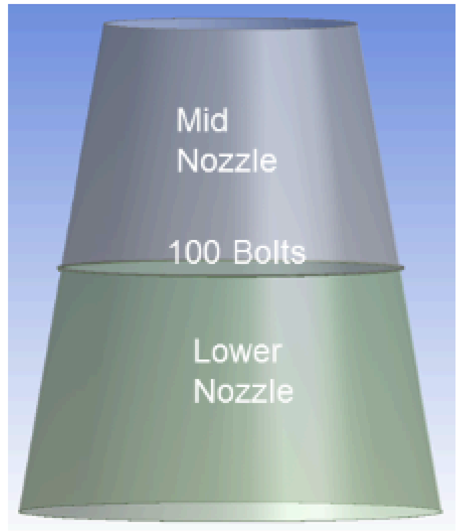
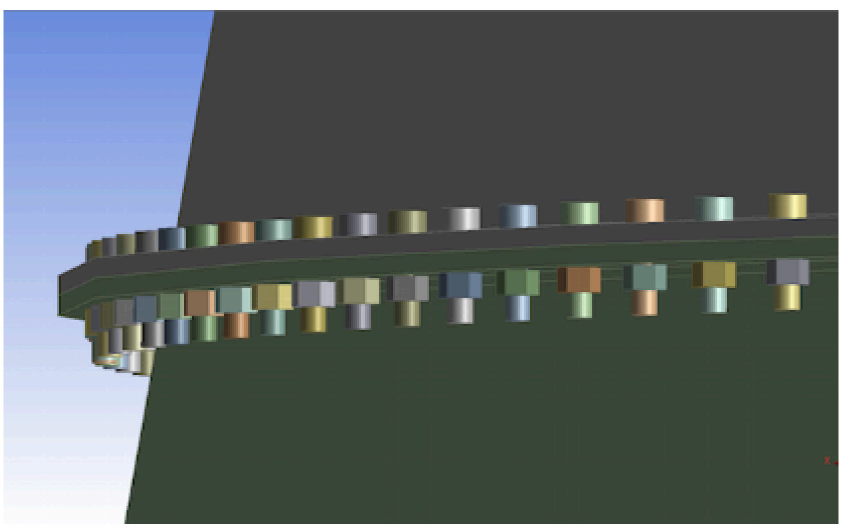
3.1 Bolted Nozzle Flange

Problem Specification

F1 engine on display at National Air and Space Museum
Photograph by Mike Peel (www.mikepeel.net)



ANSYS model of mid and lower nozzle



Close-up view of the bolted flange joint connecting the mid and lower parts of the nozzle.

We'll analyze this bolted joint by building a non-linear finite element model in ANSYS. Our main objective is to assess the margin of safety of the flange bolts. We'd also like to determine the gaps that develop between the jointed parts when the assembly is loaded. [De Laval Nozzle]

The material properties are given in the following table:

	Material	Young's Modulus (psi)	Poisson's Ratio	Coefficient of Thermal Expansion (per deg. F)
Nozzle	300 Series Stainless Steel	2.9E+7	0.27	1E-5
Bolt and Nut	A-286 Steel	2.9E+7	0.31	9.5E-6

The pressure due to the exhaust gas in the nozzle is calculated using 1D gas dynamics. It is assumed to vary linearly along the nozzle axis. The pressure at the exit (z=0) is 12.17 psi and the pressure at the entrance to the mid-nozzle is 47.72 psi.

The regeneration channels are omitted in the model. In exchange, a free body diagram is used to deduce the equivalent forces on the mid nozzle and lower nozzle (the upper nozzle is not modeled here). This force pair is modeled as two separate forces, each of 1000 lbf. The gas temperature is 700 F which causes thermal strain. We assume that bolt is pre-loaded to 50% of its breaking strength.

Pre-Analysis for ANSYS (Bolt & Nut Case)

Mathematical Model

- Based on **3D elasticity** (same as bike crank case, but more complex).
- Domain: **4 parts** → mid nozzle, lower nozzle, bolt, nut.
- Geometry simplified: **half bolt + nut** (symmetry assumption).
- **4 coupled boundary value problems** (interfaces/contact surfaces).

Governing Equations

- **Force balance** in x, y, z → 3 equations.
- Unknowns: 6 stresses (3 normal + 3 shear).

- Additional equations:
  - **Constitutive model** (Hooke's law, 3D version).
  - **Strain-displacement relations** (3 normal + 3 shear).
- Total: **15 equations** for 15 unknowns (σ, ε, u).

Additional Effects

- **Thermal strain**:
  - α = thermal expansion coefficient
  - ΔT = temperature change (from hot gas in nozzle.
  - Causes normal stresses, not shear.
- **Bolt preload**:
  - Strain added when tightening bolt.
  - Modeled analogous to thermal strain.
  - Appears only in bolt domain.

Assumptions

- Linear material properties: E, ν, α = constants.
- Small displacements & small strains.

Boundary Conditions in Bolt & Nozzle Model (ANSYS Pre-Analysis)

General Rule

- At every boundary point → must define **displacement (essential BC)** or **traction (natural BC)**.
- Applies to **normal** + **tangential** directions.

Essential Boundary Conditions (Displacement BCs)

1. **Symmetry planes (periodicity cuts)**
   - Normal displacement = 0
   - Tangential traction = 0
   - Equivalent to **ANSYS "frictionless" support**
2. **Top of mid-nozzle (connection to upper nozzle)**
   - Normal displacement = 0
   - Tangential traction = 0
   - Approximates constraint from upper nozzle

Natural Boundary Conditions (Traction BCs)

1. **Propellant pressure on nozzle interior**
   - Normal traction from hot-gas pressure
   - Pressure **varies axially** (not uniform)
   - Computed via **1D gas dynamics** (Excel calculations provided)
2. **Forces from regenerative cooling channels**
   - Cooling channel pressure → pulls joint apart
   - Modeled as applied forces on relevant surfaces (channels not explicitly modeled)
3. **Contact surfaces between parts**
   - Contact traction depends on **displacement** (nonlinear):
     - If not in contact → traction = 0
     - If in contact → traction develops
   - Nonlinearity arises since traction can switch between 0 and finite value.
4. **Other boundaries**
   - Default = **0 traction** (normal + tangential).

Key Nonlinear Effect

- **Contact mechanics**: Traction depends on displacement → introduces **high nonlinearity** into FE solution.

