Forecasting the Equity Risk Premium

Deadline: 6 October 2025 at 23h59

Can we forecast the equity risk premium? In this **group assignment** you will analyze whether different candidate predictor variables can forecast the equity risk premium. You address this question in three parts: First, you analyze whether the predictor variables can be used to forecast the equity risk premium in-sample. Second, you analyze their predictive performance out-of-sample. Finally, you evaluate whether combining the variables improves the out-of-sample forecasting performance.

General Setup

The dataset *predictor_data.csv* contains the (quarterly) Welch and Goyal (2008) predictor variables. In this assignment, you use $N = 14$ out of the 15 variables considered by Welch and Goyal (2008) and Rapach et al. (2010) as predictor variables:

1. Log dividend-price ratio, D/P : Difference between the log of a 12 -month moving sum of dividends paid on the S&P 500 index and the log of stock prices (S&P 500 index).
2. Log dividend yield, D/Y: Difference between the log of a 12-month moving sum of dividends and the log of lagged stock prices.
3. Log earnings-price ratio, E/P: Difference between the log of a 12 -month moving sum of earnings on the S&P 500 and the log of stock prices.
4. Log dividend-payout ratio, D/E: Difference between the 12 month moving sums of dividends and earnings.
5. Stock Variance, SVAR: Monthly sum of squared daily returns on the S&P 500.
6. Book-to-market ratio, B/M: Ratio of the book value to the market value of the Dow Jones Industrial Average.
7. Net equity expansion, NTIS: Ratio of a 12-month moving sum of net equity issues by NYSElisted stocks to the total end-of-year market capitalization of NYSE stocks.
8. Treasury bill rate, TBL: Interest rate on a three-month treasury bill on the secondary market.
9. Long-term yield, LTY: Long-term yield of government bonds.
10. Long-term returns, LTR: Return on long-term government bonds.
11. Term spread, TMS: Difference between the long-term yield and the Treasury bill rate.
12. Default yield spread, DFY: Difference between BAA- and AAA-rated corporate bond yields.
13. Default return spread, DFR: Difference between long-term corporate bond and long-term government bond returns.

14. Inflation, INFL: Inflation is calculated from the CPI (all urban consumers). Following Welch and Goyal (2008) and Rapach et al. (2010) you use $x_{i,t-1}$ for inflation since the data are released in the following month.

The empirical framework is based on Welch and Goyal (2008), Rapach et al. (2010), and Rapach et al. (2016) and relies on the predictive regression model for the equity premium given by

$$r_{t+1} = \alpha_i + \beta_i x_{i,t} + \varepsilon_{t+1} \tag{1}$$

where r_{t+1} is the return of the stock market index in excess of the risk-free interest rate, $x_{i,t}$ is the predictive variable of interest, and ε_{t+1} denotes the disturbance term. Furthermore, i indexes the forecast variable of interest.

The Assignment

The assignment consists of the following tasks

1. Perform an in-sample estimation of the predictive regression in equation (1) for each $i = 1, \dots, N$. In your estimation, take the negative of the predictor variables NTIOS, TBL, LTY, and INFL. Test the null hypothesis $H_0 : \beta_i = 0$ against the one-sided alternative $H_A : \beta_i > 0$. Make sure to report the estimated coefficients, heteroscadesticity and autocorrelation consistent t-statistics and the corresponding p-values. Report the adjusted R^2 of your regressions. **(2.5 points)**

2. Perform and out-of-sample estimation of your model. Use an expanding window. For this part, divide the total sample of T observations into an in-sample period consisting of the first m observations, a holdout out-of-sample period consisting of p observations, and an out-of-sample period composed of the last q observations. In general, the equity premium forecast based on equation (1) is given by

$$\hat{r}_{i,t+1} = \hat{\alpha}_{i,t} + \hat{\beta}_{i,t} x_{i,t} \tag{2}$$

with $\hat{\alpha}_{i,t}$ and $\hat{\beta}_{i,t}$ denoting the OLS estimates of α_i and β_i, respectively, based on data up to and including period t. Thus, using the OLS estimates $\hat{\alpha}_{i,m+p}$ and $\hat{\beta}_{i,m+p}$ of α_i and β_i, respectively, the first out-of-sample forecast is given by

$$\hat{r}_{i,m+p+1} = \hat{\alpha}_{i,m+p} + \hat{\beta}_{i,m+p} x_{i,m+p} \tag{3}$$

In particular, $\{r_t\}_{t=2}^{m+p}$ is regressed on a constant, and $\{x_{i,t}\}_{t=1}^{m+p-1}$ following equation (1). The second out-of-sample forecast is generated similarly, by regressing $\{r_t\}_{t=2}^{m+p+1}$ on a constant and $\{x_{i,t}\}_{t=1}^{m+p}$ which yields

$$\hat{r}_{i,m+p+2} = \hat{\alpha}_{i,m+p+1} + \hat{\beta}_{i,m+p+1} x_{i,m+p+1} \tag{4}$$

