Prof. Dr. Leif Kobbelt
Gregor Kobsik, Tim Elsner

Visual Computing Institute | RWTH AACHEN UNIVERSITY

Theoretical Exercise 3

Date Published: May 10th 2023

This is the sample solution.

Exercise 1 Geodesic Distance

Given is a triangle with the 2D points $x_1 = (3,4)^T, x_2 = (5,2)^T, x_3 = (1,1)^T$. We already know the distances $d_1 = 2, d_2 = 1$. What is d_3?

Solution

1. Move x_3 to the origin:
$\tilde{x_1} = (2,3)^T, \tilde{x_2} = (4,1)^T, \tilde{x_3} = (0,0)^T$

2. Combine points into matrix:
$X = [x_1, x_2] = \begin{pmatrix} 2 & 4 \\ 3 & 1 \end{pmatrix}, \vec{d} = (2,1)$
$Q = (X^TX)^{-1} = \begin{pmatrix} 13 & 11 \\ 11 & 17 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{17}{100} & \frac{-11}{100} \\ \frac{-11}{100} & \frac{13}{100} \end{pmatrix}$

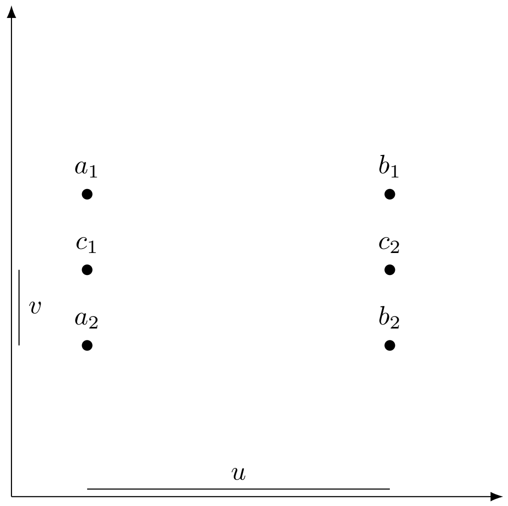
3. Solve equation system:
$$p^2 IQI^T - 2pIQ\vec{d}^T + \vec{d}Q\vec{d}^T = 1$$
$$\Leftrightarrow p^2 \frac{8}{100} - 2p\frac{14}{100} + \frac{37}{100} = 1$$
$$\Leftrightarrow p^2 - \frac{28}{8}p + \frac{37}{8} = \frac{100}{8}$$
$$\Leftrightarrow (p - \frac{14}{8})^2 = \frac{100}{8} - \frac{37}{8} + \frac{196}{64}$$
$$\Leftrightarrow (p - \frac{14}{8})^2 = \frac{175}{16}$$
$$\Leftrightarrow p =^{+}_{-} \sqrt{\frac{175}{16}} + \frac{14}{8}$$
$$\Leftrightarrow p \approx^{+}_{-} 3.31 + \frac{14}{8}$$
$$\Leftrightarrow p \approx -1.56, 5.06$$

Distance value is the bigger of the two solutions: 5.06

Exercise 2 Shape Distances

Give an example of three point clouds, where the triangle inequality is not fulfilled for the Chamfer distance.

Solution



$dist(AB) = \frac{2u}{2} + \frac{2u}{2} = 2u$
$dist(AC) = dist(CB) = \frac{2h}{2} + \frac{h+\sqrt{h^2+u^2}}{2}$
$dist(AC) + dist(CB) = 3h + \sqrt{h^2+u^2}$ goes to u if h goes to zero
$\Rightarrow dist(AC) + dist(CB) = u < 2u = dist(AB)$

Exercise 3 Histogram Distances

You are given two datasets of shapes P and Q, where each shape is represented by a shape descriptor (we ignore questions of transformation invariance). Assume we want to know how similar these datasets are. What measures do you know for this? What are the benefits and downsides?

Solution

Histograms:
- we need to find reasonable bins.
- problematic in high dimensions.

EMD:
+ takes distance into account
- expensive to compute

KLD:
- not a real distance
- weird scale
- does not take distance into account
- is not symmetric (unintuive). This is fixed by JSD

Parametric:
Frechet distance:
+ no problem with binning
- assumes Gaussian distribution (in general not true)

Non-parametric:
+ no assumptions over distribution and distance space

Coverage:
- not symmetric
- only regards nearest neighbour (brittle)
- absolute distance does not matter

Minimum Matching Distance:
- similar downsides to coverage
+ now takes distance into account