Numerical Solution strategy

- **Linear FEM:**
  $K\mathbf{u} = \mathbf{f}$
  Equations are linear; stiffness matrix `K` is constant.
- **Nonlinear FEM:**
  - Appears at **interfaces/contacts** where **traction depends on displacement**.
  - Leads to terms like products of displacements → equations become nonlinear.
  - Cannot be written as simple `K u = f`
- **Solution strategy** [Newton-Raphson](#)**:**
  1. Start with initial guess for displacements.
  2. Compute **residual (force imbalance)**.
  3. Linearize using slope (derivative) and update guess.
  4. Repeat until residual is below tolerance.
- **Interpretation in ANSYS:**
  - Solver iterates → residual forces decrease.
  - Convergence checked in the **force convergence plot**.
  ✅ In short: *Nonlinearity comes from displacement-dependent boundary conditions (traction/contact). Newton–Raphson iteratively corrects displacements until equilibrium is satisfied.*

Hand Calculations

$$\text{Average gas pressure} = \frac{12.17 + 47.72}{2} = 30\ psi$$

$$\begin{aligned}
\text{Top radius} &= 41.75\ \ inch \\
Bottomradius &= 69.50\ \ inch \\
\text{Projected area in Z-direction} \\
\pi(69.5^2 - 41.75^2) &= 9699\ \ inch^2 \\
\text{Net reaction force in Z-direction on } 1/400\text{th model} \\
\frac{30 * 9699}{400} &= 720\ \text{lbf}
\end{aligned}$$

In Z-direction, the vertical component of the forces exerted due to gas pressure are modelled here. The projected area in Z-direction is captured and multiplied by the average pressure exerted by gases over the nozzle cone area. As we are analyzing half bolt, and there are 200 bolts, it is 1/400 for each half bolt.

$$\begin{aligned}
\text{At the exit p=12.71 psi} \\
\text{r} &= \text{69.5 inch} \\
\text{t} &= \text{0.5inch} \\
\sigma_\theta &= \frac{pr}{t} \\
\sigma_\theta &= 1692 psi
\end{aligned}$$

For the circumferential stress, which is along the circumference direction like on the pressure vessel. We can compare this hand calculated value with the numerical solution

Mesh

1. Sizing on Mid Nozzle and Lower Nozzle - 0.3 inch
2. Sizing on Bolt and Nut - 0.075 inch
   Due to this sizing, there are 25273 elements generated with 47648 nodes. It is way above the student license limit of 30000 nodes. By using

Method of HEX Dominant,, the number of elements and nodes are brought down to 4255 and 26134 respectively.

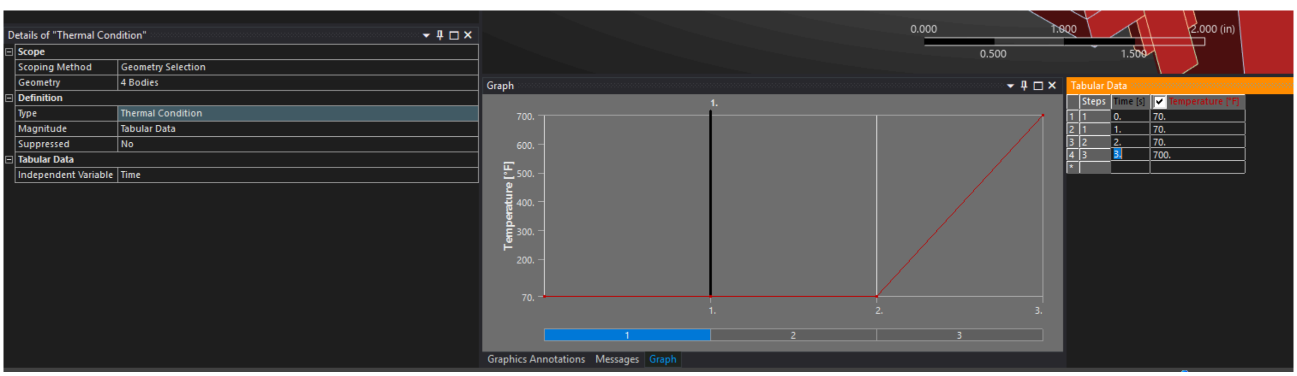
With Mesh Metric->Element Quality, we can see the type of mesh elements generated. Mesh metric->Skewness we can see which elements are skewed and by how much margin. This help us to identify the bad elements, which could influence the results.

Material

Create the material as per given data. Isotropic secant and instantaneous co-efficient of thermal expansion is selected.
Add the Youngs modulus and poison's ratio and thermal expansion co-efficient.
Update the materials inside geometry for each parts. Nozzle 300 series stainless steel and bolt and nut A 286 steel

Boundary Conditions

1. **Analysis setting:** - Load Steps counts set to 3. Load steps are set to 3
2. **Thermal Condition:** -



   Magnitude-> tabular. Set the temperature for each load step. From 2nd load step to 3rd the temperature will ramp up to 700F.
   Change in length due to thermal expansion

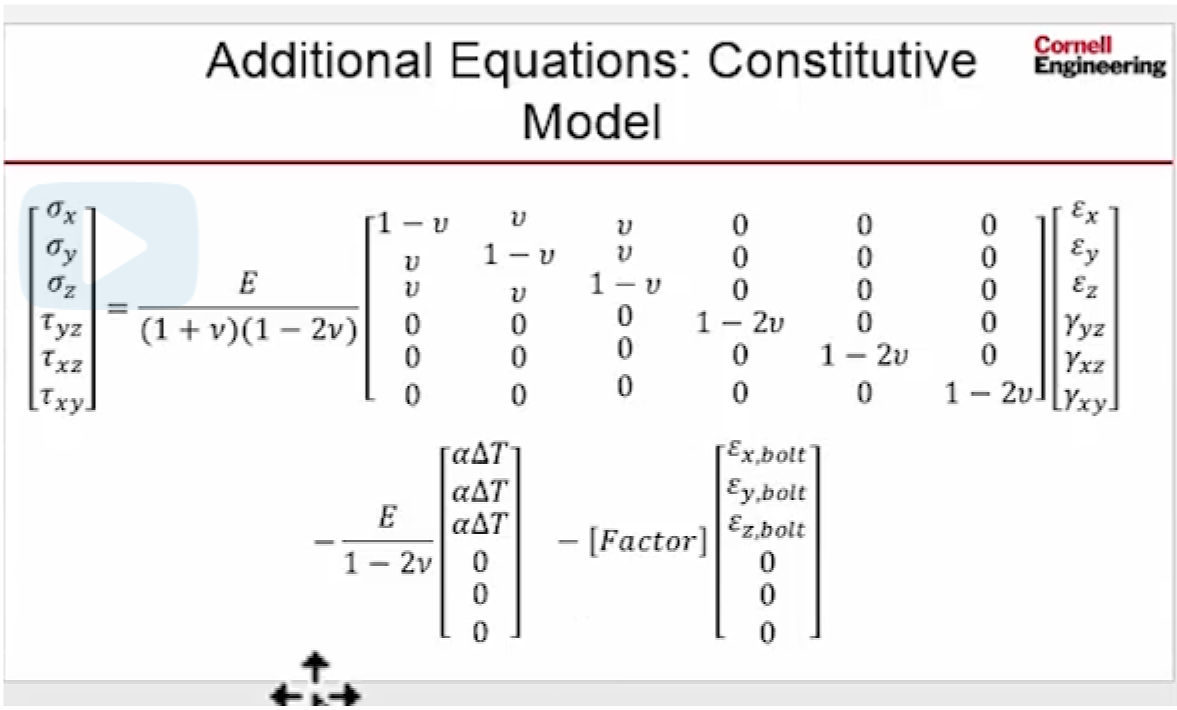
$$\triangle l = \alpha * \triangle T * L = 0.00001 * (700 - 70) * 160 = 1.008\text{inch}$$

3. **Bolt Preload:** - due to tightening of bolt. The values are calculated in the excel sheet provided.

asset-v1_CornellX+ENGR2000X+2T2016+type@asset+block@F-1_Engine__model_calculations__Non-Proprietyary_Non-ITAR_Rev2.xlsx

Preload calculated value is 2320 lbs. for half Bolt. 4640 lbs. for full bolt. Add the Bolt pretension in static analysis and in the table add the preload value for the first load step. The further load steps are locked to show that it is only done at first load step.

Bolt preload sometimes needs to be applied in sub steps to allow the solver to converge to solution easily. Deciding the load steps in each case is a very important task.



