Vector Practice Sheet

(1) find the magnitude of following
(a) $2\hat{i}-3\hat{j}+4\hat{k}$ (b) $\hat{i}+2\hat{j}-\hat{k}$ (c) $2\hat{i}-\hat{k}$

(2) If Vectors $2\hat{i}-\hat{j}+4\hat{k}$ and $p\hat{i}+4\hat{j}-3\hat{k}$ are prependicular to each other find value of p.

(3) find $\vec{a}\cdot\vec{b}$ and $\vec{a}\times\vec{b}$ for Vectors
(a) $\vec{a}=2\hat{i}+\hat{j}+4\hat{k}$
$\vec{b}=\hat{i}-3\hat{j}-2\hat{k}$
(b) $2\hat{i}-3\hat{j}+6\hat{k}$
$\hat{i}+2\hat{j}-3\hat{k}$

(4) find angle b/w the vectors $\hat{i}-2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$

(5) Show that the Vectors $(2\hat{i}+3\hat{j}+6\hat{k})$ and $(3\hat{i}-6\hat{j}+2\hat{k})$ are prependicular to each other.

(6) find the magnitude of two Vectors $\vec{a}$ and $\vec{b}$ having same magnitude such that the angle b/w them is $60°$ and their scalar product is $\frac{1}{2}$.

(7) find $|\vec{x}|$ for $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=12$ where $\vec{a}$ is Unit Vector.

(8) Show that the Vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ form the Vertices of right angle

(9) find angle b/w Vectors $\vec{a}$ and $\vec{b}$ with magnitude $\sqrt{3}$ and 2 respectively having $\vec{a}\cdot\vec{b}=\sqrt{6}$

(10) find $|\vec{a}\times\vec{b}|$ if $\vec{a}=\hat{i}-7\hat{j}+7\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+2\hat{k}$

(11) Show that $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b}) = 2(\vec{a}\times\vec{b})$

(12) find λ and μ if

$(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k}) = 0$

Answers -

1 (a) $\sqrt{29}$ (b) $\sqrt{6}$ (c) $\sqrt{5}$

2 8

3 (a) -9

4 $\cos^{-1}(5/7)$

6 1

7 $\sqrt{13}$

9 45°

10 $19\sqrt{2}$

12 $\mu = 27/2$, $\lambda = 3$

Solutions

⑥ (a) $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$$|\vec{a}| = \sqrt{2^2 + (-3)^2 + 4^2}$$
$$= \sqrt{4 + 9 + 16}$$
$$= \sqrt{29}$$

② $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ $\perp^r$

and $\vec{b} = p\hat{i} + 4\hat{j} - 3\hat{k}$

then $\vec{a} \cdot \vec{b} = 0$

$$(2\hat{i} - 1\hat{j} + 4\hat{k}) \cdot (p\hat{i} + 4\hat{j} - 3\hat{k}) = 0$$

$$2p - 4 - 12 = 0$$
$$2p = 16$$
$$\boxed{p = 8}$$

⑦

~~③~~ ④ $\vec{a} = 1\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

$$\vec{a} \cdot \vec{b} = 3 + 4 + 3 = 10$$

Now $|\vec{a}| = \sqrt{1 + 4 + 9} = \sqrt{14}$, $|\vec{b}| = \sqrt{9 + 4 + 1} = \sqrt{14}$

$$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$$
$$10 = \sqrt{14} \times \sqrt{14} \cos\theta$$
$$\text{So } \cos\theta = \frac{\cancel{10}\ 5}{\cancel{14}\ 7}$$

$$\boxed{\theta = \cos^{-1}(5/7)}$$

⑥ $|\vec{a}| = |\vec{b}|$

and $\vec{a}\cdot\vec{b} = \frac{1}{2}$, $\theta = 60^\circ$

$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$

$\frac{1}{2} = |\vec{a}| \times |\vec{a}| \cos 60$

$\frac{1}{\cancel{2}} = |\bar{a}|^2 \times \frac{1}{\cancel{2}}$

so $|\bar{a}|^2 = 1$

$\boxed{|\vec{a}| = 1}$ and $\boxed{|\bar{b}| = 1}$

⑦ $(\vec{x} - \vec{a})\cdot(\vec{x} + \vec{a}) = 12$ and $|\vec{a}| = 1$

$|\vec{x}|^2 + \cancel{\vec{x}\cdot\vec{a}} - \cancel{\vec{a}\cdot\vec{x}} - |\vec{a}|^2 = 12$

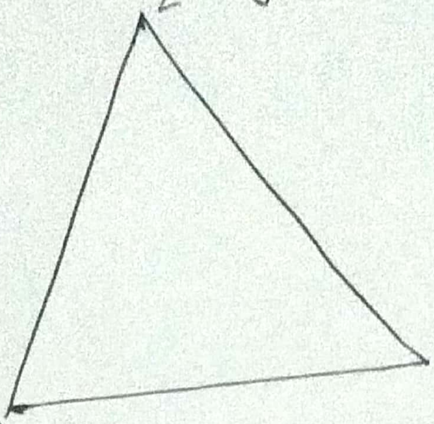
so $|x|^2 - 1^2 = 12$

$|\bar{x}|^2 = 13$

so $\boxed{|\vec{x}| = \sqrt{13}}$

(8)

A $2\hat{i} - \hat{j} + \hat{k}$



C $3\hat{i} - 4\hat{j} - 4\hat{k}$

B $\hat{i} - 3\hat{j} - 5\hat{k}$

$$\vec{AB} = \vec{OB} - \vec{OA}$$
$$= \hat{i} - 3\hat{j} - 5\hat{k} - 2\hat{i} + \hat{j} - \hat{k}$$
$$= -\hat{i} - 2\hat{j} - 6\hat{k}$$
$$|\vec{AB}| = \sqrt{1+4+36} = \sqrt{41}$$

Now

$$\vec{BC} = \vec{OC} - \vec{OB}$$
$$= 3\hat{i} - 4\hat{j} - 4\hat{k} - \hat{i} + 3\hat{j} + 5\hat{k}$$
$$= 2\hat{i} - \hat{j} + \hat{k}$$
$$|\vec{BC}| = \sqrt{4+1+1} = \sqrt{6}$$

Now

$$\vec{CA} = \vec{OA} - \vec{OC}$$
$$= 2\hat{i} - \hat{j} + \hat{k} - 3\hat{i} + 4\hat{j} + 4\hat{k}$$
$$= -\hat{i} + 3\hat{j} + 5\hat{k}$$
$$|\vec{CA}| = \sqrt{1+9+25} = \sqrt{35}$$

$AB = \sqrt{41}$, $BC = \sqrt{6}$, $CA = \sqrt{35}$

$AB^2 = 41$

$BC^2 + CA^2 = 6 + 35$
$= 41$

So $AB^2 = BC^2 + CA^2$

by P.T ΔABC is
Right angle Δ.

(9) $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$, $\bar{a} \cdot \bar{b} = \sqrt{6}$

Now $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$

$\sqrt{6} = \sqrt{3} \times 2 \cos\theta$

So $\cos\theta = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{2}}{\cancel{2}\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}} = \cos 45$

$\boxed{\theta = 45°}$