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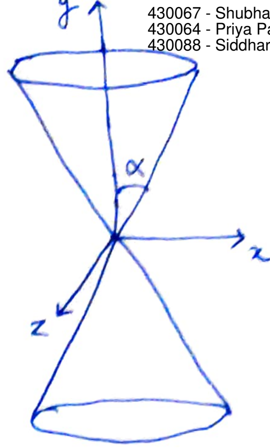
Q1) a)




b) Here, the radius of cone is $y \tan\alpha$

Now, $F(x,y,z) = \cancel{x^2+y}$
$$= x^2+z^2-r^2$$
$$= x^2+z^2-(y\tan\alpha)^2$$
$$= x^2+z^2-y^2\tan^2\alpha$$

c) The matrix for the quadric is
$F(x,y,z) = x^2+z^2-y^2\tan^2\alpha$

So,

Here, $\begin{matrix} x \\ y \\ z \end{matrix}$ $(x,y,z,1) \cdot Q_c \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$

$$= \sum Q_c \cdot \begin{pmatrix} x^2 & xy & xz & x \\ xy & y^2 & yz & y \\ xz & yz & z^2 & z \\ x & y & z & 1 \end{pmatrix}$$

$$\Rightarrow Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\tan^2\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

d) $$Q = \begin{pmatrix} 2a & b & c & d \\ b & 2e & f & g \\ c & f & 2h & i \\ d & g & i & 2j \end{pmatrix}$$

So,
$$F(x,y,z) = (x,y,z,1) \cdot Q \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$= 2ax^2 + bxy + cxz + dx + bxy + 2ey^2 + fyz + yg + cxz + fyz + 2hz^2 + iz + dx + gy + iz + 2j$$

$$\frac{\partial F}{\partial x} = 4ax + by + cz + d + by + cz + d \quad = 4ax + 2by + 2cz + 2d$$

$$\frac{\partial F}{\partial y} = 2bx + 4ey + 2fz + 2g$$

$$\frac{\partial F}{\partial z} = 2cx + 2fy + 4hz + 2i$$

so,

$$G = \begin{pmatrix} 4a & 2b & 2c & 2d \\ 2b & 4e & 2f & 2g \\ 2c & 2f & 4h & 2i \end{pmatrix}$$

e)

$$F_e(x,y,z) = x(x-2) + y(y-4) + 3z(2\sqrt{3} - z) - 4 = 0$$

$$= x^2 - 2x + y^2 - 4y + 6\sqrt{3}z - 3z^2 - 4 = 0$$

So, here

$$Q_e = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -3 & 3\sqrt{3} \\ -1 & -2 & 3\sqrt{3} & -4 \end{pmatrix}$$

f) Normal at $p = (1, 5, 2\sqrt{3}, 1)^T$ of $Q_e$

$$n(x,y,z) = \frac{\nabla F(x,y,z)}{\|\nabla F(x,y,z)\|}$$

$$= \frac{G.P}{\|G.P\|}$$

$$\cancel{G.P = \begin{pmatrix} 4 & 0 & 0 & -4 \\ 0 & 4 & 0 & -8 \\ 0 & 0 & -12 & 12\sqrt{3} \end{pmatrix} . \begin{pmatrix} 1 \\ 5 \\ 2\sqrt{3} \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \\ 12\sqrt{3} \end{pmatrix}}$$

$$\cancel{\|G.P\| = \sqrt{8^2 + 12^2 + (12\sqrt{3})^2}}$$

$$=$$

$$G = \begin{pmatrix} 2 & 0 & 0 & -2 \\ 0 & 2 & 0 & -4 \\ 0 & 0 & -6 & 6\sqrt{3} \end{pmatrix}, \quad P = \begin{pmatrix} 1 \\ 5 \\ 2\sqrt{3} \\ 1 \end{pmatrix}$$

$$G.P = \begin{pmatrix} 0 \\ 6 \\ -6\sqrt{3} \end{pmatrix} \quad \text{and} \quad \|G.P\| = \sqrt{6^2 + (6\sqrt{3})^2} = 12$$

$$\text{so, } n = \begin{pmatrix} 0 \\ 1/2 \\ -\sqrt{3}/2 \end{pmatrix}$$

Excercise 2!

