Correlation Coefficient: Simple Definition, Formula, Easy Steps

Correlation coefficients are used in statistics to measure how strong a relationship is between two variables. There are several types of correlation coefficient: Pearson's correlation (also called Pearson's *R*) is a **correlation coefficient** commonly used in linear regression. If you're starting out in statistics, you'll probably learn about Pearson's *R* first. In fact, when anyone refers to **the** correlation coefficient, they are usually talking about Pearson's.

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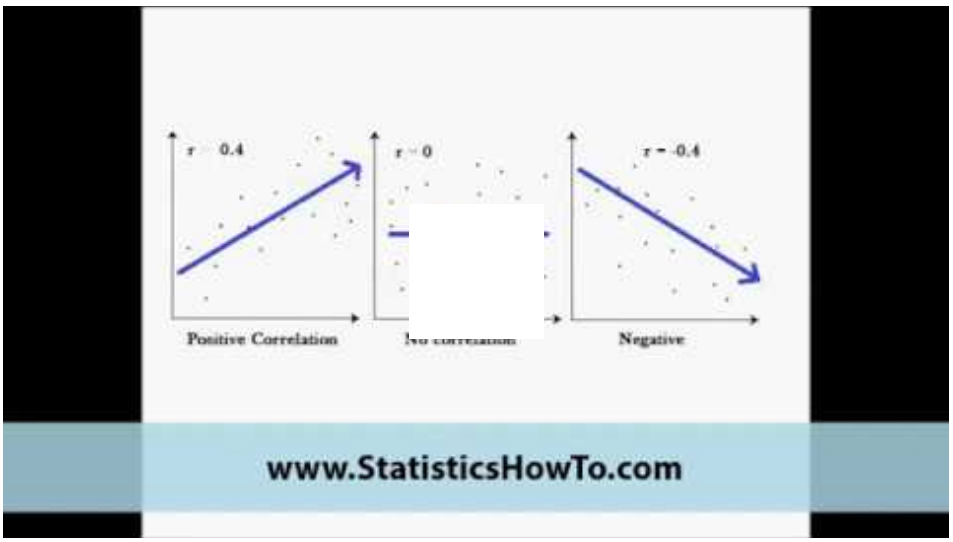
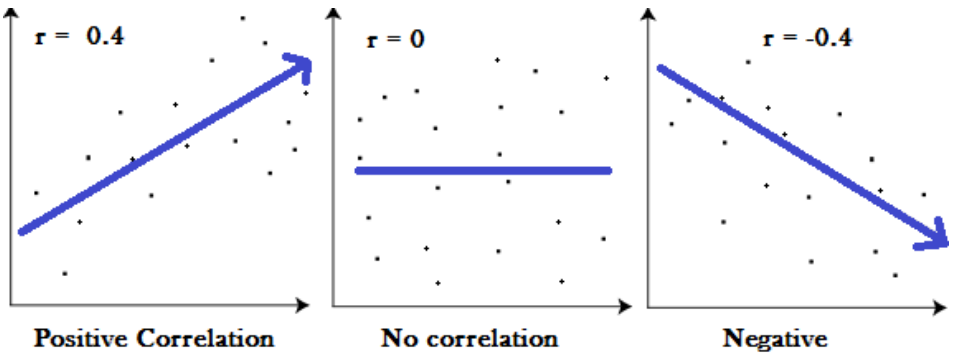
Correlation Coefficient Formula: Definition

Watch the video or read the article below:



Correlation coefficient formulas are used to find how strong a relationship is between data. The formulas return a value between -1 and 1, where:

- 1 indicates a strong positive relationship.
- -1 indicates a strong negative relationship.
- A result of zero indicates no relationship at all.



Graphs showing a correlation of -1, 0 and +1

Meaning

- A correlation coefficient of 1 means that for every positive increase in one variable, there is a positive increase of a fixed proportion in the other. For example, shoe sizes go up in (almost) perfect correlation with foot length.
- A correlation coefficient of -1 means that for every positive increase in one variable, there is a negative decrease of a fixed proportion in the other. For example, the amount of gas in a tank decreases in (almost) perfect correlation with speed.
- Zero means that for every increase, there isn't a positive or negative increase. The two just aren't related.

