokes equations,[19] which requires attributing a certain viscosity η to it. We can also model the fluid as a collection of a large number of molecules, dynamically interacting with each other via some potential energy function. There will be no principled way to draw a sharp boundary between those molecular-dynamical models that count as agreeing with the Navier-Stokes model—and hence attributing to it the viscosity η—and those that don't.

• *Effective theories.* There are systematic recipes available in physics (so-called renormalization group methods) for constructing a simple model (with a small number of parameters) starting from a much more complex model (with a large number of parameters). Under these recipes, many different complex models will 'flow' (i.e., be taken by the renormalization procedure) to the same simple model.[20] Such simpler models, often called *effective theories*, come with *cutoffs*, which determine a regime of validity—i.e., these models/theories will only be applicable when the appropriate inputs to these models are not beyond the cutoff values. These cutoffs exist because the simple model, by construction, is

17. See Albert (2000) for a classic philosophical discussion concerning the definitions of and the relations between macrostates and microstates.

18. The vagueness of the relation between macrostates and microstates is the starting point of Chen (2022). He uses this vagueness to argue that the fundamental laws might be vague. In contrast, my focus stays on the non-fundamental, and my goal here is to use such examples to construct a novel explanation of vagueness instead of arguing for the existence of a novel kind of vagueness as Chen (2022) does. See also, Miller (2021), for a different argument for imprecision at the fundamental level.

19. See, e.g., Thorne and Blandford (2017, pp. 711-713).

20. See Williams (2023, Sec. 4.3) for a philosophical introduction to these ideas.

insensitive to the precise details of the goings-on beyond the cutoff—goings-on which would require the use of the more complex models. The renormalization group procedure folds-in the beyond-cutoff details into the parameters and structure of the simple model.

That said, the renormalization group recipes offer no principled way to draw a sharp boundary between those complex models that flow to the same simple model and those that don't, i.e., there is no way to determine a precise value as *the* cutoff.[21] And what's more, that there be no precise cutoff value for the validity of an effective theory is a central normative requirement of renormalization group methods: one explicitly works to get rid of any dependence on the precise cutoff value when using the renormalization group.[22]

4 Ordinary language use as continuous with scientific modeling

So far we have been focused on models in physics. Below, I will explain how vagueness yields fault-tolerance. I want my explanation to extend beyond physics, to ordinary descriptive language as well. For my explanation to be so extendable, I need to argue that when we use ordinary descriptive language, we model the world in a way that is relevantly similar to the way in which we model the world in physics. This will allow us to see, in a relatively unified way, how vagueness supplies fault-tolerance in both physics and ordinary language use, for they can both be seen as instances of modeling in general.

Let me now argue that our use of descriptive language—especially the kind of language that we prototypically take to be vague—is at least partially aimed at modeling the world. Consider declarative sentences, using which a speaker says something is such-and-so, e.g., "Jack is bald" or "Anne is tall". This kind of language models the world by *categorizing* objects—i.e., by slotting objects into different categories. We have to be careful with this statement though. If we take categorizing an object to be the same thing as describing a *set* which contains the object, then this leads to the question whether vague predicates like "tall" and "bald" pick out a well-defined set. To avoid this concern, I'll

21. Note that effective theories have imprecision in them well-before the cutoff scale. This is a well-known point in physics; see, e.g., Miller (2021, pp. 2905-2907) for a place in the philosophical literature where this is discussed.

22. One plausible inference from the arguments of my paper is that fault-tolerance might explain why we have this normative requirement in renormalization group.

appeal to a notion of categorization that is broader than placing objects into mathematically respectable sets—the notion I'm appealing to being what we tend to employ in most acts of collecting, classifying, organizing, taxonomizing, and so on: an activity that doesn't require appeal to the machinery of set theory.

These ordinary acts of categorization are continuous with scientific taxonomy. These categories frequently contain implicit theories: as in, by categorizing an object as such-and-so, we commit ourselves to categorizing that and other objects in certain ways. So, for example, if I say "John is tall", I'm committed to thereby categorizing anybody who is taller than John as also tall and to not categorizing John as short. This is akin to how, in chemistry, if I categorize a particular sample as metallic copper, then I'm committed to it being conductive.[23] That categorization comes with a network of commitments strengthens the analogy between ordinary description and scientific modeling. The inferential implications of our categorization will be helpful in seeing the way in which these models are fault-tolerant.[24]

Let me briefly consider a couple of points of difference between models in science and models behind ordinary language use. These differences will not matter much for my main point, concerning fault-tolerance. Nevertheless, I will briefly consider them and show that these differences aren't as dramatic as one might initially think. This will help us in forming a clear, unified picture of scientific modeling and ordinary language use.[25]

