

Mamdani FIS Combined with LU Decomposition Method and Two-Level LWT for Image Watermarking Technique

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Abstract—Fuzzy theory and linear algebra have a substantial role in various sciences and the biggest effect was in the share of computer science. In the field of image processing, the process of digital image watermarking represents the inclusion of a secret logo or an image called watermark into a host image to get a watermarked image to guarantee copyright protection or ownership. The watermarking will be acceptable if it achieves specific qualities like imperceptibility and robustness. In this paper, Mamdani fuzzy inference system (FIS) was employed on the original image for obtaining the controller parameter which works on finding a trade-off between imperceptibility and robustness, and on the other hand, is combined with the algebraic matrix decomposition method LU for proposing a digital watermarking technique in which the watermark is included in the LH2 band produced by applying two-level of the lifting wavelet transform (LWT) using the modular arithmetic (mod). The experimental results show that the proposed algorithm resistant to many attacks.

Keywords—Mamdani fuzzy inference system (FIS), lower–upper decomposition (LU), lifting wavelet transform (LWT), discrete cosine transform (DCT), image watermarking.

I. INTRODUCTION

Digital watermarking works on hiding a watermark related to the original content of the digital cover. Watermarking represents one of the preferable proposed solutions for digital images copyright protection [1] [2] [3]. In digital images watermarking, the watermarks are included in the cover images in a manner that remains inside these images [4].

In general, mathematics was of remarkable importance in the development of many scientific trends of a technological nature. In particular, fuzzy theory and linear algebra play an integral role in image watermarking in the image processing field. The SVD method is the familiar matrix decomposition method used in image watermarking but this does not prevent that there are other matrix decomposition methods works successfully in this field of science. The working of the matrix decomposition methods is like doing transfers. By using the decomposition the image transfers to another domain but in algebraic qualities. On the other hand, the systems of fuzzy logic are capable of explaining inaccurate information and explaining their decisions. The straightforward manner to perform the Fuzzy Logic is the fuzzy inference system (FIS). The inputs to the FIS are taken from the Human Visual System model. In parallel, matrix decomposition methods in linear

algebra may be used as a transformation to translate an image to the frequency domain and especially used to extract the important features of the image. A. M. Latif [5] used the Fuzzy logic and tabu search for an adaptive digital image watermarking technique. In this technique, the cover image is separated into blocks, and for each block, the transform of parametric slant-Hadamard is implemented. This type of transformation involved several parameters that can be managed for controlling the watermarking requirements like imperceptibility and robustness. M. A. Hajjaji et al. [6] proposed a medical image watermarking scheme. In this scheme, the cover medical image is firstly transformed using the two-level discrete wavelet transform (DWT), then for improving the robustness, Karhunen Loeve Transform (KLT) is implemented on the 8×8 sub-blocks of the second level middle-frequency and low-frequency subbands. After that, for enhancing the factor of imperceptibility, the optimal value of the visibility factor should be determined depending on many parameters of KLT and DWT, and this is done by setting up and then applying the FIS using as input the features that are extracted from Cooccurrence matrix for determining an initial value of visibility factor to each block. B. Jagadeesh et al. [7] presented a blind, imperceptible and robust image watermarking scheme in which the FIS, backpropagation neural networks (BPNN), and discrete cosine transform (DCT) are used in the embedding and extraction procedures. This scheme utilizes the parameters of the human visual system as inputs to the FIS and BPNN. The obtained result from these artificial intelligence techniques is utilized as a factor of weighing that is used to embed the watermark with optimal imperceptibility and energy. L. Zhang and D. Wei [8] proposed an invisible and robust scheme of watermarking in which firstly, the cover and watermark images are transformed using the invariant integer wavelet transform, then, the singular values are found based on QR decomposition and singular value decomposition (SVD) with less time complexity. After that, for improving the robustness of the watermarking against common attacks, the watermark is included in the high frequency (HH) and low frequency (LL) of the transformed cover image, Additionally, for ensuring the security, the double encryption scheme and more keys are utilized in this proposed watermarking scheme[18].

In this paper, the Mamdani FIS and the LU matrix decomposition method are applied to the original image with two-level of lifting wavelet transform (LWT) to embed the

watermark in the LH2 band. The organization of this paper is as follows. Section 2 is devoted to giving previously well-known information and needed in the rest of the paper. The methodology including the embedding and the extraction algorithms are given in section 3. An experimental result is given in section 4 to show the advantages of the proposed technique. In section 5, the main conclusions are offered.

