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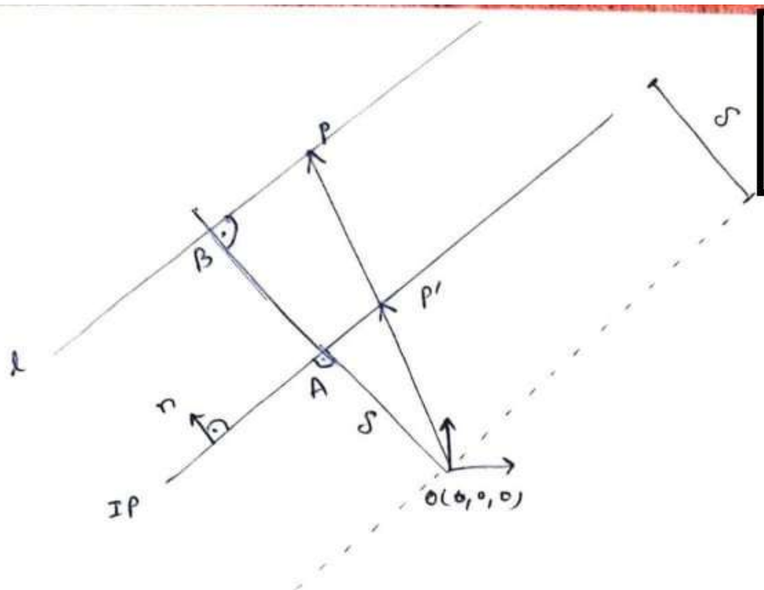
Exercise 1

(a)



⇒ we draw a line OB perpendicular to image plane such that OA = δ and line l is $\parallel$ to IP.

⇒ We observe that $\Delta\, OAP' \sim \Delta\, OBP$
as $\angle OAP' = \angle OBP = 90°$ and $\angle P'OA$ and $\angle POB$ are common angles.

By property of similar Δs,

$$\boxed{\frac{OP'}{OP} = \frac{\delta\ (OA)}{OB}} \quad -(1)$$

Now, OB is the projection of vector $\bar{P}$ on the normal of IP as OB $\perp$ to IP.

So, $\boxed{OB = \bar{n}^T . P}$, as $\|\bar{n}\| = 1$ $\quad -(11)$

from (1) and (11) we have,

$$\frac{OP'}{OP} = \frac{\delta}{n^T.P}$$

$$\Rightarrow P' = \frac{\delta}{n^T.P} P \quad [\text{as } O = (0,0,0)]$$

$$\Rightarrow \boxed{P' = \frac{P}{\frac{n^T P}{\delta}}}$$

✓

We observe that the point P' is formed by scaling the vector $\bar{P}$ by a factor of $\frac{n^T P}{\delta}$.

In Homogeneous co-ordinate system

$$\text{if } P = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \quad \text{then } P' = \begin{pmatrix} x \\ y \\ z \\ \frac{n^T P}{\delta} \end{pmatrix}$$

Assuming the projection matrix as M, this can be written as:

$\boxed{MP = P'}$ where M is a 4x4 matrix

Since there is no change in linear and translation part, the eqn becomes:

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline n_1 & n_2 & n_3 & 0 \end{array}\right) \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ \frac{n^T P}{\delta} \end{pmatrix}$$

where we have to calculate n_1, n_2 and n_3

$$\Rightarrow \begin{bmatrix} 1 \cdot x + 0 \cdot y + 0 \cdot z + 0 \cdot 1 \\ 0 \cdot y + 1 \cdot y + 0 \cdot z + 0 \cdot 1 \\ 0 \cdot x + 0 \cdot y + 1 \cdot z + 0 \cdot 1 \\ n_1 x + n_2 y + n_3 z + 0 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ \frac{n^T P}{\delta} \end{bmatrix} \quad \left\{ \begin{array}{l} \text{we know} \\ n = \begin{bmatrix} 0 \\ -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix} \end{array} \right.$$

$$\begin{bmatrix} x \\ y \\ z \\ n_1 x + n_2 y + n_3 z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ \frac{0}{\delta} + -\frac{\sqrt{2}}{2\delta} y + \frac{-\sqrt{2}}{2\delta} z \end{bmatrix}$$