$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{bmatrix} = \frac{E}{(1+v)(1-2v)}\begin{bmatrix}1-v & v & v & 0 & 0 & 0\\ v & 1-v & v & 0 & 0 & 0\\ v & v & 1-v & 0 & 0 & 0\\ 0 & 0 & 0 & 1-2v & 0 & 0\\ 0 & 0 & 0 & 0 & 1-2v & 0\\ 0 & 0 & 0 & 0 & 0 & 1-2v\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{bmatrix} - \frac{E}{1-2v}\begin{bmatrix}\alpha\Delta T\\ \alpha\Delta T\\ \alpha\Delta T\\ 0\\ 0\\ 0\end{bmatrix} - [Factor]\begin{bmatrix}\varepsilon_{x,bolt}\\ \varepsilon_{y,bolt}\\ \varepsilon_{z,bolt}\\ 0\\ 0\\ 0\end{bmatrix}$$

With thermal condition and bolt pretension we added the strain conditions of the matrix for temperature and bolt pretension

4. **Friction less Support**: at the top end of the mid nozzle.

5. **Hot gas Pressure**: - First geometry is selected. The internal faces of the nozzle and the contact surfaces between the mid and lower nozzle. Under tabular data independent variable Z is selected and in global co-ordinate system. At the bottom level 0 the pressure is 12.71 psi and at top at mid nozzle entry 160 inch height the pressure is 47.72 psi. In graph controls the X-axis , time is selected and in the steps the load added at 2nd step until the end of 3rd. Graph shows the clear view. Variable pressure in Z-direction and application of pressure in time steps.

6. **Force from regeneration channels:** - This force is pulling the mid and lower nozzle apart. 1000 lbs. Force is applied at the contact surfaces the the nozzles. Make sure the direction of the force. Magnitude is tabular and the force, like others, comes into act in 2nd load step.
7. **Contact Surfaces**: - [Contacts surfaces]() have already been setup. This is very important step in setting the model. ## 🚀 Rocket Nozzle Pressure & Regeneration Channel Loads

Estimating the pressure load and regen forces

1. Pressure Load in Nozzle

- Start with **chamber pressure** (~70 atm or 1000 psi for Saturn V).
- Use **nozzle geometry + area ratio** → apply the **area–Mach relation** to get Mach number and pressure variation along nozzle.
- Can use Excel to calculate pressure distribution, then apply it in ANSYS.
- Simplification: apply **uniform pressure** at mid-section first, then refine with full distribution.

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \$\$ **2. Regeneration Channel Force$$

- Cooling channels not modeled directly → effect estimated using **free body diagram (FBD)**.
- Estimated **regen pressure ≈ chamber pressure** (1000 psi).
- Each tube area ~1 in² → force = pressure × area = **1000 lbs. per side**.
- Equal & opposite forces act across flanges → represent load due to regen channels.
  Equation (force):

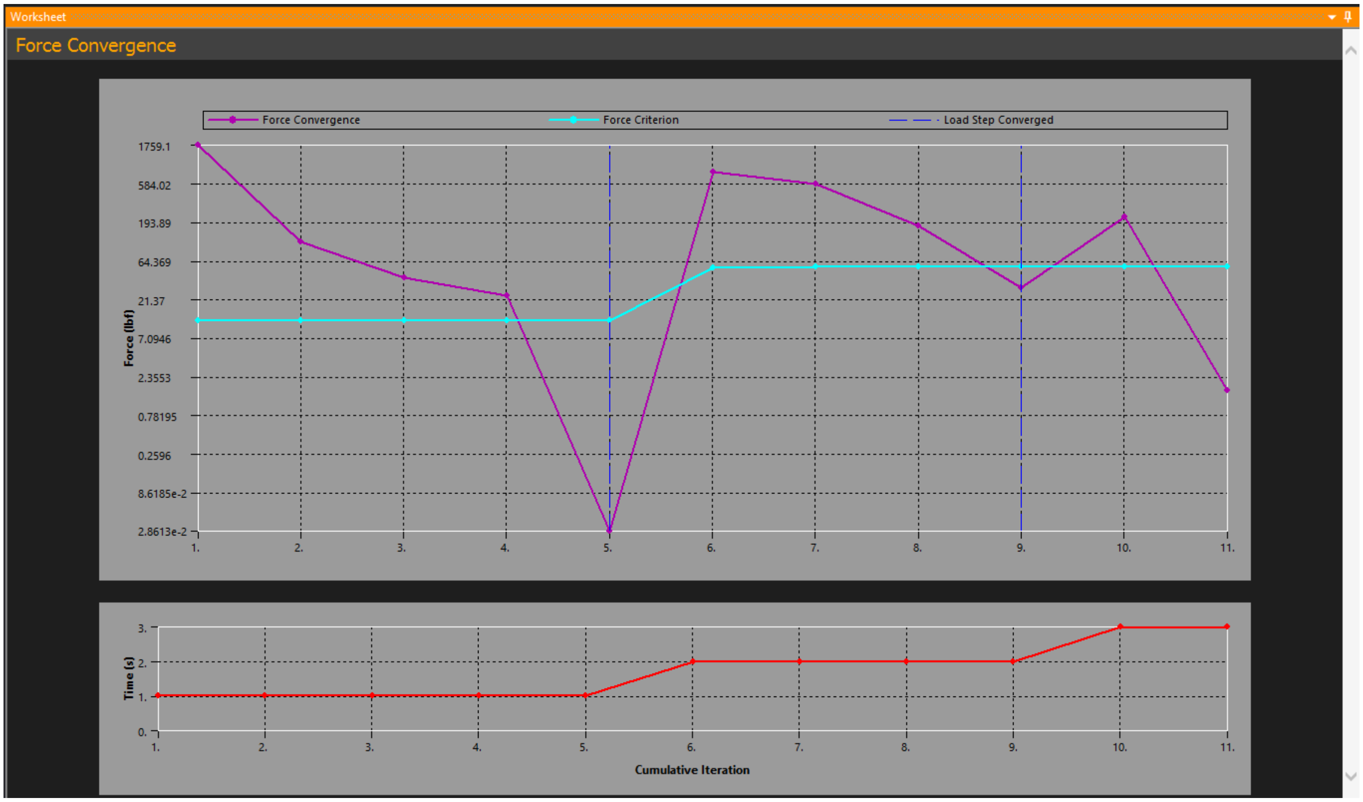
$$F = P \cdot A$$

3. Contacts surfaces

Numerical Solution

Warning - One or more bodies may be under constrained and experiencing rigid body motion. Weak springs have been added to attain a solution. Refer to Troubleshooting in the Help System for more details.

