Trigonometric Identities

Ist

$$\sin^2\theta + \cos^2\theta = 1$$

OR

$$\cos^2\theta = 1 - \sin^2\theta$$

OR

$$\sin^2\theta = 1 - \cos^2\theta$$

IInd

$$\sec^2\theta - \tan^2\theta = 1$$

OR $$\sec^2\theta = 1 + \tan^2\theta$$

IIIrd

$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

OR $$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$

$$= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{2+2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{2\cancel{(1+\sin A)}}{\cancel{(1+\sin A)}\cos A}$$

$$= \frac{2}{\cos A}$$

$$= 2\sec A = RHS$$

(iv)

LHS

$$\frac{1+\sec A}{\sec A}$$

$$= \frac{1}{\sec A} + \frac{\cancel{\sec A}}{\cancel{\sec A}}$$

$$= \cos A + 1$$

RHS

$$\frac{\sin^2 A}{1-\cos A}$$

$$= \frac{1^2-\cos^2 A}{1-\cos A} \quad (\because \sin^2 A = 1-\cos^2 A)$$

$$= \cancel{(1-\cos A)}(1+\cos A)$$

2016, 2017

(vi)

LHS

$$\sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1^2-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{}}$$

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(viii) $$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta - 1)}$$

$$= \frac{\tan\theta\,(1 - 2\sin^2\theta)}{2(1 - \sin^2\theta) - 1}$$

$$= \frac{\tan\theta\,(1 - 2\sin^2\theta)}{2 - 2\sin^2\theta - 1}$$

$$= \frac{\tan\theta\,\cancel{(1 - 2\sin^2\theta)}}{\cancel{1 - 2\sin^2\theta}}$$

$= \tan\theta$ R

2015, 2016, 2017, 2019

(viii)

LHS $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + \cos^2 A + \sec^2 A$
$\quad + 2\sin A \operatorname{cosec} A + 2\cos A \sec A$

$= 1 + (1 + \cot^2 A) + (1 + \tan^2 A)$
$\quad + 2\cancel{\sin A} \times \frac{1}{\cancel{\sin A}} + 2\cancel{\cos A} \times \frac{1}{\cancel{\cos A}}$

$= 7 + \cot^2 A + \tan^2 A$