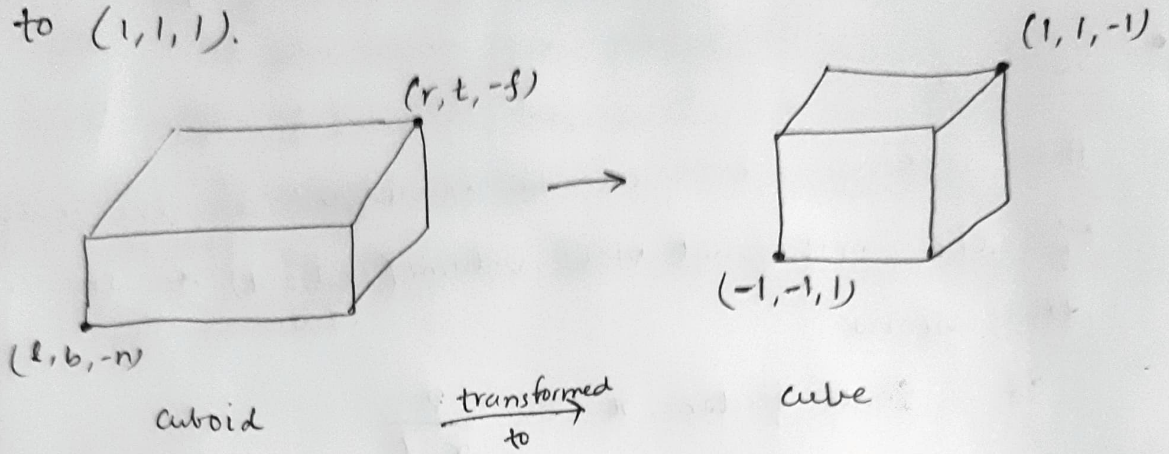
(a) If both, the near and the far plane are parallel to the image plane, the viewing frustum of an orthogonal projection takes the geometric shape of a cube.

The resulting shape is a cube due to the parallel projection. As we move the projection center to infinity, the four edges of frustum, intersecting at the projection center, now become parallel. So, the resulting viewing frustum distort to a cube formed by far and near plane being parallel.

Exercise 2

(b) for an orthogonal projection, the view volume is a cuboid, specified as $(l, r, b, t, n, f)$

we need to transform it to a cube from $(-1,-1,1)$ to $(1,1,1)$.




This transformation is a parallel projection and can be done with the help of translation and scaling.

In such a projection :

- <u>Translation</u> :

Translation lines up midpoints

So, midpoint of $x = \frac{r+l}{2}$

midpoint of $y = +\frac{(t+b)}{2}$

mid of $z = +\frac{(f+n)}{2}$

So, the translation factors become,

$$\begin{array}{c} x \\ y \\ z \end{array} \approx \begin{pmatrix} -(r+l)/2 \\ -(t+b)/2 \\ +(f+n)/2 \end{pmatrix}$$

So,

$$\text{Translation Matrix } (T) = \begin{bmatrix} 1 & 0 & 0 & -(r+l)/2 \\ 0 & 1 & 0 & -(t+b)/2 \\ 0 & 0 & 1 & -(f+n)/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Scaling**

The scaling factor can be considered as the ratio of the orthogonal view volume (cuboid) to the cube dimensions.

So, Scale factor, $x = \frac{2}{r-l}$

$sf, y = \frac{2}{t-b}$

$sf, z = \frac{2}{f-n}$

$$\text{Scaling Matrix } (S) = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & \frac{2}{f-n} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Combining T and S, final projection matrix M

$$M = \cancel{TS}\ S \cdot T = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$