II. BACKGROUND

A. Mamdani FIS

The method of Mamdani FIS was presented to innovate a control system by composing a set of linguistic control rules that were received from the human proficiencies employees. With the system of Mamdani, in each rule, the output is a fuzzy set. The advantages of the Mamdani FIS are intuitive, a more interpretable rule base, have widespread acceptance. Because the systems of Mamdani are very easier and intuitive for understanding the bases of rule, they are very suitable for the applications of the expert systems in which the rules are generated from humans' expert knowledge.

The process of fuzzy inference formulates the mapping from a given input to an output utilizing the fuzzy logic. The fuzzy inference process includes the whole pieces that are described in Logical Operations, Membership Functions, and If-Then Rules [9]. Fig. 1 shows the interpreting of the fuzzy inference diagram.

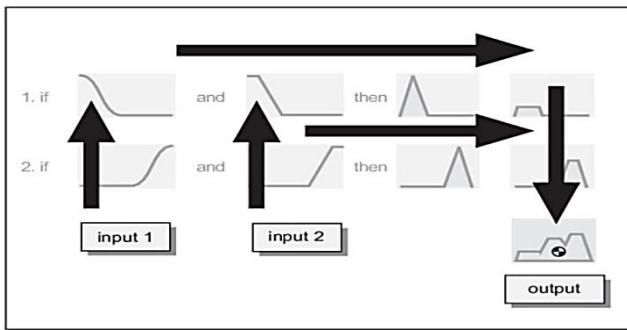


Fig. 1. Interpreting the fuzzy inference diagram.

In this paper, Mamdani fuzzy inference is used as shown in Fig. 2, with the following steps [10]:

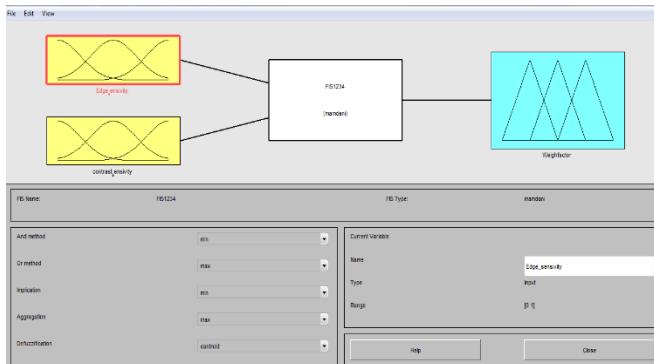
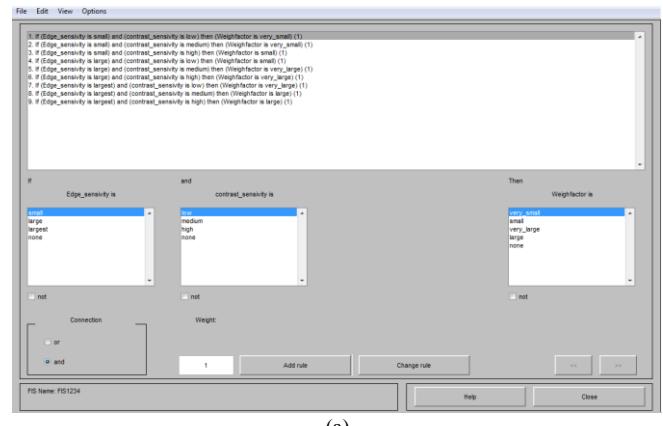


Fig. 2. Mamdani FIS of two inputs and one output.

- STEP 1: The input variables represented by two Human visual parameters the Edge sensitivity and the Contrast sensitivity are computed to fuzzify.

- STEP 2: The result of the antecedent evaluation represents a single number is obtained by the fuzzy operator AND that used in the nine If-Then rules in the next step.
- STEP 3: The nine If-Then rules are representing by choosing the minimum implication operator other than prod. Fig. 3 shows the GUI for these rules.



(a)

[1. If (Edge_sensitivity is small) and (contrast_sensitivity is low) then (Weighfactor is very_small) (1)
2. If (Edge_sensitivity is small) and (contrast_sensitivity is medium) then (Weighfactor is very_small) (1)
3. If (Edge_sensitivity is small) and (contrast_sensitivity is high) then (Weighfactor is small) (1)
4. If (Edge_sensitivity is large) and (contrast_sensitivity is low) then (Weighfactor is small) (1)
5. If (Edge_sensitivity is large) and (contrast_sensitivity is medium) then (Weighfactor is very_large) (1)
6. If (Edge_sensitivity is large) and (contrast_sensitivity is high) then (Weighfactor is very_large) (1)
7. If (Edge_sensitivity is largest) and (contrast_sensitivity is low) then (Weighfactor is very_large) (1)
8. If (Edge_sensitivity is largest) and (contrast_sensitivity is medium) then (Weighfactor is large) (1)
9. If (Edge_sensitivity is largest) and (contrast_sensitivity is high) then (Weighfactor is large) (1)]

(b)

Fig. 3. (a) The GUI, (b) The 9 Rules.