The absolute value of the correlation coefficient gives us the relationship strength. The larger the number, the stronger the relationship. For example, |-.75| = .75, which has a stronger relationship than .65.

> *Like the explanation? Check out the Practically Cheating Statistics Handbook, which has hundreds of step-by-step, worked out problems!*

Types of correlation coefficient formulas.

There are several types of correlation coefficient formulas.

One of the most commonly used formulas in stats is Pearson's correlation coefficient formula. If you're taking a basic stats class, this is the one you'll probably use:

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[\, n\sum x^2 - (\sum x)^2 \,][\, n\sum y^2 - (\sum y)^2 \,]}}$$

Pearson correlation coefficient

Two other formulas are commonly used: the sample correlation coefficient and the population correlation coefficient.

Sample correlation coefficient

$$r_{xy} = \frac{s_{xy}}{s_x s_y}$$

S_x and s_y are the sample standard deviations, and s_{xy} is the sample covariance.

Population correlation coefficient

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

The population correlation coefficient uses σ_x and σ_y as the population standard deviations, and σ_{xy} as the population covariance.

Check out my Youtube channel for more tips and help with statistics!

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What is Pearson Correlation?

Correlation between sets of data is a measure of how well they are related. The most common measure of correlation in stats is the Pearson Correlation. The full name is the **Pearson Product Moment Correlation (PPMC)**. It shows the linear relationship between two sets of data. In simple terms, it answers the question, *Can I draw a line graph to represent the data?* Two letters are used to represent the Pearson correlation: Greek letter rho (ρ) for a population and the letter "r" for a sample.

Potential problems with Pearson correlation.

The PPMC is not able to tell the difference between dependent variables and independent variables. For example, if you are trying to find the correlation between a high calorie diet and diabetes, you might find a high correlation of .8. However, you could also get the same result with the variables switched around. In other words, you could say that diabetes causes a high calorie diet. That obviously makes no sense. Therefore, as a researcher you have to be aware of the data you are plugging in. In addition, the PPMC will not give you any information about the slope of the line; it only tells you whether there is a relationship.

Real Life Example

Pearson correlation is used in thousands of real life situations. For example, scientists in China wanted to know if there was a relationship between how weedy rice populations are different genetically. The goal was to find out the evolutionary potential of the rice. Pearson's correlation between the two groups was analyzed. It showed a positive Pearson Product Moment correlation of between 0.783 and 0.895 for weedy rice populations. This figure is quite high, which suggested a fairly strong relationship.

If you're interested in seeing more examples of PPMC, you can find several studies on the National Institute of Health's Openi website, which shows result on studies as varied as breast cyst imaging to the role that carbohydrates play in weight loss.
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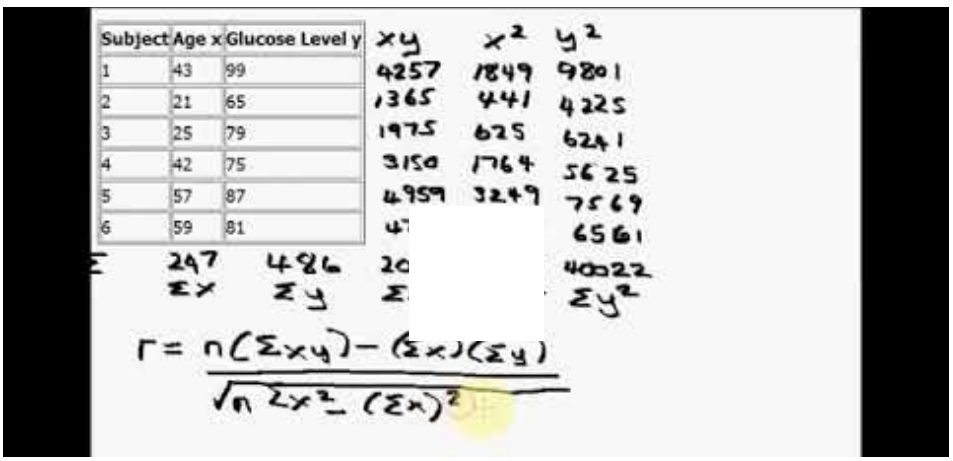
How to Find Pearson's Correlation Coefficients

By Hand



Example question: Find the value of the correlation coefficient from the following table:

SUBJECT	AGE X	GLUCOSE LEVEL Y
1	43	99
2	21	65
3	25	79
4	42	75
5	57	87
6	59	81

Step 1:*Make a chart.* Use the given data, and add three more columns: xy, x^2, and y^2.