Equating corresponding parts of the matrices, we have,

$$n_1 = 0, \quad n_2 = \frac{-\sqrt{2}}{2\delta}, \quad n_3 = \frac{-\sqrt{2}}{2\delta}$$

So, our final projection matrix becomes,

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & \frac{-\sqrt{2}}{28} & \frac{-\sqrt{2}}{28} & 0 \end{array}\right]$$

(b) We are given the line $\alpha(\lambda) = o + \lambda d$. Let the co-ordinates of o as $(x, y, z)^T$ and direction coordinates $(dx, dy, dz)^T$

So, any point on line is given by :

$$\alpha(\lambda) = \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \lambda \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} x + \lambda dx \\ y + \lambda dy \\ z + \lambda dz \end{bmatrix}$$

Projection on the plane, $n : \left(0, -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)^T$ can be given as,

Projected Point, $p' = P \cdot \alpha(\lambda)$

$$p' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x + \lambda dx \\ y + \lambda dy \\ z + \lambda dz \\ 1 + 0 \end{bmatrix}$$

$\left\{ \begin{array}{l} \text{homogeneous coordinates} \\ 1 \text{ for point,} \\ 0 \text{ for vector} \end{array} \right.$

$$p' = \begin{bmatrix} x + \lambda dx \\ y + \lambda dy \\ z + \lambda dz \\ -(y + \lambda dy) - (z + \lambda dz) \end{bmatrix}$$

Now, dehomogenising the projected point p' to Euclidean coordinates, we have,

$$p' = \begin{bmatrix} -(x + \lambda dx) / [(y + z) + \lambda(dy + dz)] \\ -(y + \lambda dy) / [(y + z) + \lambda(dy + dz)] \\ -(z + \lambda dz) / [(y + z) + \lambda(dy + dz)] \end{bmatrix}$$

factoring out $1/\lambda$, we have, in all terms:

$$p' = \begin{bmatrix} -\dfrac{\cancelto{0}{\frac{x}{\lambda}} + dx}{\cancelto{0}{\frac{(y+z)}{\lambda}} + (dy+dz)} \\[2ex] -\dfrac{\cancelto{0}{\frac{y}{\lambda}} + dy}{\cancelto{0}{\frac{(y+z)}{\lambda}} + (dy+dz)} \\[2ex] -\dfrac{\cancelto{0}{\frac{z}{\lambda}} + dz}{\cancelto{0}{\frac{(y+z)}{\lambda}} + (dy+dz)} \end{bmatrix}$$

as we compute vanishing point $\lambda \to \infty$.
Hence $\frac{1}{\lambda} \to 0$.

The above equation then becomes,

$$p' = \begin{bmatrix} -\dfrac{dx}{dy+dz} \\[2ex] -\dfrac{dy}{dy+dz} \\[2ex] -\dfrac{dz}{dy+dz} \end{bmatrix}$$

which is the V.P. of line: $0 + \lambda d$ in 3D.

(C) In the above equation, the VP only exists if the terms in the denominator are NOT equal to 0.

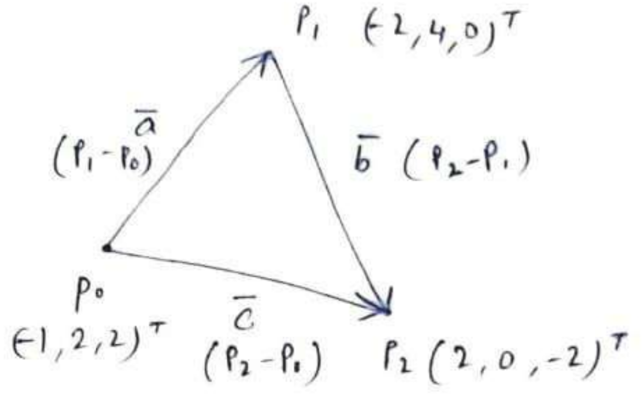
$$\Rightarrow \quad dy + dz \neq 0$$

$$\Rightarrow \quad \boxed{dy \neq -dz} \rightarrow \text{VP exists}$$

$$\text{if} \quad \boxed{dy = -dz} \rightarrow \text{VP doesn't exist}$$

Here dy and dz represent the direction of line in y and $+z$ direction

(d) ~~Ato to~~ According to question,



We draw vectors $\bar{a}, \bar{b}$ and $\bar{c}$.