The first point of difference is *explicitness*. Models in science often explicitly specify which inferences are permissible and impermissible. Many models in science, especially in physics, are explicitly *mathematical*, such as the models we saw above, and by being mathematical, they provide a sharp and explicit delineation of which inferences are allowed and disallowed. E.g., in thermal physics, there is often an explicit specification of how temperature is constrained by other quantities, such as pressure or volume: e.g., $PV = nRT$ for ideal gases. Whereas, in ordinary language, permissible inferences and constraints are rarely explicitly specified, let alone mathematically. The Oxford English Dictionary

23. The latter example is from Brandom (2019). One might develop this line into an inferentialist account of language—as Brandom does—on which the meanings of terms just is a network of inferential relations. I do not however need to endorse such an account of language.

24. Similar ideas have been explored by some psychologists, who have argued that concepts can be thought of as *theories*, akin to scientific theories, and that these concepts change in child development in much the same way theories change as science develops. See, in particular, the *theory theory* of concepts defended by Gopnik and Meltzoff (1998). See also Carey (2009) and Keil (1992).

25. Let me also note that the following are definitely not exhaustive of the differences between scientific modeling and ordinary language use.

(2023) entry for *bald* is “Having no hair on some part of the head where it would naturally grow; hairless”. So this explicitly specifies that uses of “bald” typically entails claims about “hair” and “head”. But such specification falls well short of any kind of detailed regimentation.

While this is true, the amount of explicitness available in science should not be overstated. In science: (a) there are many potentially undiscovered and unarticulated relations in science; e.g., the temperature of a gas is related to the colors of spectral lines it emits, but this observation wasn’t articulable with classical statistical mechanics but required the advent of quantum mechanics; and (b) scientific terms bear a host of relations to ordinary-language concepts which are rarely made explicit; the relation between the ordinary-language notion of temperature and the scientific concept of temperature is complex and rarely fully articulated.[26] More generally, a great deal of scientific knowledge and procedure is unsystematized and implicit, available only to those embedded in research communities.[27] This makes many aspects of scientific models more implicit, and hence closer to ordinary language, than one might initially think.

The other point of dissimilarity is that while scientific models are deliberately constructed to describe and explain certain systems, ordinary language emerges from a web of social practices and so has a structure that is rarely the outcome of conscious deliberation and decision to describe some class of systems. One might think that this point is particularly relevant for the arguments to come since appeal to ideas like fault-tolerance suggests that we are saying our models possess the kind of virtues one would expect of engineered artifacts. While scientific modeling might plausibly be deliberately engineered in certain ways to embody certain virtues[28], it is perhaps harder to see why our ordinary language would also embody such engineered virtues if they only emerge out of a largely unconscious web of social practices.

To address this point, first note that scientific models aren’t quite so deliberately engineered as one might think. Such models are also, in significant part, the outcome of a complex social and evolutionary process.[29] Similarly, languages and

26. See Wilson (1982, pp. 564-566) for a brief discussion of this point. The historical and conceptual complexities of the scientific notion of temperature are discussed in Chang (2004).

27. This is a frequent theme in philosophy of science. A couple of useful entry points here are Thomas Kuhn’s notion of a *disciplinary matrix* (Kuhn and Hacking 2012) and Michael Polanyi’s notion of *tacit knowledge* (Polanyi 1958; Polanyi and Sen 1966).

28. See, e.g., Wimsatt (2007) for, *inter alia*, an extended defense of the use of engineering metaphors in the philosophy of science.

29. See Hull (1988) for an articulation of how scientific development can be seen as an evolutionary process. See also Wilson (2006) for similar themes.

the cognition supporting them can be seen as evolved. This could be biological evolution (see, e.g., Pinker (2007)), cultural evolution (see, e.g., Everett (2012) and Heyes (2018)), or some combination of the two thereof (see, e.g., Deacon (1997)).[30] Once we see that language (and science) are evolved—their features selected for on the basis of some kind of 'fitness'—then we have a plausible route for the emergence of design virtues, such as fault-tolerance.[31][32]

5 Fault tolerance and vagueness

From the development in Sec. 3 it should be clear how entering a borderline zone accompanies model breakdown. Vagueness arose whenever the three-body model predicted an orbit for the inner planet not entirely well-modeled by a given two-body model, while at the same time, the two-body model was not so bad a fit that we had a clear reason to say that the three-body model does *not* agree with the two-body model. In short, vagueness arose when the two-body model was breaking down in its task of capturing the three-body predictions.