Force Convergence Plot



Solution Information->Solution Output-> Force Convergence

This plot shows the Newton-Raphson method working in real. $g(u) - f = \triangle f$, here there are 80,000 equations for F , i.e., 80K DOFs. And The force convergence value is the aggregate of the Delta f $\triangle f$ and the force criterion is the tolerance value. When purple line is below blue line, we have a good solution and ansys stops calculating. But then load steps keeps increasing, and we have new tolerance. Dotted Blue vertical line shows that at this point we have a good solution. Delta_f can never be zero, if so a close to zero value is reached the system is considered to be in equilibrium

Numerical Results

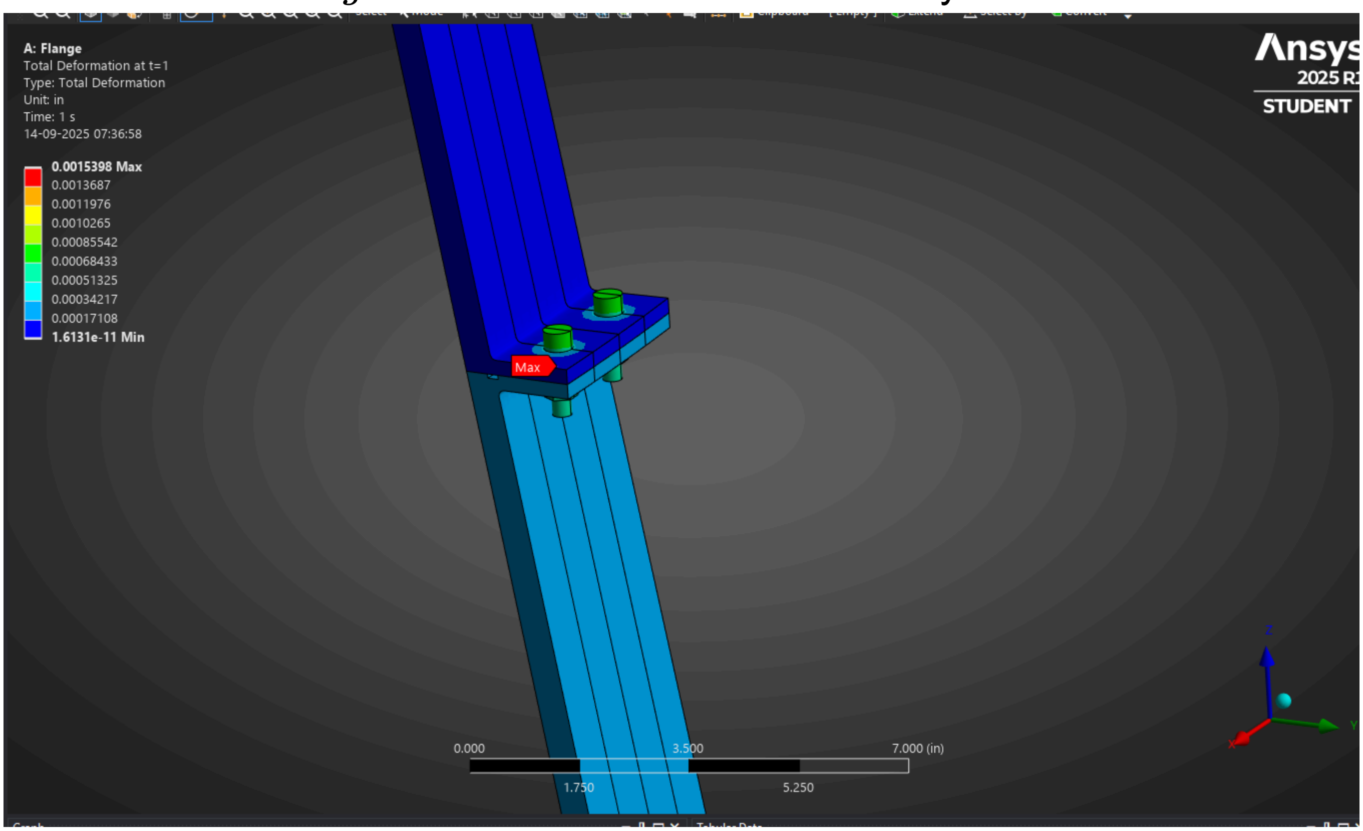
Add total deformation at time step 1 2 3. Equivalent stress is also added. To measure the gap opening between the contact surface of the mid and lower nozzle, Contact tool is added. Contact of the nozzle is selected and Gap function is added. Gap at each time step is duplicated.

View Full Bolt

Turn on Beta options - Workbench->tools->options->appearance->turn on beta options

In mechanical->symmetry->graphical expansion 1->number of repeat 2-> method half (half of bolt is present)-> $\triangle Z = 1e - 5$ to add slight distance. Just a need of software to work. ->Co-ordinate plane, about which the duplicate will form, plane 7.

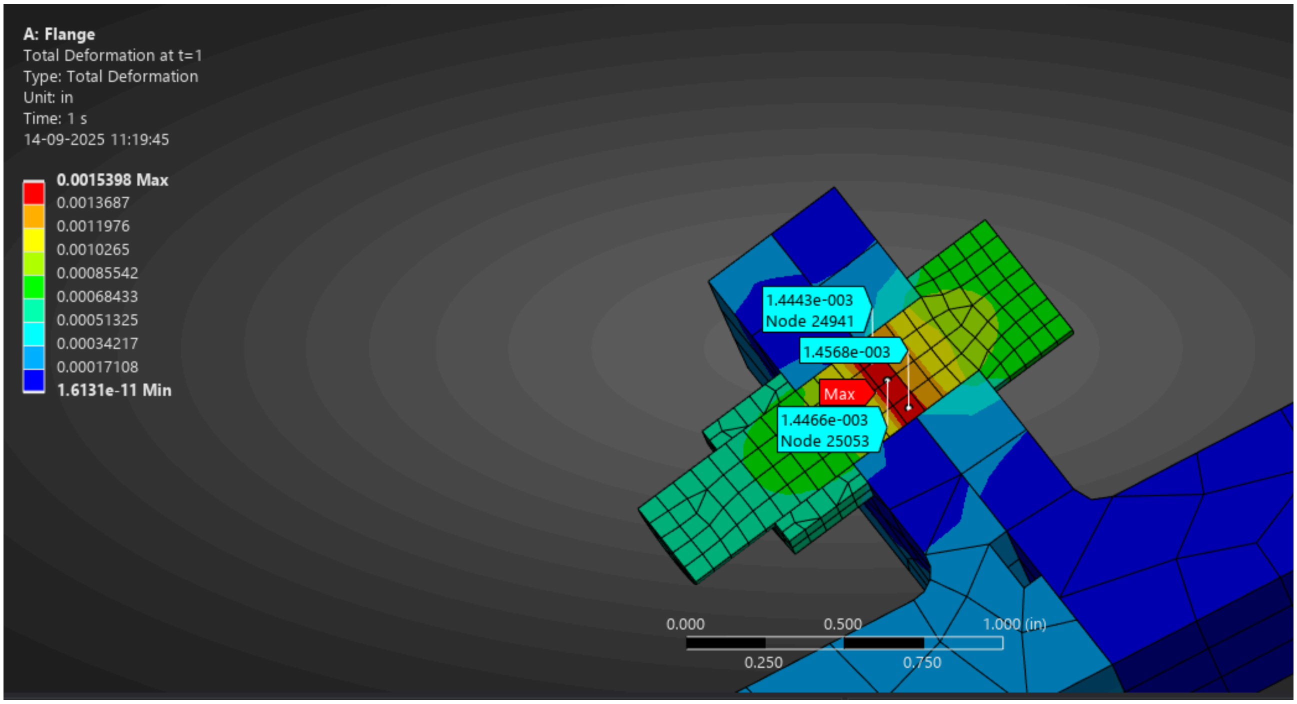
To repeat second bolt->graphical expansion 2-> number of repeat 2->type Polar-> $\triangle\theta = 1.8deg$ as this is 1.5 slice of the entire system.



