

## Minimum Spanning Tree

Aim: To implement minimum spanning tree using prim's and kruskal's algorithm

Problem Statement: Represent graph of your college campus using adjacency list/matrix. Nodes represent various departments and links should represent the distance between them. Find minimum spanning tree using

a) kruskal's Algorithm

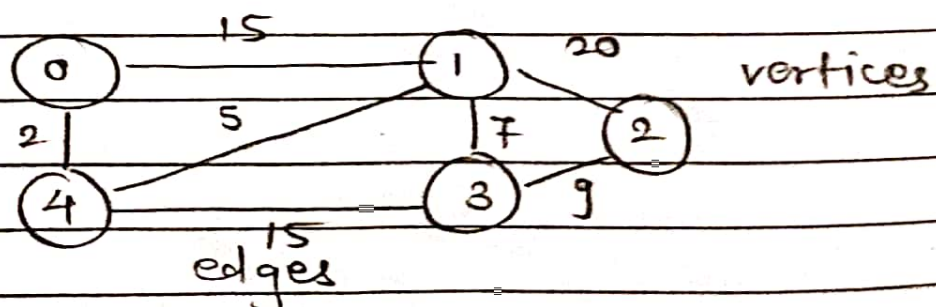
b) prim's algorithm

Analyse time & space complexity

Theory:

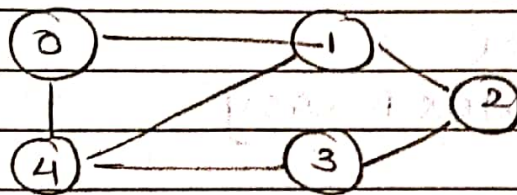
i) Graph:

It is a non-linear data structure consisting of nodes and edges





- e) Graph representation using 2D Array Matrix
- Adjacency matrix is a 2D Array of size  $V \times V$  where  $V$  is the number of vertices in graph.
  - let 2D Array be  $adj[i][j]$ , then  $adj[i][j] = 1$  if there is an edge from  $V$  to  $V_2$  i to  $j$ .
  - for undirected graph matrix is symmetric



Adjacency matrix:

	0	1	2	3	4
0	0	1	0	0	0
1	1	0	1	1	0
2	0	1	0	1	0
3	0	1	1	0	1
4	0	0	0	1	0

Advantages:

- 1) Easier representation
- 2) Removing an edge takes  $O(1)$  time

Disadvantages:

- 1) Consume more space
- 2) More time to add an vertex



Graph representation using adjacency list:

- An array of list is used.
- Size is equal to no of vertex.
- Let  $arr[i]$  be an array. An entry  $arr[i]$  represents list of vertices adjacent to vertex

Advantages:

- 1) Saves Space
- 2) Adding vertex is easy

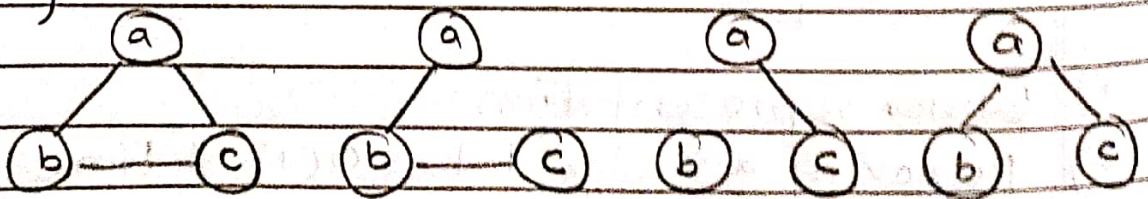
Application of graph:

- 1) Maps
- 2) Computer Network
- 3) Social networking sites
- 4) To represent flow of computations

Spanning Tree:

A spanning tree is subset of Graph  $G$ , which has all the vertices covered with minimum possible no of edges

eg



Minimum Spanning Tree:

A minimum spanning tree is spanning tree with weight less than or equal to weight of every other spanning tree. The weight is sum of weight on each edge of spanning tree.



## Uses of MST:

- 1) Telephone wiring
- 2) Electronic circuits
- 3) Computer Networks

## Algorithms:

### 1) Prim's algorithm

- It starts with tree  $T$ , consisting of single vertex  $v$ .
- Then it finds shortest edge emanating from  $v$  that connects  $T$  to rest of the graph.
- It adds this edge & new vertex to tree  $T$ .
- It then picks the shortest edge emanating from the revised tree  $T$  that also connects  $T$  to the rest of graph & repeats the process till all vertices are visited.

### 2) Kruskal's algorithm

- Sort all edges in ascending order of weights.
- Pick the smallest edge check if it forms cycle with spanning tree. If not include this edge else discard it.
- Repeat (b) till there are  $(v-1)$  edges.

## Validations:

- No. of vertices and edges should be positive.
- Start & end vertices are within the no. of vertices provided by user.



### Test cases :

- Complete undirected graph with no loops, parallel edges.
- Connected undirected graph with no loops, parallel edges.

### Conclusion :

- Time complexity of prim's algorithm  $O(V^2)$  for adjacency matrix  
For adjacency list  ~~$O(E^2)$~~   $O(E \log V)$   
It works faster with dense graphs
- Time complexity of kruskal's algorithm  $O(E \log E)$  or  $O(E \log V)$

It runs faster in sparse matrix.