Following this procedure, you obtain q out-of-sample forecasts $\{\hat{r}_t\}_{t=m+p+1}^{T}$. Set m to 20 years and p to 10 years in your implementation. In your forecast evaluation, you compare these forecasts with the historical average of the excess returns over the risk-free rate, which serves as a benchmark. In particular, the benchmark return in period $t+1$ is given by

$$\bar{r}_{t+1} = \frac{1}{t}\sum_{j=1}^{t} r_j \tag{5}$$

To evaluate the out-of-sample performance of the different forecasts, you use the out-of-sample R^2, R^2_{OS} (Campbell and Thompson, 2008). The out-of-sample R^2 compares the forecast based on the predictive regression model in equation (1) $\hat{r}_{t+1}$, with the benchmark $\bar{r}_{t+1}$. In particular, it measures the proportional reduction in MSPE for $\hat{r}_{t+1}$ relative to $\bar{r}_{t+1}$:

$$R^2_{OS} = 1 - \frac{\sum_{k=p+1}^{p+q}\left(r_{m+k} - \hat{r}_{m+k}\right)^2}{\sum_{k=p+1}^{p+q}\left(r_{m+k} - \bar{r}_{m+k}\right)^2} \tag{6}$$

where q are the out-of-sample observations (see above). A positive out-of-sample R^2 indicates that the forecast $\hat{r}_{t+1}$ outperforms the historical average forecast in terms of MSPE. Report the out-of-sample R^2. Which variables outperform the historical benchmark? **(3.5 points)**

3. Consider the out-of-sample performance of a so-called "kitchensink" regression in which you include all predictor variables. How does the model perform? **(0.5 points)**

4. Stay in the out-of-sample setting. Individual forecasts may be improved by combining all individual forecasts into a combined forecast. You combine the individual forecasts for period $t+1$ computed based on equation 1 as follows

$$\hat{r}_{c,t+1} = \sum_{i=1}^{N} \omega_{i,t}\hat{r}_{i,t+1}, \tag{7}$$

where $\omega_{i,t}$ denotes the ex-ante combination weight for the forecast based on variable i at time t. Theoretically, $\omega_{i,t}$ can vary over t, however, they do not need to.

First, consider averaging methods of the individual forecast. For the mean combination forecast, set the combination weights in equation (7) $\omega_{i,t} = 1/N$ for each $i = 1, \ldots, N$. Next, for the median combination forecast, select the median of $\{\hat{r}_{i,t+1}\}_{i=1}^{N}$. That is, set $\omega_{j,t} = 1$ for the forecast variable $j \in \{1, \ldots, N\}$ associated with the individual forecast being the median forecast $\hat{r}_{j,t+1}$ of $\{\hat{r}_{i,t+1}\}_{i=1}^{N}$. Otherwise, set $\omega_{j,t} = 0$.

Using the discount mean square prediction error (DMSPE) method allows the combination weights to depend on the forecast performance of the individual variables over the holdout out-of-sample period (Stock and Watson, 2006). In particular, the combining weights are calculated as

$$\omega_{i,t} = \Phi_{i,t}^{-1} \Big/ \sum_{j=1}^{N} \Phi_{j,t}^{-1}, \quad \text{where} \quad \Phi_{i,t} = \sum_{s=m}^{t-1} \theta^{t-1-s}\left(r_{s+1} - \hat{r}_{i,s+1}\right)^2, \tag{8}$$

with $\theta > 0$ being a discount factor, $m + 1$ being the first period in the holdout out-of-sample period, and the individual forecast $\hat{r}_{i,s+1}$ is based on data up to and including period s. Therefore, the DMSPE method attaches greater weight to forecast variables whose forecasts yielded lower MSPEs over the holdout out-of-sample period. Furthermore, θ controls the degree of discounting. Report results for $\theta = 0.9$ and $\theta = 1$. Comment on potential differences. Do the combination methods improve forecast performance? **(2 points)**

5. Reflect on your findings from questions $1 - 4$. Do the variables that perform well in-sample also perform well out-of-sample? Based on your economic intuition, do you expect the forecast combination to improve on the individual forecasts? Explain. Does the forecast combination you performed under 4 perform better than the "kitchen sink" model under 3? Explain your results. Which combination method do you consider the most appropriate? **(1.5 points)**

Submission

You are expected to hand in two different files: first, a PDF file in which you write your report and provide the results; second, a file that includes all the coding. The latter file should replicate all your reported results and should run on any computer with only changing the working directory. Solutions are only accepted if *all reported results* can be replicated from the submitted code. Your submission is only considered complete if all files are included. The report should be maximum of seven pages. We recommend between four and seven pages. Please submit the report with a font of 11 pt and use 1.2 line spacing. If you encounter any problems, please explain them as well as your solution in your submission.

References

Campbell, J. Y. and Thompson, S. B. (2008). Predicting excess stock returns out of sample: Can anything beat the historical average? *The Review of Financial Studies*, 21(4):1509–1531.

Rapach, D. E., Ringgenberg, M. C., and Zhou, G. (2016). Short interest and aggregate stock returns. *Journal of Financial Economics*, 121(1):46–65.

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Stock, J. H. and Watson, M. W. (2006). Forecasting with many predictors. *Handbook of Economic Forecasting*, 1:515–554.

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