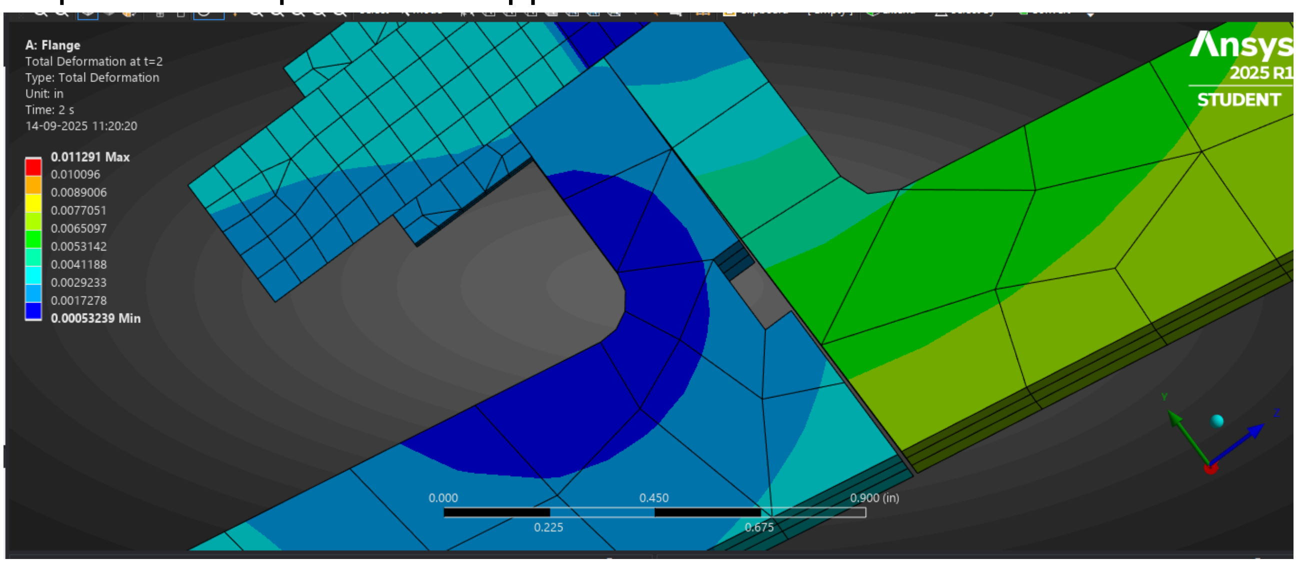
Discussion of results

Deformation results

1. Bolt preload deformation is present as applied in 1st load step. Which is close to zero.



2. At second time step we can see a gap between the nozzles. This is the step where the pressure is applied.



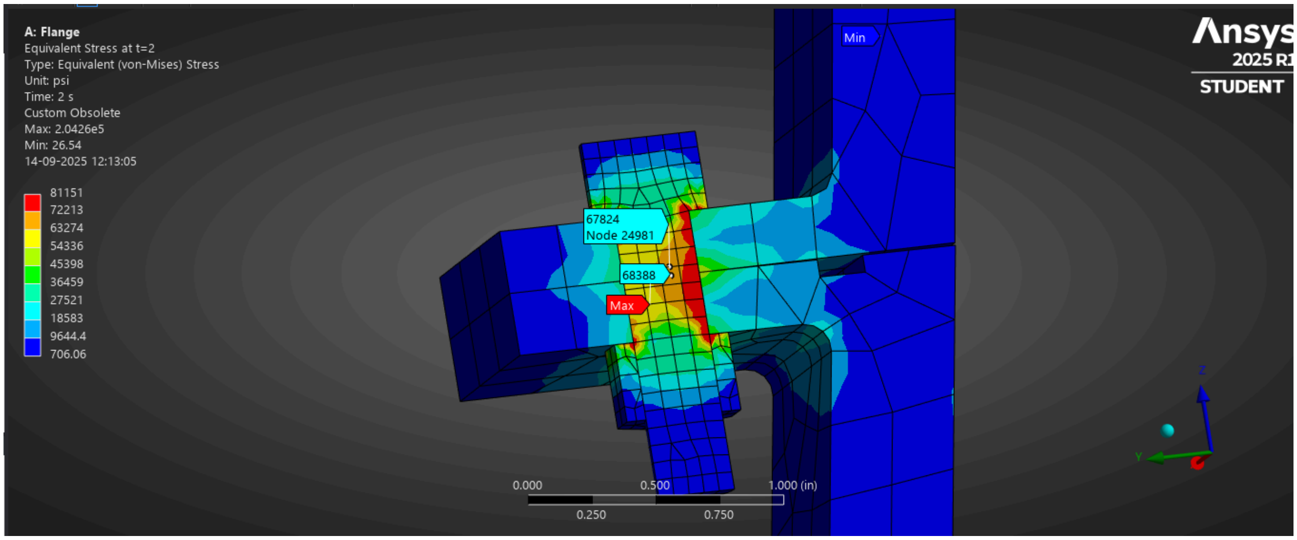
3. In third load step, the hot gas temperature comes into picture which causes the nozzle to expand due to high temperatures. The nozzle moves down and axially outward.

Equivalent stress

1. Bolt pretension hand calc -

$$Force = 2320lbs \;\; Area = 0.038161inch^2 \;\; stress = 60800\frac{lbs}{inch^2}\$\$Pastedimage$$

2. Pressure acting load step.

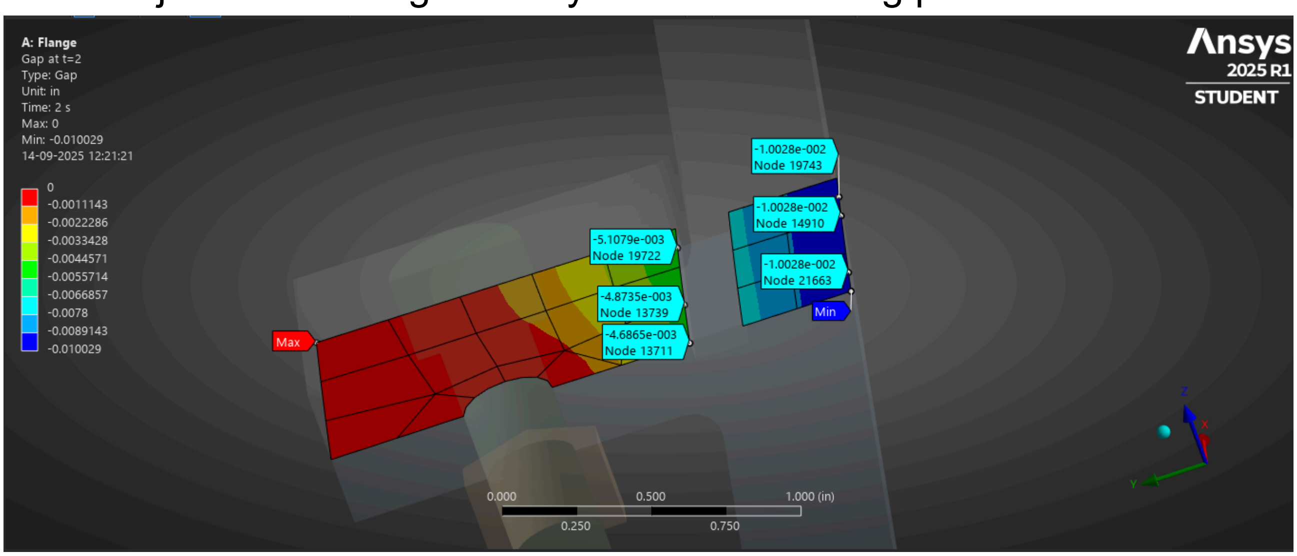


10% increase in the loads at the bolt

3. With the temperature acting in the third load step the stresses remain roughly same, just the molecules expand and move.

