②

BIYNOMIAL EXTRA

Q① If α, β, γ are the zeroes of $p(x) = x^3 - 3x^2 - 3x + 1$
Then find the value of (i) $\alpha + \beta + \gamma$ (ii) $\alpha\beta + \beta\gamma + \gamma\alpha$
(vi) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ (iv) $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$
(v) $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$ (iii) $\alpha\beta\gamma$

Soln

$$p(x) = x^3 - 3x^2 - 3x + 1$$

(i) $\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{(-3)}{1} = 3$

(ii) $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = -3$

(iii) $\alpha\beta\gamma = -\frac{d}{a} = -\frac{1}{1} = -1$

(iv) $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$

$= \alpha\beta\gamma(\alpha + \beta + \gamma) = -1 \times 3$
$= -3$

(v) $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$

$$= \frac{\gamma + \alpha + \beta}{\alpha\beta\gamma} = \frac{3}{-1} = -3$$

(vi) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

$$= \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{-3}{-1} = 3$$

② Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time and the product of its zeroes as $2, -7, -14$ respectively.

Soln -

$$\alpha+\beta+\gamma = 2$$

$$\alpha\beta+\beta\gamma+\gamma\alpha = -7 \quad \text{and} \quad \alpha\beta\gamma = -14$$

$$p(x) = x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma$$

$$= x^3 - 2x^2 + (-7)x - (-14)$$

$$= x^3 - 2x^2 - 7x + 14 \quad \underline{\text{Ans}}$$

③ If the zeroes of $p(x) = x^3 - 3x^2 + x + 1$ are $a-b, a, a+b$. find a and b.

Soln -

$$p(x) = x^3 - 3x^2 + x + 1$$

$$\alpha = a-b,\ \beta = a,\ \gamma = a+b$$

$$\because \alpha+\beta+\gamma = -\frac{(-3)}{1}$$

$$a-\cancel{b}+a+a+\cancel{b} = 3$$

$$3a = 3$$

$$\boxed{a = 1}$$

$$\because \quad \alpha\beta\gamma = -\frac{d}{a}$$

$$(a-b)\,a\,(a+b) = -\frac{1}{1}$$

$$(1-b)\times 1\times(1+b) = -1$$

$$\Rightarrow \quad 1-b^2 = -1$$

$$\Rightarrow \quad 2 = b^2$$

$$\boxed{b = \pm\sqrt{2}}$$

④ If the squared difference for Polynomial $P(x) = x^2 - px + 45$ is 144. find p.

Soln

$$P(x) = x^2 - px + 45$$

$$\alpha+\beta = -\frac{(-p)}{1} = p$$

$$\alpha\beta = 45$$

$$(\alpha-\beta)^2 = 144$$

$$\because \quad (\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$$

$$144 = p^2 - 4\times 45$$

$$144 + 180 = p^2$$

$$324 = p^2$$

So $\boxed{p = 18}$

⑤ A sum of the square of zeroes of $p(x) = x^2 - 8x + k$ is 40. find k.

Soln

$\alpha + \beta = -\frac{(-8)}{1} = 8$

$\alpha\beta = \frac{k}{1} = k$

$\alpha^2 + \beta^2 = 40$

$(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$8^2 = 40 + 2k$

$64 - 40 = 2k$

$\cancel{24}^{12} = \cancel{2}k$

$\boxed{k = 12}$

⑥ If the two zeroes of $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$ find other zeroes.

Soln $\alpha = 2 + \sqrt{3}$, $\beta = 2 - \sqrt{3}$

poly $g(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

So $g(x) = x^2 - (2+\sqrt{3}+2-\sqrt{3})x$

$+ (2+\sqrt{3})(2-\sqrt{3})$

$= x^2 - 4x + 2^2 - (\sqrt{3})^2$

$= x^2 - 4x + 1$

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & \cancel{x^4} - 6x^3 - 26x^2 + 138x - 35 \\
- & \cancel{x^4} \mp 4x^3 \pm x^2 \\
\hline
 & -2\cancel{x^3} - 27x^2 + 138x \\
 & \underset{+}{-}2x^3 \underset{-}{+} 8x^2 \underset{+}{-} 2x \\
\hline
 & -\cancel{35x^2} + \cancel{140x} - \cancel{35} \\
 & \underset{+}{-}35x^2 \underset{-}{+} 140x \underset{+}{-} 35 \\
\hline
 & 0
\end{array}
$$

Now

$x^2 - 2x - 35$

$= x^2 - (7-5)x - 35$

$= x^2 - 7x + 5x - 35$

$= x(x-7) + 5(x-7)$

$= (x-7)(x+5)$

Other zeroes

$x - 7 = 0$

$x = 7$

$x + 5 = 0$

$x = -5$

$\{7, -5\}$

⑦ If the polynomial $x^4-6x^3+16x^2-25x+10$ is divided by x^2-2x+k the remainder come out $x+a$. find k and a.

Soln

$$
\begin{array}{r|l}
 & x^2-4x+(8-k) \\
\hline
x^2-2x+k & x^4-6x^3+16x^2-25x+10 \\
 & -\;(x^4-2x^3+kx^2) \\
\hline
 & -4x^3+(16-k)x^2-25x \\
 & -\;(-4x^3+8x^2-4kx) \\
\hline
 & (8-k)x^2+(4k-25)x+10 \\
 & -\;\left((8-k)x^2+(-16+2k)x+8k-k^2\right) \\
\hline
\end{array}
$$

$$R = (2k-9)x + k^2-8k+10$$

Remainder given $= 1x + a$

On Comparing

$2k-9=1$

$\boxed{k=5}$

$a = k^2-8k+10$

$= 5^2-8\times5+10$

$\boxed{a=-5}$