(i).. For edge, P_0P_1 specified by $\bar{a}$, the eqn of line can be written as,

$$L_{P_0P_1} = \begin{bmatrix} -1 + \lambda a_x \\ 2 + \lambda a_y \\ 2 + \lambda a_z \end{bmatrix}$$

where
$a_x = (-2+1) = -1$
$a_y = 4 - 2 = 2$
$a_z = 0 - 2 = -2$

Now, from our formula derived in (b), $\left\{ p' = \begin{bmatrix} -dx/dy+dz \\ -dy/dy+dz \\ -dz/dy+dz \end{bmatrix} \right.$

Since Projection matrix P is same,

we get vanishing point of $L_{P_0P_1}$ as:

$$P'_{P_0P_1} = \begin{bmatrix} \frac{+1}{2+(-2)} \\ \frac{-2}{2+(-2)} \\ \frac{2}{2+(-2)} \end{bmatrix} = \begin{bmatrix} 1/0 \\ -2/0 \\ 2/0 \end{bmatrix}$$

Divide by zero!!! No VP exists

Also, from (c) if $\boxed{dy = -dz}$ VP does not exist

So, No VP exists for edge P_0P_1 of triangle.

(ii)... for edge, P_1P_2, specified by $\bar{b}$, the eqn of line can be written as:

$$L_{P_1P_2} = \begin{bmatrix} -2 + \lambda b_x \\ 4 + \lambda b_y \\ 0 + \lambda b_z \end{bmatrix}$$

where
$b_x = 2-(-2) = 4$
$b_y = 0-4 = -4$
$b_z = -2-0 = -2$

Now, from our formula in (b) ..

$$P'_{P_1P_2} = \begin{bmatrix} \frac{-4}{-4-2} \\ \frac{+4}{-4-2} \\ \frac{+2}{-4-2} \end{bmatrix} = \begin{bmatrix} 2/3 \\ -2/3 \\ -1/3 \end{bmatrix}$$

So, vp for edge P_1P_2 exists at $[2/3 \;\; -2/3 \;\; -1/3]^T$.

(iii)... for edge P_0P_2, specified by $\bar{c}$, the eqn of line can be written as

$$L_{P_0P_2} = \begin{bmatrix} -2 + \lambda c_x \\ 4 + \lambda c_y \\ 0 + \lambda c_z \end{bmatrix}$$

where
$c_x = 2-(-1) = 3$
$c_y = 0-2 = -2$
$c_z = -2-2 = -4$

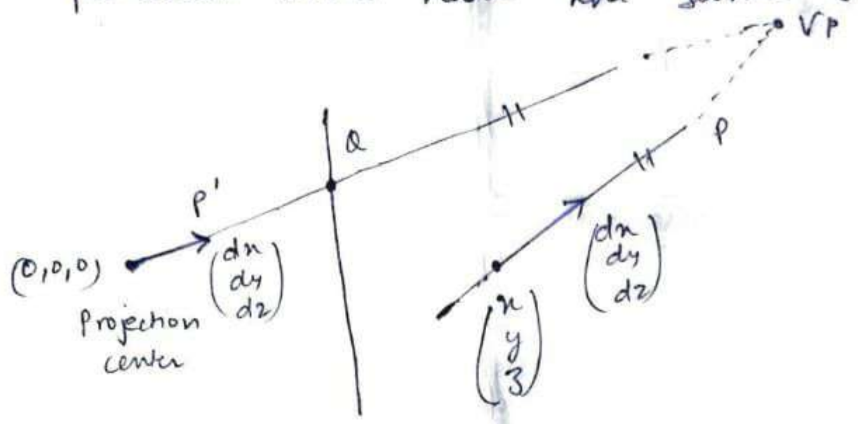
Now, from our formula in (b)..

$$P'_{P_0P_2} = \begin{bmatrix} -3/-2-4 \\ +2/-2-4 \\ +4/-2-4 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/3 \\ -2/3 \end{bmatrix}$$

So, vp for edge P_0P_2, exists at $[1/2, -1/3, -2/3]^T$

(e) The vanishing point of a line given a plane does not depend upon the starting point.

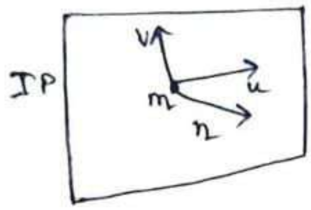
All parallel lines have the same VP.



