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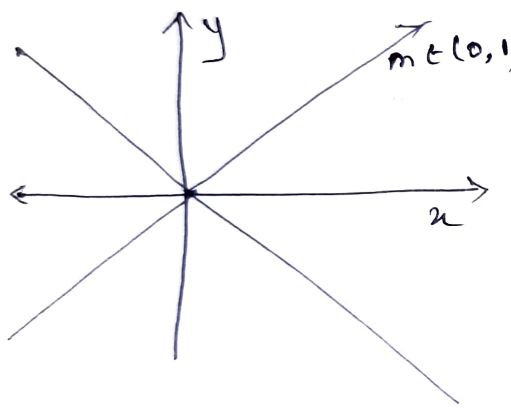
Exercise 1:

a) Given $x_0 = (3,1)^T$
$x_1 = (2,5)^T$

then, $m = \frac{5-1}{2-3} = -4$

First we map the points on line $(x,y)^T$ such that the slope is positive.
we know that if we divide the space where line segment exist according to slope, we get 8 similar sectors.



So, if we just change the sign of any one of the values of coordinate point, my slope transforms to (+)ve.

So, transform
$$(x,y)^T \to (+x,-y)^T$$

this gives me a slope of $m = 4$, when applied to x_0, x_1.

Now, as $m > 1$, we change x value to y and y to x to change the quadrant, without changing the position of (-)ve sign,
So, final transformation map,

$$\boxed{(x,y)^T \to (+y,-x)^T}$$

Now, Applying the map $(x,y)^T \longrightarrow (y,-x)^T$

we get to x_0:

$$(3,1)^T \rightarrow (1,-3)^T$$

Similarly, Applying the map to x_1:

$$(2,5)^T \rightarrow (5,-2)^T$$

So,

$$\Delta x = 5-1 = 4$$

$$\Delta y = -2+3 = 1$$

$$\text{slope}, m = \frac{\Delta y}{\Delta x} = 1/4 = 0.25 \in (0,1).$$

b) Now using the transformed x_0 and x_1 from 1(a), we compute the initial value of decision variable d, ΔE and ΔNE,

$x_0 \rightarrow (1,-3)^T \quad x_1 \rightarrow (5,-2)^T$

$\Delta x = 5-1 = 4$

$\Delta y = -2+3 = 1$

Initial value of decision variable $d = 2 * \Delta y - \Delta x$

$= 2 * 1 - 4 = -2$

Initial value of:

$\Delta E = 2 * \Delta y = 2$

$\Delta NE = 2(\Delta y - \Delta x) = 2(1-4) = -6$

Now, we use the above calculated value to complete the table, using Bresenham algorithm.

We know if $d < 0$, then ~~d += ΔE~~ update $d += \Delta E$ and $x = x+1$

else update $d += \Delta NE$ and $y = y+1$ and $x = x+1$.

Step	x	y	d	decision (E or NE)
0	1	-3	-2	E
1	2	-3	0	NE
2	3	-2	-6	E
3	4	-2	-4	E
4	5	-2	-2	END

$d = d + \Delta E$
$= -2 + 2$
$= 0$

$d = d + \Delta NE$
$= 0 + (-6) = -6$

$d = d + \Delta E$
$= -6 + 2 = -4$

$d = -4 + 2 = -2$

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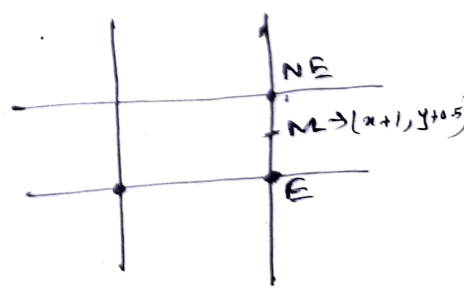
Exercise 2

(a) $y = f(x) = ax^2 + bx + c$

$f(x, y) = y - f(x) = y - ax^2 - bx - c$

Now, for decision function $d(x, y)$ we calculate $f(x+1, y+0.5)$ as decision variable is calculated at the midpoint of the next pixel.



so, $d(x, y) = f(x+1, y+0.5)$

$= (y + 0.5) - f(x+1)$

$= y + 0.5 - (a(x+1)^2 + b(x+1) + c)$

$\cancel{y+0.5 + y/a}$

Now for, $\Delta y = y_{i+1} - y = f(x+1) - f(x)$ $\left[\text{As } y = f(x)\right]$

so, $f(x+1) = f(x) + \Delta y$.