Contact Results

1. Load step one - the gap are nearly zero as no load is applied.
2. Load step 2 the gaps increase to **0.005 inch**, i.e., 5000th of an inch which is just on the verge for any seal or metal ring present there.



3. In third load step the structure just shift due to high temperature and the gap roughly remains the same.

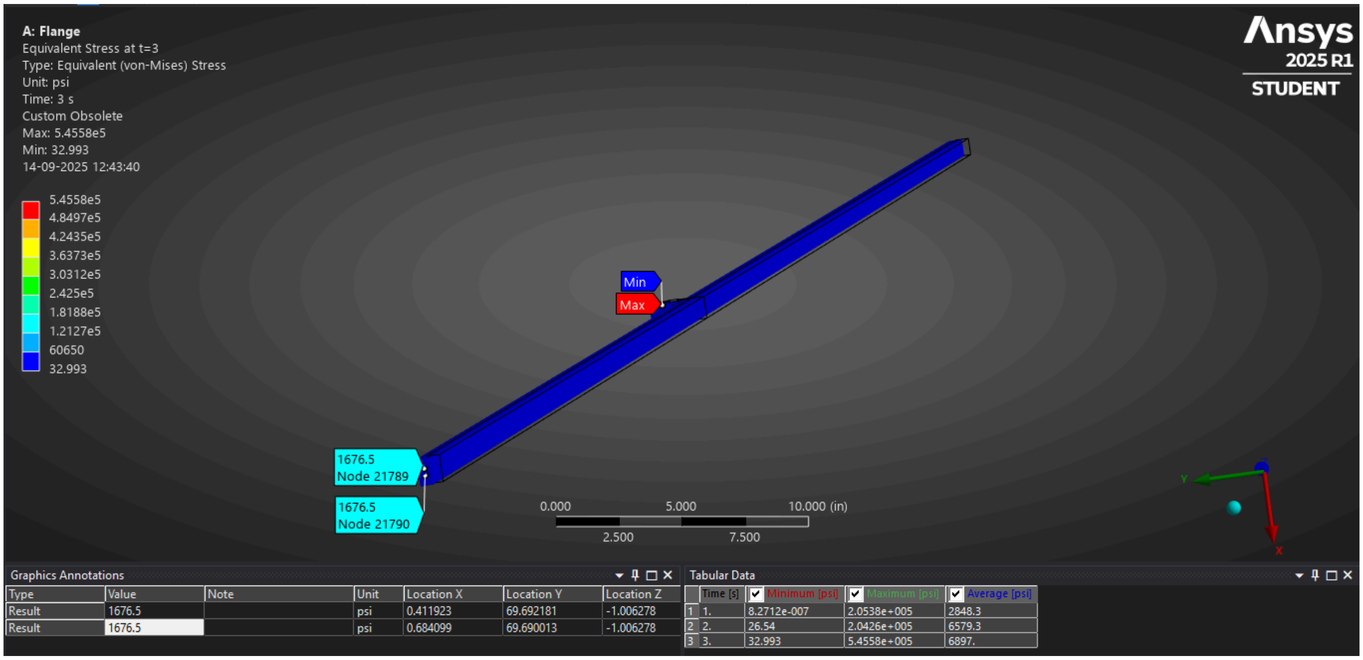
- If the gap is too large due to some reasons, to reduce the gap the thickness of the protruding flange can be increased.
- Add larger radius to the curve of flange
- Add washer for the bolt and increase the preload.
- Another reason stated was that the regen forces used in the model are just an assumption and estimation. They could be off by factor of 2x or 3x. Just a conservative approach which would cause the gap to enlarge.

An important factor to understand to keep in mind is to understand the sensitivity of the parameters and boundary conditions. Which parameter could influence the results. One analysis is not enough. Multiple analysis with different boundary conditions need to be done to understand their effect

Verification

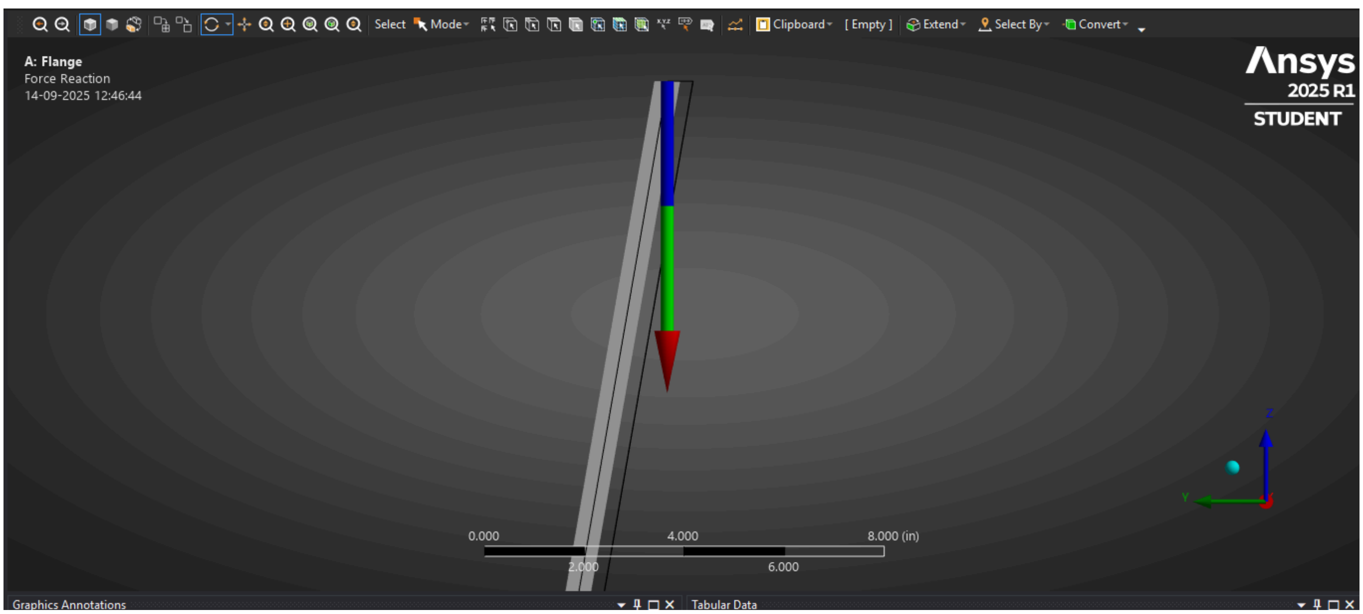
1. Hoop stress at the end of lower nozzle

$$\begin{aligned} \text{At the exit p=12.71 psi} \\ \text{r} = 69.5 \text{ inch} \\ \text{t}= 0.5\text{inch} \\ \sigma_\theta = \frac{pr}{t} \\ \sigma_\theta = 1692psi \end{aligned}$$



For verification with hand calculations we can see the hoop stress is fairly close enough to value calculated 1692 psi