Here the line P with point $(x, y, z)^T$ and direction $(dx, dy, dz)^T$ has the same VP as the line P' originating from $(0,0,0)^T$ and having direction as $(dx, dy, dz)^T$.

The VP of line P can be found by the intersection point of line P' with Image plane at Q.

(f)



translating point $m\,(x\ y\ z\ w)^T$ to origin we derive translation matrix:

$$M_t = \begin{pmatrix} 1 & 0 & 0 & 0-x \\ 0 & 1 & 0 & 0-y \\ 0 & 0 & 1 & 0-z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & -x \\ 0 & 1 & 0 & -y \\ 0 & 0 & 1 & -z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

So, we derive the M_t which can translate m to origin, such as

$$m' = M_t \cdot m$$

Now, we assume we have $\vec{u}$ as $\begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$, $\vec{v}$ as $\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$ and $\vec{n}$ as $\begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}$.

We know that for orthonormal basis $B^T = B^{-1}$ and to transform basis we use formula

$$P' = B^{-1} C P = B^T C m'$$ (m' is the translated point at origin)

let's B be the orthonormal vector with u, v and n vectors so, we get

$$B = \begin{pmatrix} u_x & v_x & n_x & 0 \\ u_y & v_y & n_y & 0 \\ u_z & v_z & n_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B^T = \begin{pmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_z & n_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

As, we are transforming standard basis to B basis.
Let's consider C to be standard basis matrix.

for standard basis:

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

so,

- $$P' = B^T C\, m'$$

$$= B^T m' \quad \left[\begin{array}{l}\text{as } C \text{ is an Identity matrix} \\ \text{of } 4\times 4\end{array}\right]$$

Now, using the value of $m' = M_t \cdot m$

$$P' = M \begin{pmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_z & n_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} m$$

$$\Rightarrow P' = \begin{pmatrix} 1 & 0 & 0 & -x \\ 0 & 1 & 0 & -y \\ 0 & 0 & 1 & -z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_z & n_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} m$$

$$\Rightarrow P' = \begin{pmatrix} u_x & u_y & u_z & -x \\ v_x & v_y & v_z & -y \\ n_x & n_z & n_z & -z \\ 0 & 0 & 0 & 1 \end{pmatrix} m$$

This provides me a 4×4 matrix is 3D,
Now dropping coordinate in n direction we get 3×4 matrix as

$$\boxed{M_{3\times 4} = \begin{pmatrix} u_x & u_y & u_z & -x \\ v_x & v_y & v_z & -y \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

$u^T m$
$v^T m$

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Q2)

a) Translation by 10 units along x-axis:

$$T = \begin{pmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Scaling by (1, 4, 16)

$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Rotation by 45° around y-axis

$$R = \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

So, the final transformation matrix:

$$T_r = T.S.R$$

$$= \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 10 \\ 0 & 4 & 0 & 0 \\ -16/\sqrt{2} & 0 & 16/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b) For the below transformations, the corresponding transformation matrices can be used as is:

i) Rotation

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ii) Translation

iii) Uniform scaling

These are angle preserving transformations.

c) After transforming a point with above transformation matrix T_r:

$$P_{t_1} = T_r . P_t$$

Before transforming, the normal is

$$n^T . P_t = 0$$

After

$$n'^T . P_{t_1} = 0$$

$$\Rightarrow n'^T . T_r . P_t = 0$$

So, $n^T . P_t = n'^T . T_r . P_t$

$$\Rightarrow n^T = n'^T . T_r$$

$$\Rightarrow n' = (T_r)^{-1} . n$$

Here, $N = (Tr^T)^{-1} = ((T.S.R)^T)^{-1} = (R^T . S^T . T^T)^{-1} = (T^T)^{-1} . (S^T)^{-1} . (R^T)^{-1}$

so,

$$n' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 1/16 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1/4 & 0 & 0 \\ \frac{-1}{16\sqrt{2}} & 0 & 1/16\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$P_1 - P_3 = \begin{pmatrix} -2 \\ 6 \\ -4 \end{pmatrix}$$

$$P_2 - P_3 = \begin{pmatrix} -2 \\ 6 \\ -4 \end{pmatrix}$$

Here, normal is

$$n = \frac{(P_1 - P_3) \times (P_2 - P_3)}{\|(P_1 - P_3) \times (P_2 - P_3)\|}$$

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