Now, using the equation

$f(x+1) = f(x) + \Delta y$ in $d(x, y)$

$d(x,y) = y + 0.5 - f(x) - \Delta y$

we know, $y = f(x)$

then, $d(x, y) = f(x) + 0.5 - f(x) - \Delta y$

$d(x, y) = 0.5 - \Delta y$

We can use the above formula to calculate decision function, but further solving for Δy

$$\Delta y = f(x+1) - f(x)$$

$$= a(x+1)^2 + b(x+1) + c - ax^2 - bx - c$$

$$= ax^2 + 2ax + a + bx + b + c - ax^2 - bx - c$$

$$\boxed{\Delta y = 2ax + a + b}$$

so, $d(x, y) = 0.5 - (2ax + a + b)$

~~Similarly we solve for $\Delta E = \Delta y$ and $\Delta NE = \Delta y - \Delta x$~~

so, if $d(x, y) < 0$, we decide to go to East E

and if $d(x, y) >= 0$, we decide to go to North East NE.

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2 (b) We know that

$$d(x, y) = f(x+1, y+0.5)$$

$$= y + 0.5 - f(x+1)$$

Now, we solve for Δy_1, Δy_2 and we know $y = f(x)$

$$\Delta y_1 = f(x+1) - f(x)$$

$$\Delta y_2 = f(x+2) - f(x+1)$$

$$\Delta^2 y = \Delta y_2 - \Delta y_1$$

$$\Rightarrow \Delta^2 y = f(x+2) - f(x+1) - \Delta y_1$$

$\Rightarrow$ Now using $f(x+1) = \Delta y_1 + f(x)$ we get

$$\Delta^2 y = f(x+2) - \Delta y_1 - f(x) - \Delta y_1$$

$$= f(x+2) - 2\Delta y_1 - f(x)$$

In 2 (a) we have derived the value of $\Delta y_1 = 2ax + a + b$

So, we use the derived value of Δy_1 in above equation.

So,

$$\Delta^2 y = f(x+2) - 2[2ax + a + b] - f(x)$$

$$\Rightarrow \Delta^2 y = a(x+2)^2 + b(x+2) + c - [4ax + 2a + 2b] - [ax^2 + bx + c]$$

$$\Rightarrow \Delta^2 y = ax^2 + 4ax + 4a + bx + 2b + c - 4ax - 2a - 2b - ax^2 - bx - c$$

$$\Rightarrow \Delta^2 y = 2a.$$

As,

$$\Delta^2 y = \Delta y_2 - \Delta y_1$$

$$\Rightarrow \Delta y_2 = \Delta^2 y + \Delta y_1$$

$$\Rightarrow \Delta y_2 = 2a + 2ax + a + b$$

So, $$\Delta y_2 = 2ax + 3a + b.$$

We now have the values of Δy_1, Δy_2 and $\Delta^2 y$.

Similar to $d(x, y)$ we compute

$d(x+1, y)$

$$= f(x+2, y+0.5) = y + 0.5 - f(x+2)$$

And, $d(x+1, y+1)$

$$= f(x+2, y+1.5) = y + 1.5 - f(x+2)$$

Now, using computed values of $d(x,y)$, $d(x+1,y)$ and $d(x+1,y+1)$

So, $\Delta E_{(x,y)} = d(x+1, y) - d(x, y)$

$$= y + 0.5 - f(x+2) - y - 0.5 + f(x+1)$$

$$= -[f(x+2) - f(x+1)]$$

$$= -\Delta y_2$$

So, $\Delta E_{(x,y)} = -[2ax + 3a + b]$

Similarly computing $\Delta NE_{(x,y)} = d(x+1, y+1) - d(x, y)$

$$= y + 1.5 - f(x+2) - y - 0.5 + f(x+1)$$

$$= 1 - [f(x+2) - f(x+1)]$$

$$= 1 - \Delta y_2$$

~~So, ΔNE_1~~ So, $\Delta NE_{(x,y)} = 1 - [2ax + 3a + b]$

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