SUBJECT	AGE X	GLUCOSE LEVEL Y	XY	X^2	Y^2
1	43	99			
2	21	65			
3	25	79			
4	42	75			
5	57	87			
6	59	81			

Step 2: *Multiply x and y together to fill the xy column. For example, row 1 would be 43 × 99 = **4,257**.*

SUBJECT	AGE X	GLUCOSE LEVEL Y	XY	X^2	Y^2
1	43	99	4257		
2	21	65	1365		
3	25	79	1975		
4	42	75	3150		
5	57	87	4959		
6	59	81	4779		

Step 3: *Take the square of the numbers in the x column, and put the result in the x^2 column.*

SUBJECT	AGE X	GLUCOSE LEVEL Y	XY	X^2	Y^2
1	43	99	4257	1849	
2	21	65	1365	441	
3	25	79	1975	625	

4	42	75	3150	1764
5	57	87	4959	3249
6	59	81	4779	3481

Step 4: *Take the square of the numbers in the y column, and put the result in the y^2 column.*

SUBJECT	AGE X	GLUCOSE LEVEL Y	XY	X^2	Y^2
1	43	99	4257	1849	9801
2	21	65	1365	441	4225
3	25	79	1975	625	6241
4	42	75	3150	1764	5625
5	57	87	4959	3249	7569
6	59	81	4779	3481	6561

Step 5: *Add up all of the numbers in the columns and put the result at the bottom of the column.* The Greek letter sigma (Σ) is a short way of saying "sum of."

SUBJECT	AGE X	GLUCOSE LEVEL Y	XY	X^2	Y^2
1	43	99	4257	1849	9801
2	21	65	1365	441	4225
3	25	79	1975	625	6241
4	42	75	3150	1764	5625
5	57	87	4959	3249	7569
6	59	81	4779	3481	6561
Σ	247	486	20485	11409	40022

Step 6: *Use the following correlation coefficient formula.*

$$r = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{\sqrt{[\,n\Sigma x^2 - (\Sigma x)^2\,][\,n\Sigma y^2 - (\Sigma y)^2\,]}}$$

The answer is: **2868 / 5413.27 = 0.529809**

Click here if you want easy, step-by-step instructions for solving this formula.

From our table:

- Σx = 247
- Σy = 486
- Σxy = 20,485

- $\Sigma x^2 = 11{,}409$
- $\Sigma y^2 = 40{,}022$
- n is the sample size, in our case = 6

The correlation coefficient =

- $6(20{,}485) - (247 \times 486) / [\sqrt{[6(11{,}409) - (247^2)] \times [6(40{,}022) - 486^2]]}$
  $= 0.5298$

The range of the correlation coefficient is from -1 to 1. Our result is 0.5298 or 52.98%, which means the variables have a moderate positive correlation.
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> *Like the explanation? Check out the Practically Cheating Statistics Handbook, which has hundreds more step-by-step explanations, just like this one!*

Correlation Formula: TI 83

If you're taking AP Statistics, you won't actually have to work the correlation formula by hand. You'll use your graphing calculator. Here's how to find r on a TI83.

Step 1: Type your data into a list and make a scatter plot to ensure your variables are roughly correlated. In other words, look for a straight line. Not sure how to do this? See: TI 83 Scatter plot.

Step 2: Press the STAT button.

Step 3: Scroll right to the CALC menu.

Step 4: Scroll down to 4:LinReg(ax+b), then press ENTER. The output will show "r" at the very bottom of the list.

Tip: If you don't see r, turn Diagnostic ON, then perform the steps again.

How to Compute the Pearson Correlation Coefficient Excel 2007

Watch the video or read the steps below:

Step 1: **Type your data into two columns** in Excel. For example, type your "x" data into column A and your "y" data into column B.