This observation about how borderline cases arise when models break down also fits ordinary-language vague terms. E.g., consider "bald". We can model a person's hair configuration using two kinds of models. We can classify them as "bald" or "not bald" (see Sec. 4) or we can model them by ascribing a certain number of hairs to their head. The person in question will be borderline bald just if it isn't clear if the *bald/not bald* model agrees with the *number of hairs* model. And this will happen precisely when the bald/not bald model starts to break down.[33]

Why are our models this way? The question is particularly puzzling when in the context of physics: Wouldn't any systematic science, especially one as enamored with precision as physics is, *want* models with clear, sharp boundaries, with no room for ambiguity about applicability? Plausibly, our modeling practices are willing to forgo the clarity of sharp boundaries because it is valuable to do

30. Also, relevant here is the work of Ruth Millikan (see, e.g., Millikan (1984, 2017)). See also Richard (2019).

31. See, e.g., the arguments of Dennett (2017) as to how evolution supplies design without designer, including for features of language.

32. These brief remarks are merely meant to assuage an initial skepticism against the emergence of design virtues in ordinary language. They are not meant to be anything like a fully articulated story of how such virtues actually emerged.

33. Viewing borderline zones as arising when models break down goes at least some way towards explaining the characteristic instability and uneasiness of our judgments when faced with borderline cases—after all, our models are breaking down, and we don't quite have the right models (until we construct intermediate models) to talk about the situation.

so. The fact that vagueness is associated with model-breakdown gives us a clue. If vagueness serves some sort of purpose, then that purpose is likely best served when our models of the world start breaking down. I propose that that purpose is fault-tolerance, a feature that becomes salient during failure.

Fault-tolerance is a central normative principle in engineering. One thing we want of a fault-tolerant system is that it continue functioning, insofar as possible, despite encountering variability. That is, we want a system that is *robust*. Moreover, when smooth functioning is no longer possible—say if the errors are too severe—then we want the system's performance to degrade proportionally to the amount of errors encountered. That is, we want the system to *degrade gracefully*. Finally, we want the system to allow itself to be *patched* in appropriate ways as it is degrading. That is, we want the system to be *error-correctable*.[34,35]

We will now see how these three desiderata of fault-tolerance—robustness, graceful degradation, and error-correctability—when applied to modeling practices, are grounded in, respectively, the phenomena characteristic of vagueness—sorites-susceptibility, the presence of borderline cases, and higher-order vagueness. Note however, I'm not suggesting that these features of fault-tolerance can all be independently realized. Just as the various characteristic features of vagueness cannot typically be realized independently of each other (as remarked above in Sec. 2), and we specify them separately only for the sake of conceptual clarity, the various features of fault-tolerance are separated only for conceptual clarity and we shouldn't assume that we can always realize them independently. So the idea here is that vagueness—which is one interdependent manifold of features—supplies the basis for fault-tolerance—another interdependent manifold of features.

Start with robustness and sorites-susceptibility. Robustness requires that if the situations we encounter differ only slightly, then we should be able to continue using the same model, much like how we want a bridge to continue being stable if just one more person steps on to the bridge. Thus, if the slightest

34. There are different ways the notion of fault-tolerance is presented in the engineering literature. For some entry points, see, e.g., Pradhan (1996) and Dubrova (2013). I have chosen three aspects that I think are relevant to explaining vagueness. Let me also emphasize that in engineering contexts, one can construct mathematical models that describe or aid in the design of fault-tolerant systems. In the paper, I do not engage with the specific mathematical details and rely more on the broader qualitative insights of that discipline, partially because the mathematical details will depend on the system in question.

35. My notion of fault-tolerance is related to Wimsatt (2007)'s notion of error-tolerance, but the value he sees in error-tolerance is somewhat different from the value I'm identifying, so I stick with my terminology. Moreover, 'fault-tolerance' is a well-established notion in engineering, which I'm drawing on.

change in a situation that we are modeling necessitated the use of a new model, then our models would not be robust. This feature is realized by the *principle of tolerance*, which attaches to prototypically vague predicates and states that there is "a degree of change too small to make any difference" (Wright 1975, p. 333). The principle of tolerance endows our models with robustness. However, it also renders them sorites-susceptible.

Let us bring out more clearly how sorites-susceptibility makes models robust, both in the context of physics and in ordinary discourse. Let's start with physics. Say I'm predicting the motion of a projectile using Newton's equations. Say I'm trying to calculate the final velocity of the projectile starting with its initial velocity (and other data). Even if I could measure, very precisely, the initial velocity of the projectile, I wouldn't be justified (in most realistic scenarios and without adding many caveats) in predicting the projectile's final velocity to an equally great degree of precision. This is because such a prediction would be fragile: a small perturbation or injection of noise (say a stray wind current or a local inhomogeneity in the gravitational field) might be enough to render the prediction inaccurate. So if I want a robust prediction, then it'll be better if I make a less precise prediction. However, this will come at the cost of sorites-susceptibility, for a robust prediction is, by design, tolerant to small changes in the target situation. But we can always chain together many small changes (each individually small enough to preserve the accuracy of the prediction) to create changes large enough to make even the robust prediction fail.