2. Reaction at frictionless support



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$$\pi(69.5^2 - 41.75^2) = 9699 \ \ inch^2$$
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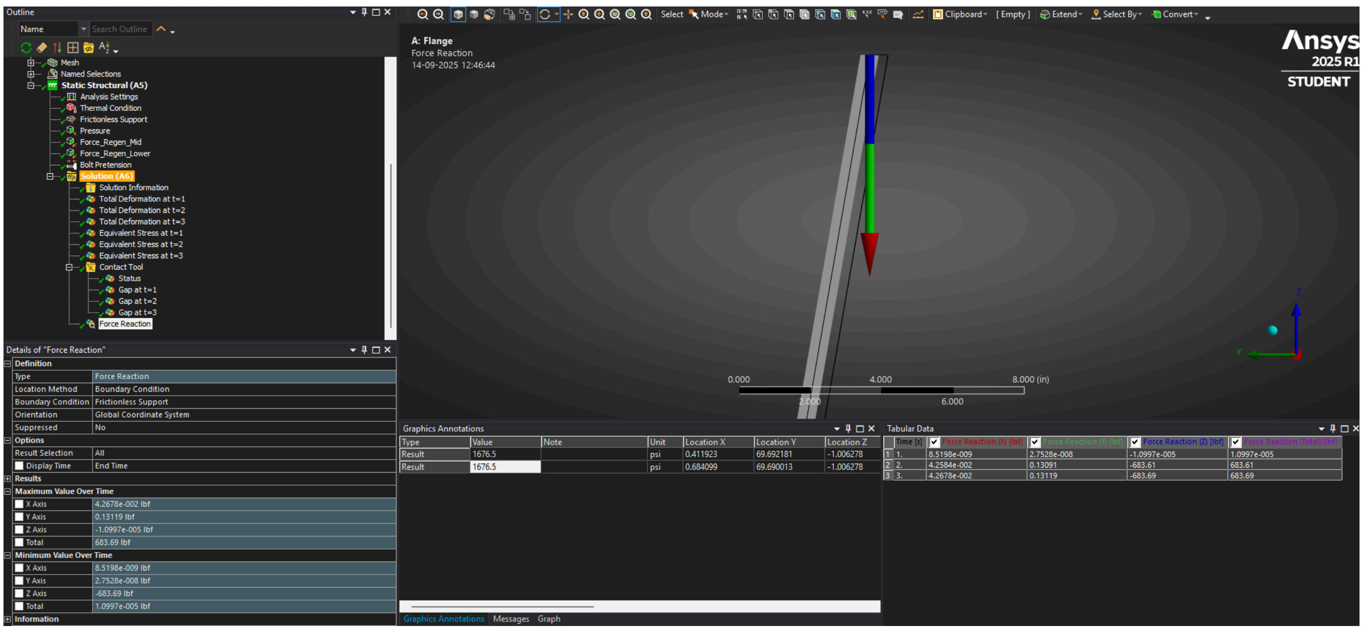
$$\frac{30 * 9699}{400} = 720 \text{ lbf}$$



Ansys model gave a reaction force of **684 lbf** in negative Z- direction. Which verifies that our model is correct and fairly close to hand calculation result.

Problem Specification

In this exercise, you will

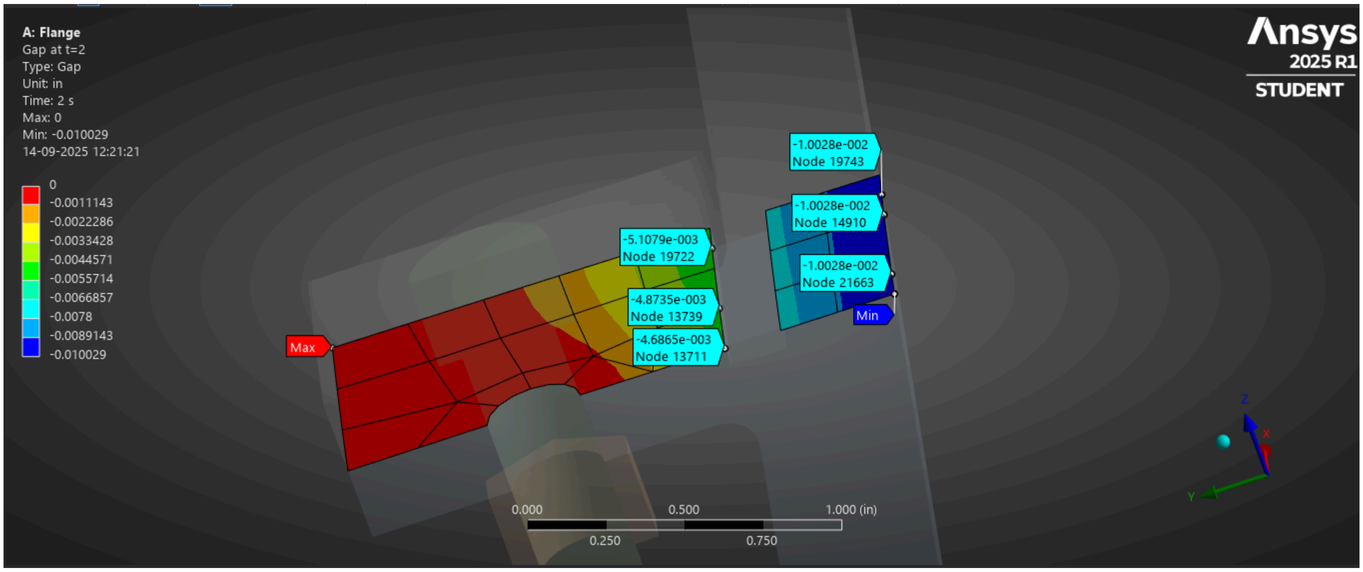
- Study the sensitivity of the bolted nozzle flange results to key input parameters
  For industrial purposes, it is often very helpful to know how sensitive the model is to certain inputs. In this homework, we will explore the sensitivity of the bolted flange model to the following inputs:

1. Load due to regeneration channels

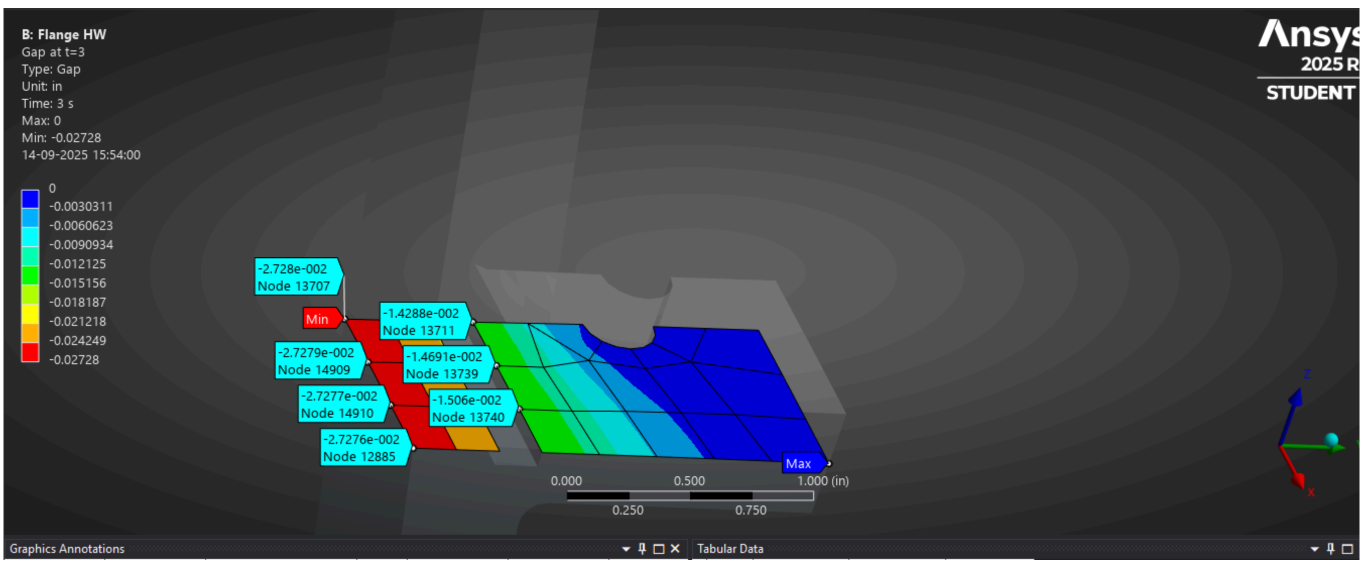
2. Bolt preload
3. Coefficient of friction between flanges

1. Change Regeneration Channel Loads

Case 1- Regen force 1000 lbf.