Step 2: **Select any empty cell.**

Step 3: **Click the function button** on the ribbon.

Step 4: **Type "correlation" into the 'Search for a function' box.**

Step 5: **Click "Go."** CORREL will be highlighted.

Step 6: **Click "OK."**

Step 7: **Type the location of your data** into the **"Array 1"** and **"Array 2"** boxes. For this example, type "A2:A10" into the Array 1 box and then type "B2:B10" into the Array 2 box.

Step 8: **Click "OK."** The result will appear in the cell you selected in Step 2. For this particular data set, the correlation coefficient(r) is -0.1316.

Caution: The results for this test can be misleading unless you have made a scatter plot first to ensure your data roughly fits a straight line. The correlation coefficient in Excel 2007 will *always* return a value, even if your data is something other than linear (i.e. exponential).

That's it!

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Correlation Coefficient SPSS: Overview.

Watch the video or read the steps below:

Step 1: **Click "Analyze," then click "Correlate," then click "Bivariate."** The Bivariate Correlations window will appear.

Step 2: **Click one of the variables** in the left-hand window of the Bivariate Correlations pop-up window. Then click the center arrow to move the variable to the "Variables:"

window. Repeat this for a second variable.

Step 3: **Click the "Pearson" check box** if it isn't already checked. Then click either a "one-tailed" or "two-tailed" test radio button. If you aren't sure if your test is one-tailed or two-tailed, see: Is it a a one-tailed test or two-tailed test?

Step 4: **Click "OK" and read the results.** Each box in the output gives you a correlation between two variables. For example, the PPMC for Number of older siblings and GPA is -.098, which means practically no correlation. You can find this information in two places in the output. Why? This cross-referencing columns and rows is very useful when you are comparing PPMCs for dozens of variables.

Tip #1: It's always a good idea to make an SPSS scatter plot of your data set *before* you perform this test. That's because SPSS will *always* give you some kind of answer and will assume that the data is linearly related. If you have data that might be better suited to another correlation (for example, exponentially related data) then SPSS will still run Pearson's for you and you might get misleading results.
Tip #2: Click on the "Options" button in the Bivariate Correlations window if you want to include descriptive statistics like the mean and standard deviation.
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Minitab

Watch this video on how to calculate **the correlation coefficient in** Minitab, or read the steps in the article below:

The Minitab correlation coefficient will return a value for r from -1 to 1.

Sample question: Find the Minitab correlation coefficient based on age vs. glucose level from the following table from a pre-diabetic study of 6 participants:

SUBJECT	AGE X	GLUCOSE LEVEL Y
1	43	99
2	21	65
3	25	79
4	42	75
5	57	87
6	59	81

Step 1: **Type your data into a Minitab worksheet**. I entered this sample data into three columns.

Data entered into three columns in a Minitab worksheet.

Step 2: **Click "Stat", then click "Basic Statistics" and then click "Correlation."**

"Correlation" is selected from the "Stats > Basic Statistics" menu.

Step 3: **Click a variable name in the left window and then click the "Select" button** to move the variable name to the Variable box. For this sample question, click "Age," then click "Select," then click "Glucose Level" then click "Select" to transfer both variables to the Variable window.

Step 4: (Optional) **Check the "P-Value" box** if you want to display a P-Value for r.

Step 5: **Click "OK".** The Minitab correlation coefficient will be displayed in the Session Window. If you don't see the results, click "Window" and then click "Tile." The Session window should appear.

Results from the Minitab correlation.

For this dataset:
Value of r: 0.530
P-Value: 0.280
That's it!

Tip: Give your columns meaningful names (in the first row of the column, right under C1, C2 etc.). That way, when it comes to choosing variable names in Step 3, you'll easily see what it is you are trying to choose. This becomes especially important when you have dozens of columns of variables in a data sheet!

Meaning of the Linear Correlation Coefficient.

Pearson's Correlation Coefficient is a linear correlation coefficient that returns a value of between -1 and +1. A -1 means there is a strong negative correlation and +1 means that there is a strong positive correlation. A 0 means that there is no correlation (this is also called **zero correlation**).