Turn now to more ordinary contexts. In the context of ordinary conversations, it is a norm that we ought not to make our statements more precise than necessary. This is one part of Grice's Maxim of Quantity, the part which exhorts us to make our contributions to a conversation no more informative than necessary (Grice 1989, p. 26), and that entails that we shouldn't make our contributions more precise than necessary.[36] So, e.g., if someone asks me, in an informal context, how much a cup of coffee costs at the coffee shop nearby and I say that it's \$3.46 and there's a 6.1% tax and at least 12% tip is expected, then I would have given an unnecessarily precise answer. One reason why such a precise answer can be criticized is that the answer is not robust. If the coffee shop decides to change the price tomorrow to \$3.55 or the state's sales tax rate changes to 6.25% or the norms around tipping change, my answer would have been invalidated. However, if I had simply said, in response to the original query, that the price is about

36. However, Grice does not discuss robustness as a potential justification for this norm. His focus is more on how unnecessary precision can be distracting.

$4, then my answer would have been much more robust to such sources of noise. But now, it is unclear how far from $4 the price of the coffee can drift before the initial statement counts as false or misleading. So, as above, robustness arises because of sorites-susceptibility.

Next, let's turn to how the presence of borderline cases allows for graceful degradability. A system degrades gracefully just if the degree to which it degrades (when it does degrade) is proportional to the degree to which errors have been encountered. That is, it retains partial functionality instead of failing catastrophically. E.g., if a bridge is slowly getting overloaded, we'd like the bridge to degrade by slowly showing cracks instead of collapsing altogether. I claim that because our models allow for borderline cases, they degrade gracefully—the model continues being useful despite its limitations. Let's first see how this works in the context of modeling in physics and then turn to models in ordinary language.

In the physics context, in the celestial mechanics example, when we see the system deviating from a two-body model—and thus when we are faced with borderline cases of agreement between the two-body model and three-body model—it is still useful to describe the system as one that is close to but deviating from a two-body model. (As mentioned in Sec. 3, we can construct new intermediate models based on this observation; intermediate models will be central to our discussion below on error-correction.) Indeed, the theoretical framework in which we state these models come equipped with resources to quantify the degree to which the behavior of the system is deviating from the behavior predicted by the two-body model: we can define appropriate distance measures between the two-body orbit and the three-body orbit, or specify the deviation of orbital parameters such as eccentricity, or quantify the amount of orbital precession, and so on.[37] You can still use the two-body model to make some inferences, but it is just that these inferences are no longer as secure as they would be if you were considering a clear case of agreement between the two-body and three-body models. We can see this as a kind of graceful degradation. As our models fail, they don't do so catastrophically, and they retain some of their usefulness as we enter the borderline zone.

In the context of ordinary language, we see a similar phenomenon. When we classify someone as borderline bald, we are neither accepting nor rejecting the classificatory model of *bald/not bald* wholesale, as we might do if we are

37. See, e.g., Fitzpatrick (2012) for details.

asked whether a building is (non-metaphorically) bald or not bald, or as we might do with someone who has an unusual hairstyle that leaves exactly one half of their scalp without hair. In contrast, for a borderline bald person, the bald/not bald model isn't entirely useless; it establishes relevant classes of cases with which to compare the case at hand. Furthermore, when you hear that someone is borderline bald, you can still draw useful inferences from that claim: you can infer that they don't have a full head of hair on their head; you can infer that some people would say this person is bald; you may infer (depending on the context) that in the future it's likely that they will become clearly bald. So it is still useful to think of that person *using* the modeling apparatus of *bald/not bald*—it's just that you are drawing inferences that are related to, but not exactly, the inferences you would draw if we simply labeled that person "bald". For such borderline cases, the *bald/not bald* model, though degraded, is gracefully degraded—it retains some usefulness despite encountering situations it isn't entirely fit to model.

