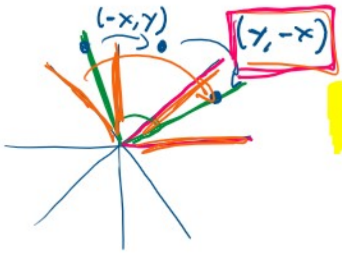
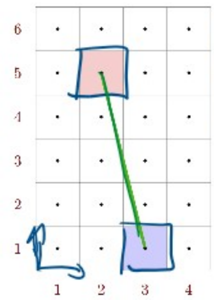
Assignment 5



Exercise 1 Line Rasterization [10 Points]

In this task you are going to rasterize the line in the figure below using the Bresenham algorithm introduced in the lecture. Keep in mind that the grid represents a set of pixels where the center of each pixel is marked by a dot and has the integer coordinates $(x, y)^\top$.




```
Δx = x₁ - x₀;
Δy = y₁ - y₀;
d = 2*Δy - Δx;
Δ_E = 2*Δy;
Δ_NE = 2*Δy - 2*Δx;

set_pixel(x₀, y₀);

for (x = x₀, y = y₀; x < x₁;)
{
  if (d < 0)  { d += Δ_E;   ++x;      }
  else        { d += Δ_NE;  ++x; ++y  }
  set_pixel(x, y);
}
```

(a) Case Reduction [3 Points]

Our line segment starts at $x_0 = (3, 1)^\top$ and ends at $x_1 = (2, 5)^\top$. **Specify the map** $(x, y)^\top \to (\ldots, \ldots)^\top$ that transforms the line into a setting with slope $m \in [0, 1]$, as seen in the lecture. **Apply this map** to x_0 and x_1.

1

a) $(x,y) \to (y,-x)$

$x_0 = (3,1) \to (1,-3)$

$x_1 = (2,5) \to \boxed{(5,-2)}$

(b) Bresenham Algorithm [7 Points]

Compute the **initial value of the decision variable** d, as well as the **updates** Δ_E and Δ_{NE} using the formulas derived in the lecture.

Then, execute the Bresenham algorithm by hand and **fill all blanks** in the following table. Make sure to use integer arithmetic only.

Hint: To check your result, you can apply the inverse transformation from part (a) to each point, and complete the above figure.

b) $\Delta x = 4$

$\Delta y = 1$

slope $m = \frac{\Delta y}{\Delta x} = \frac{1}{4} \in [0,1]$ ✓

$d = 2\Delta y - \Delta x = 2 - 4 = \boxed{-2}$

$\Delta E = 2\Delta y = \boxed{2}$

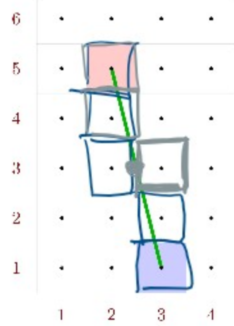
$\Delta NE = 2\Delta y - 2\Delta x = \boxed{-6}$

$\boxed{d \leq 0 \; ? \; E : NE}$

Step	x	y	d	decision (E or NE)
0	1	-3	-2	E
1	2	-3	0	NE E
2	3	-2	-6	E NE
3	4	-2	-4	E E
4	5	-2	-2	End

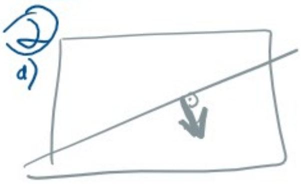
inv. trasf.

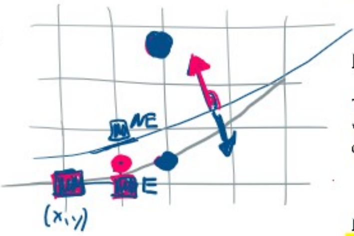
x	y
3	1
3	2
2	3
2	4
2	5



2

d)





Exercise 2 Rasterization of Quadratic Polynomials [10 Points]

The idea of the Bresenham algorithm, *Digital Differential Analysis* (DDA), can be generalized to the step-wise evaluation of arbitrary polynomials. In this task you will adapt this algorithm to the rasterization of quadratic polynomials that have the form

$$\boxed{y = f(x) = ax^2 + bx + c}$$

For this question, we assume that the polynomial is rasterized for $x \in \{x_0, x_0 + 1, \ldots, x_0 + n\}$, and that $0 \leq f'(x) \leq 1$, for all $x \in [x_0, x_0 + n]$.

By re-arranging terms, we can turn the above parametric formulation into an implicit one, telling us whether a point lies above, on, or below the 2D curve:

$$F(x, y) = y - f(x) = y - ax^2 - bx - c.$$

>0 above
<0 below

(a) Decision Variable [4 Point]

Suppose that we finished drawing the pixel $(x, y)^\top$. For the next pixel $x + 1$, the algorithm will have to choose between either east (E) or north east (NE). **Give a formula** for the decision function $d(x, y)$ and **explain** how $d(x, y)$ can be used to decide whether to go east or north east.

a)

$d(x,y) = F(x+1, y+\frac{1}{2})$

$= \boxed{y + \frac{1}{2} - a(x+1)^2 - b(x+1) - c}$

$\boxed{d(x,y) < 0 \; ? \; NE : E}$

(b) Decision Variable Updates [6 Point]

Instead of computing $d(x, y)$ for each pixel from scratch, we can use DDA to compute $d(x + 1, y)$, or $d(x + 1, y + 1)$ for the next pixel as an increment of $d(x, y)$. **Derive** the increment for both cases, i.e. $\Delta_E(x, y)$ and $\Delta_{NE}(x, y)$. Note that both expressions are now linear functions in x and y.

You can stop here, i.e. you don't have to compute the constant updates for those linear updates in this task. Also, it is okay that your expressions contain non-integer arithmetic.

b) $\Delta E(x) = d(x+1, y) - d(x,y)$

$= F(x+2, y+\frac{1}{2}) - F(x+1, y+\frac{1}{2})$

$= \cancel{y+\frac{1}{2}} - a(x+2)^2 - b(\cancel{x}+2) - \cancel{c}$

$- (\cancel{y+\frac{1}{2}} - a(x+1)^2 - b(\cancel{x}+1) - \cancel{c})$

$= -a\left((x+2)^2 - (x+1)^2\right) - b \cdot 1$

$= -a(\cancel{x^2} + 4x + 4 - \cancel{x^2} - 2x - 1) - b$

$= -a(2x+3) - b$

$$= -a\left(\cancel{x^2} + 4x + 4 - \cancel{x^2} - 2x - 1\right) - b$$

$$= -a(2x+3) - b$$

$$\Delta NE(x) = d(x+1, y+1) - d(x, y)$$

$$= F\left(x+2, y+\frac{3}{2}\right) - F\left(x+1, y+\frac{1}{2}\right)$$

$$= \cancel{y} + \boxed{\frac{3}{2}} - a(x+2)^2 - b(x+2) - \cancel{c}$$

$$-\left(\cancel{y} + \boxed{\frac{1}{2}} - a(x+1)^2 - b(x+1) - \cancel{c}\right)$$

$$= \boxed{-a(2x+3) - b \qquad +1}$$