This can initially be a little hard to wrap your head around (who likes to deal with negative numbers?). The Political Science Department at Quinnipiac University posted this useful list of the meaning of Pearson's Correlation coefficients. They note that these are "**crude estimates**" for interpreting strengths of correlations using Pearson's Correlation:

r value =	
+.70 or higher	Very strong positive relationship
+.40 to +.69	Strong positive relationship
+.30 to +.39	Moderate positive relationship
+.20 to +.29	weak positive relationship
+.01 to +.19	No or negligible relationship
0	No relationship [zero correlation]
-.01 to -.19	No or negligible relationship
-.20 to -.29	weak negative relationship
-.30 to -.39	Moderate negative relationship

-.40 to -.69	Strong negative relationship
-.70 or higher	Very strong negative relationship

It may be helpful to see graphically what these correlations look like:

Graphs showing a correlation of -1 (a negative correlation), 0 and +1 (a positive correlation)

The images show that a strong negative correlation means that the graph has a downward slope from left to right: as the x-values increase, the y-values get smaller. A strong positive correlation means that the graph has an upward slope from left to right: as the x-values increase, the y-values get larger.

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Cramer's V Correlation

Cramer's V Correlation is similar to the Pearson Correlation coefficient. While the Pearson correlation is used to test the strength of linear relationships, Cramer's V is used to calculate correlation in tables with more than 2 x 2 columns and rows. Cramer's V correlation varies between 0 and 1. A value close to 0 means that there is very little association between the variables. A Cramer's V of close to 1 indicates a very strong association.

Cramer's V	
.25 or higher	Very strong relationship
.15 to .25	Strong relationship
.11 to .15	Moderate relationship
.06 to .10	weak relationship
.01 to .05	No or negligible relationship

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Where did the Correlation Coefficient Come From?

A correlation coefficient gives you an idea of how well data fits a line or curve. Pearson wasn't the original inventor of the term correlation but his use of it became one of the most popular ways to measure correlation.

Francis Galton (who was also involved with the development of the interquartile range) was the first person to measure correlation, originally termed "co-relation," which actually makes sense considering you're studying the relationship between a couple of different

variables. In Co-Relations and Their Measurement, he said "The statures of kinsmen are co-related variables; thus, the stature of the father is correlated to that of the adult son,..and so on; but the index of co-relation ... is different in the different cases." It's worth noting though that Galton mentioned in his paper that he had borrowed the term from biology, where "Co-relation and correlation of structure" was being used but until the time of his paper it hadn't been properly defined.

In 1892, British statistician Francis Ysidro Edgeworth published a paper called "Correlated Averages," Philosophical Magazine, 5th Series, 34, 190-204 where he used the term "Coefficient of Correlation." It wasn't until 1896 that British mathematician Karl Pearson used "Coefficient of Correlation" in two papers: Contributions to the Mathematical Theory of Evolution and Mathematical Contributions to the Theory of Evolution. III. Regression, Heredity and Panmixia. It was the second paper that introduced the Pearson product-moment correlation formula for estimating correlation.

The Pearson Product-Moment Correlation equation.

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Correlation Coefficient Hypothesis Test

If you can read a table — you can **test for correlation coefficient.** Note that correlations should only be calculated for an entire range of data. If you restrict the range, *r* will be weakened.

Sample problem: test the significance of the correlation coefficient r = 0.565 using the critical values for PPMC table. Test at α = 0.01 for a sample size of 9.

Step 1: *Subtract two from the sample size to get df, degrees of freedom.*
9 – 7 = 2

Step 2: *Look the values up in the PPMC Table.* With df = 7 and α = 0.01, the table value is = **0.798**

Step 3: *Draw a graph, so you can more easily see the relationship.*

r = 0.565 does not fall into the "reject" region (above 0.798), so there isn't enough evidence to state a strong linear relationship exists in the data.

Related Articles / More Correlation Coefficients

Other similar formulas you might come across that involve correlation (**click for article**):

- Concordance Correlation coefficient.
- Intraclass Correlation.
- Kendall's Tau.
- Moran's I.
- Partial Correlation.
- Phi Coefficient.
- Point Biserial Correlation.
- Polychoric Correlation.
- Spearman Rank Correlation.
- Tetrachoric Correlation.
- Zero-Order Correlation.

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