

Trigonometric Identities

$$\begin{array}{l} \text{I}^{\text{st}} \\ \sin^2 \theta + \cos^2 \theta = 1 \\ \text{OR} \\ \cos^2 \theta = 1 - \sin^2 \theta \\ \text{OR} \\ \sin^2 \theta = 1 - \cos^2 \theta \end{array} \left. \vphantom{\begin{array}{l} \sin^2 \theta + \cos^2 \theta = 1 \\ \cos^2 \theta = 1 - \sin^2 \theta \\ \sin^2 \theta = 1 - \cos^2 \theta \end{array}} \right\}$$

$$\begin{array}{l} \text{II}^{\text{nd}} \\ \sec^2 \theta - \tan^2 \theta = 1 \\ \text{OR} \\ \sec^2 \theta = 1 + \tan^2 \theta \end{array} \left. \vphantom{\begin{array}{l} \sec^2 \theta - \tan^2 \theta = 1 \\ \sec^2 \theta = 1 + \tan^2 \theta \end{array}} \right\}$$

$$\begin{array}{l} \text{III}^{\text{rd}} \\ \operatorname{cosec}^2 \theta - \cot^2 \theta = 1 \\ \text{OR} \\ \operatorname{cosec}^2 \theta = 1 + \cot^2 \theta \end{array} \left. \vphantom{\begin{array}{l} \operatorname{cosec}^2 \theta - \cot^2 \theta = 1 \\ \operatorname{cosec}^2 \theta = 1 + \cot^2 \theta \end{array}} \right\}$$

$$\left. \begin{array}{l} \operatorname{cosec} \theta = \frac{1}{\sin \theta} \\ \sec \theta = \frac{1}{\cos \theta} \\ \cot \theta = \frac{1}{\tan \theta} \end{array} \right\}$$

$$\left. \begin{array}{l} \tan \theta = \frac{\sin \theta}{\cos \theta} \\ \cot \theta = \frac{\cos \theta}{\sin \theta} \end{array} \right\}$$

$$(ii) \quad \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A) \cos A}$$

$$= \frac{\cancel{\cos^2 A} + 1 + \cancel{\sin^2 A} + 2\sin A}{(1 + \sin A) \cos A}$$

$$= \frac{1 + 1 + 2\sin A}{(1 + \sin A) \cos A}$$

$$= \frac{2 + 2\sin A}{(1 + \sin A) \cos A}$$

$$= \frac{2 \cancel{(1 + \sin A)}}{\cancel{(1 + \sin A)} \cos A}$$

$$= \frac{2}{\cos A}$$

$$= 2 \sec A = \text{RHS}$$

(iv)

$$\begin{aligned} & \text{LHS} \\ & \frac{1 + \sec A}{\sec A} \\ &= \frac{1}{\sec A} + \frac{\sec A}{\sec A} \\ &= \underline{\underline{\cos A + 1}} \end{aligned}$$

RHS

$$\begin{aligned} & \frac{\sin^2 A}{1 - \cos A} \\ &= \frac{1 - \cos^2 A}{1 - \cos A} \quad (\because \sin^2 A = 1 - \cos^2 A) \\ &= \cancel{(1 - \cos A)} (1 + \cos A) \end{aligned}$$

2016, 2017
(vi)

$$\begin{aligned} & \text{LHS} \\ & \sqrt{\frac{1 + \sin A}{1 - \sin A} \times \frac{1 + \sin A}{1 + \sin A}} \\ &= \sqrt{\frac{(1 + \sin A)^2}{1^2 - \sin^2 A}} \\ &= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}} \\ &= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin A}} \end{aligned}$$

$$(vii) \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta}$$

$$= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)}$$

$$= \frac{\tan \theta (1 - 2 \sin^2 \theta)}{2(1 - \sin^2 \theta) - 1}$$

$$= \frac{\tan \theta (1 - 2 \sin^2 \theta)}{2 - 2 \sin^2 \theta - 1}$$

$$= \frac{\tan \theta (1 - 2 \sin^2 \theta)}{1 - 2 \sin^2 \theta}$$

$$= \tan \theta \quad \underline{B}$$

2015, 2016, 2017
(Viii) 2019

LHS

$$\begin{aligned} & (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 \\ &= \sin^2 A + \operatorname{cosec}^2 A + \cos^2 A + \sec^2 A \\ &\quad + 2 \sin A \operatorname{cosec} A + 2 \cos A \sec A \\ &= 1 + (1 + \cot^2 A) + (1 + \tan^2 A) \\ &\quad + 2 \sin A \times \frac{1}{\sin A} + 2 \cos A \times \frac{1}{\cos A} \\ &= 7 + \cot^2 A + \tan^2 A \end{aligned}$$