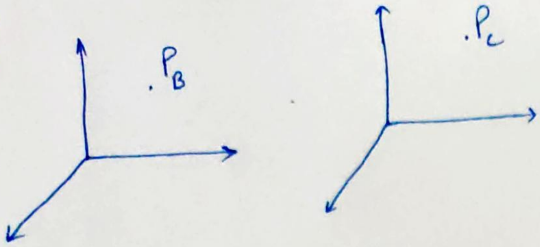
Q1)

a)



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$$P_{WC} = [b_1\ b_2\ b_3] P_B$$

$$P_{WC} = [C_1\ C_2\ C_3] P_C$$

so, $[b_1\ b_2\ b_3] P_B = [C_1\ C_2\ C_3] P_C$

$$\Rightarrow P_c = [C_1\ C_2\ C_3]^{-1} [b_1\ b_2\ b_3] P_B$$

so, $M_1 = [C_1\ C_2\ C_3]^{-1} [b_1\ b_2\ b_3]$ ✓

This is derived from the fact that, multiplying a coordinate vector with a matrix can be interpreted as the transformation of basis for that coordinate.

b) ? −1

c) For orthonormal bases:

$$\|b_1\| = \|b_2\| = \|b_3\| = 1$$

Using dot product with angular expression;

$$b_1 \cdot b_2 = \|b_1\| . \|b_2\| . \cos 90^\circ = 0$$

$$b_1 \cdot b_1 = \|b_1\| . \|b_1\| \cos 0^\circ = 1$$

and $b_1 \cdot b_3 = \|b_1\| . \|b_3\| \cos 90^\circ = 0$

so, for orthonormal basis vectors where I is (3×3) identity matrix

$$B^T . B = I_{3\times3} = B . B^T$$

$$B^T . B = I$$

$$\Rightarrow B^T . B . B^{-1} = B^{-1}$$

$$\Rightarrow B^T = B^{-1} \cdot \quad [B . B^{-1} = I]$$ ✓

d) ? −2

e) From change of basis derived in (a).

$$P_C = [C_1\ C_2\ C_3]^{-1} [b_1\ b_2\ b_3] P_B$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

From 1(c), we get for orthonormal bases, $C^T = C^{-1}$

so, $$C^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

Hence, $P_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & -1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

✓

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Q2) $P_i = \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}$ for any i is a point.

a) $P_1 - 2P_2 + P_3$

This results in a vector. As, in extended coordinates

$a_1\lambda_1 + a_2\lambda_2 + a_3\lambda_3$

$= 1\cdot 1 + (-2)\cdot 1 + 1\cdot 1$

$= 0$ $\rightarrow$ Hence, it's a vector

b) $P_0 + \sum_{i=1}^{n} (P_i - P_0)$

This results in a point. As,

$\lambda_0 + \sum_{i=1}^{n} (\lambda_i - \lambda_0)$

$= 1 + \sum_{i=1}^{n} (1 - 1)$

$= 1$ $\rightarrow$ Hence, it's a ~~vecto~~ point.

c) $\alpha P_0 + \beta P_1$, where $\alpha, \beta \in \mathbb{R}$

This is not geometrically meaningful. As,

$\alpha\lambda_0 + \beta\lambda_1$

$= \alpha\cdot 1 + \beta\cdot 1$

$= \alpha + \beta$ which is not defined properly

d) $\alpha P_0 + \beta P_1 + (1-\alpha-\beta)P_2$, where $\alpha, \beta \in \mathbb{R}$

This yields a point. As,

$\alpha\lambda_0 + \beta\lambda_1 + (1-\alpha-\beta)\lambda_2$

$= \alpha\cdot 1 + \beta\cdot 1 + (1-\alpha-\beta)\cdot 1$

$= \alpha + \beta + (1-\alpha-\beta)$

$= 1$ $\rightarrow$ Hence, it's a point.

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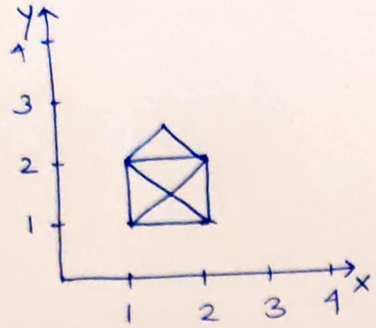
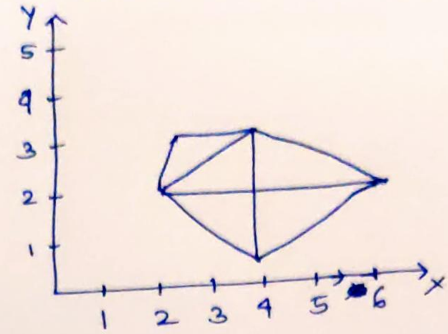
Q3)
a)



$\rightarrow$



The order of transformation is

$$Tr(2) . S. R. Tr(1)$$

so, $Tr(1) = \begin{pmatrix} 1 & 0 & -1.5 \\ 0 & 1 & -1.5 \\ 0 & 0 & 1 \end{pmatrix}$ $\rightarrow$ Moving house to center

$$R = \begin{pmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ $\rightarrow$ Rotate by 45°

$$S = \begin{pmatrix} \frac{4}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{2}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ $\rightarrow$ Scaling the house

$$Tr(2) = \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$ $\rightarrow$ Moving to final position

so, final transformation matrix:

$$Tr(F) = Tr(2) . S . R . Tr(1) = \begin{pmatrix} 2 & -2 & 4 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

b) Transformation of origin & basis by above mapping:

$$Tr(F). e_1 = \begin{pmatrix} 2 & -2 & 4 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$Tr(F). e_2 = \begin{pmatrix} 2 & -2 & 4 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

$$Tr(F). e_3 = \begin{pmatrix} 2 & -2 & 4 \\ 1 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} . \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$$