So we have seen how sorites-susceptibility and borderline cases confer, respectively, robustness and graceful degradability.[38] Let us now turn to higher-order vagueness, and see how it confers error-correctability, the third feature of fault-tolerance I have listed above. A system is error-correctable just if it has affordances that allow one to repair the system when it is encountering faults. When a bridge begins to display cracks, we want to be able to patch those cracks. In the context of our discussion, error-correctability is the requirement that our models allow for the construction of intermediate models—models that work better than the initial models—when our initial models are breaking down. Higher-order vagueness, I claim, provides for iterated error-correctability: it allows us to error-correct not just the initial failing models, but also the new intermediate models we construct because they too will inevitably fail in certain circumstances.

Let's see how the process of constructing intermediate models works, first in physics and then in ordinary language. Suppose we are in a situation where a model is breaking down: Say that according to a three-body model, the inner planet is in an orbit which is deviating from an ellipse. In this regime, a two-body model isn't in adequate agreement with the three-body model. A natural and standard move here is to consider a new, corrected model. So let's say the orbit of the inner planet according to the three-body model is deviating from an

38. Let me re-emphasize that we are individuating these features only for conceptual clarity; in reality, they all run together.

ellipse because the outer planet is too massive to have its influence approximated away. This would perturb the orbital parameters (such as the eccentricity and inclination of the orbit) of the inner planet so that they are no longer constants in time, as they would be under a two-body model. To incorporate this, we can generate a new model that supplies equations that tell us *how* the orbital parameters of the inner planet change due to the perturbation.[39] These equations would constitute an *intermediate* model: a model on which the deviation from an elliptical orbit that we earlier deemed a symptom of the breakdown of the two-body description, is now a clearly explicable case according to the new model.

We can construct similar intermediate models in ordinary language as well. When we encounter situations that our categories—which carry with them their implicit models—cannot capture, we are able to add *corrections* to our pre-existing categories to create new penumbral categories. Our language contains tools to do that. These are adverbial phrases such as *kind of*, *sort of*, *almost*, *nearly*, and so on. So if someone is above-average height but not clearly tall, we often say "they're *kind of* tall". The kinds of inferences we are licensed to draw with such modified models are related to, but not exactly the same as, the inferences we could draw from the unmodified models. Such linguistic tools are of a piece with perturbation theory in physics; while not as precise as perturbation theory, they do offer us the ability to *extend* our current models to nearby domains where our current models don't work so well.

So we see how we can extend our pre-existing models to handle their breakdown regimes. But what happens when the newer, extended models start breaking down in their own turn, when faced with certain situations? So, e.g., maybe the variation in the orbital parameters according to the three-body model is deviating even more so than what is predicted by the intermediate model. Or, e.g., if we encounter someone about whom we cannot clearly decide whether or not they are kind of tall. These situations where intermediate models break down are higher-order borderline cases. To deal with such situations, we can iterate the process of error-correction again and generate newer higher-order intermediate models to deal with the novel situations. In the physics case, the standard way to do this is to add more terms to one's perturbation series yielding new models that can capture finer detail in the behavior of the orbit. In ordinary language we can do this by chaining the adverbial phrases *kind of*, *sort of*, and

39. This would be a perturbation theory in the ratio of the mass of the outer planet to the mass of the central star. See Fitzpatrick (2012, Ch. 9) for the gory details.

so on. So if I say, "He's kind of tall", someone can reasonably respond to me and say, "Well, he's only kind of sort of tall", and we might both agree with that characterization.[40]

If we have vagueness at higher and higher orders, then these new intermediate, penumbral models can all be fault-tolerant, i.e., these intermediate models too are robust, degrade gracefully, and are error-correctable. Insofar as we can keep extending our fault-tolerant models in fault-tolerant ways, this is made possible by higher and higher orders of vagueness.[41]

6 Conclusion

The world is a complicated place, and we have to construct many different models to navigate this complicated place. Different models have different regimes of validity, i.e., different kinds of situations for which they can serve as a good model. It turns out that there are no sharp boundaries between those kinds of situations that a model is a good model for and those kinds of situations that it isn't. And it turns out there is at least one good reason for this unsharpness: It makes our models fault-tolerant.

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40. How well a model vs. its extended version fits a given situation depends on the context of use. If sorting a building as *tall* vs. *kind of tall* makes a significant difference (say because of a legal dispute), then we may be finicky; however, in casual conversation, we may easily concede that the building is only kind of tall. Similarly, we require a much more precise fit, and hence use extended models, for launching a spacecraft from one planet to land on another; however, we would settle for a simpler model, if we only care about coarser patterns of the solar system. (While fixing the context might help in the selection of a relevant model, this wouldn't eliminate all vagueness.)

41. There is a debate in the literature concerning whether vagueness will always carry through to arbitrarily high orders or whether it will terminate at some order or other: see Williamson (1999), Mahtani (2008), and Dorr (2015) for a particular thread of this debate. I don't need to take a stand on this issue here.

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