Case 2: New force 2000 lbf.



Case 1: gap at t=3 **0.005107 inch** and **0.010028 inch**

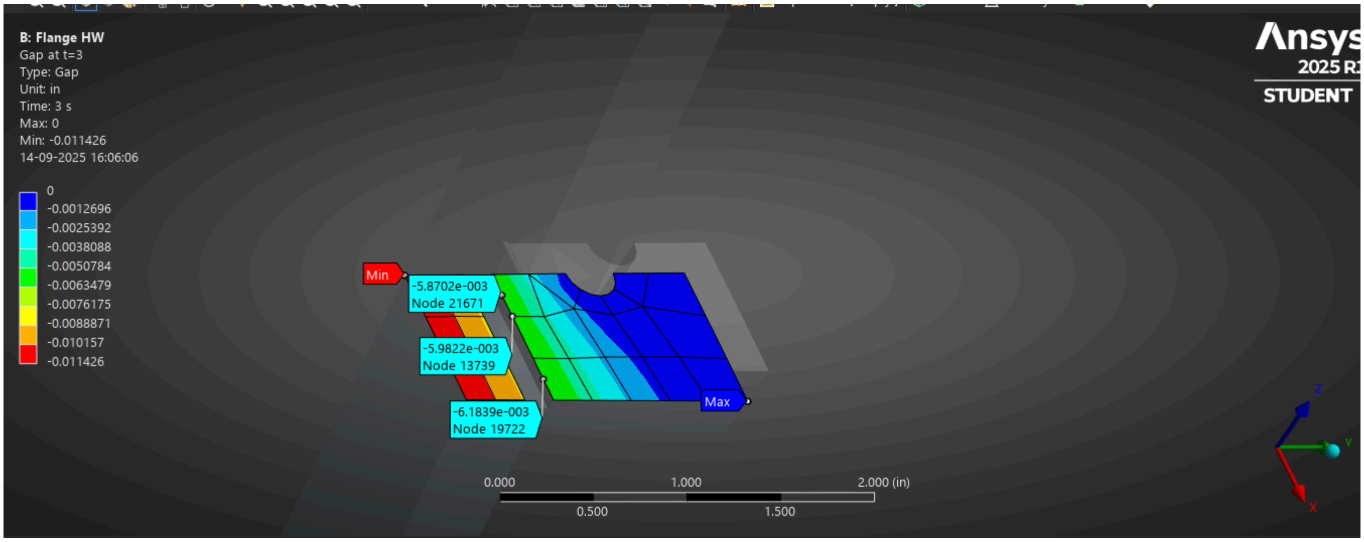
Case 2: gap at t=3 **0.01469 inch** and **0.027277 inch**

The Gap increased by factor of 3 when the regen force is doubled

2. Change Bolt Preload

Next, let's explore how changing the preload applied to the bolt influences the gapping between the two flanges. First, change the forces applied on the flanges by the regeneration channels back to 1,000 lbf.

In our base case, the bolt preload applied was 2,320 lbf. In this homework, let's cut that preload in half to 1,160 lbf as shown in the figure below. Leaving all other inputs the same, re-solve the model.



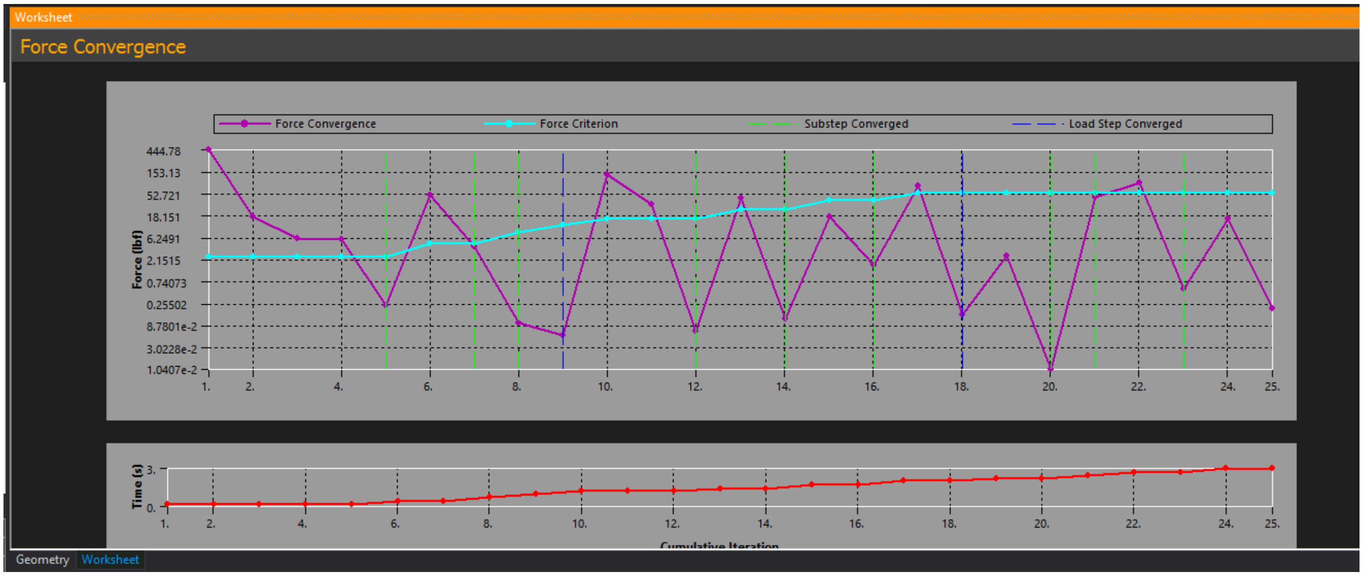
Case 1: gap at t=3 **0.005107 inch**

Case 2: gat at t=3 **0.0061839 inch**

When the preload of the bolt was reduce the gap increased by a factor of 1.2, i.e., 20%

3. Change to Frictional Contact

In the base case covered in the Bolted Nozzle Flange module, we modeled the contact between the two flanges as frictionless. In reality, there will be friction at the contact surfaces that will prevent them from sliding freely. To model this, change the contact between the two flanges from frictionless to frictional with a coefficient of friction = 0.5 (see figure below). In reality, the coefficient of friction will depend heavily on the specific material and the surface quality of the flanges. Return all other inputs (regeneration channel force, bolt preload) to the values in the base case. Re-solve the model.



How did the total number of iterations required to reach a final solution change between the frictionless and frictional cases?

- Case 1: frictionless contact between mid and lower nozzle. No. Of iterations required were 11
- Case 2: Frictional contact between nozzles with friction co-efficient of 0.5 the no. Of iterations increased to 25.

Effect on Boundary Conditions

When we change the contact between the flanges from frictionless to frictional, the only aspect of the mathematical model that changes is the natural boundary condition at that particular contact surface.

True