- STEP 4: The aggregation method is applying the functions sum, max, and probabilistic OR, but the max function is chosen in this work to aggregate all the results of the rules in step 3 because it is well accepted and more straightforward.
- STEP 5: The defuzzification method is the last step of the fuzzy inference process. The most popular defuzzification method, the centroid technique, is used in this work. This method works on finding a point denoting the center of the area of the aggregated fuzzy set.

The triangular membership function (see Fig. 4) is used in the above steps among many other functions and defined as follows:

$$\mu(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{m-a}, & a \leq x \leq m \\ \frac{b-x}{b-m}, & m \leq x \leq b \\ 0, & x \geq b \end{cases} \quad (1)$$

Or more concise $\mu_A(x) = \max(\min\left(\frac{x-a}{m-a}, \frac{b-x}{b-m}\right), 0)$ (2)

where a, m, b represent the x coordinates of the three vertices of $\mu_A(x)$ in a fuzzy set A (a : lower boundary and b : upper boundary, the degree of membership is 0, m : the center, the degree of membership is 1).

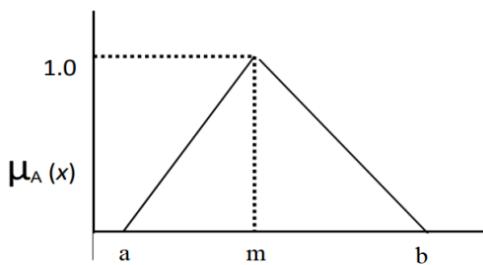


Fig. 4. The Triangular Membership Function.

B. LU Decomposition

In linear algebra [11] [12] and numerical analysis, lower-upper decomposition (LU) (or LU factorization) operators a matrix A as the product of a lower triangular matrix (L) and an upper triangular matrix(U), as well as sometimes the product includes a permutation matrix. In 1948 Turing [13] gives LU decomposition which is a fundamental adjusted manner of Gaussian elimination. Usually, LU decomposition is used to solve the square systems of linear equations. It is a significant operation when computing the matrix determinant or matrix inverting, such as; for a 3×3 matrix A, LU decomposition can be given as follows:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$A = LU = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$$

$$= \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{pmatrix} \quad (3)$$

C. Discrete Cosine Transform (DCT)

The DCT represents a technique for converting the signal from time-domain representation to frequency band form. For a given image A of size n×n, in digital image processing, the two-dimensional DCT is given as:

$$C_{nm} = \alpha n \alpha m \sum_{i=0}^{I-1} \sum_{j=0}^{J-1} Z_{ij} \left(\frac{\cos(2\pi i+1)n}{2I} \right) \left(\frac{\cos(2\pi j+1)m}{2J} \right), \text{ for } 0 \leq n \leq I-1 \text{ and } 0 \leq m \leq J-1 \quad (4)$$

$$\alpha n = \begin{cases} \frac{1}{\sqrt{J}}, & n = 0 \\ \frac{2}{\sqrt{J}}, & 1 \leq n \leq J-1 \end{cases} \quad \alpha m = \begin{cases} \frac{1}{\sqrt{I}}, & m = 0 \\ \frac{2}{\sqrt{I}}, & 1 \leq m \leq I-1 \end{cases}$$

DCT is characterized by the property that most of the important optical functions are concentrated around the image in a few DCT parameters [14], therefore, we observe the use of DCT frequently in image compression applications [15].

D. Lifting Wavelet Transform (LWT)

The LWT has many differences compared with the classical wavelet (DWT) because 1- LWT can compute more efficiently. 2- The LWT doesn't need big memory space, 3- LWT has integer coefficients, 4- DWT capable to address the impairment of quantization errors from the classical wavelet transform. LWT facilitates the case by immediately analyzing

the case in the special domain [16]. The major precept of the LWT is to establish a new wavelet with improved characteristics depend on an ease wavelet that represents the basic key of lifting. LWT turns into a robust scheme for the different applications used in image processing: image compression [17], watermarking, and pattern recognition. The lifting wavelet transform (LWT) and its inverse transform (ILWT) are of one dimensional (ID) signals. An obvious method to use LWT for two-dimensional signals like images is to use row-column (horizontal-vertical) or column-row (vertical-horizontal) passes which the corresponding one-dimensional LWT. In general, this strategy contains three essential phases:

- Splitting: The cover image Im is dismantled into even and odd non-overlapping elements of $Im_e(x)$ and $Im_o(x)$.
- $$Im_e(x) = Im(2x), \quad Im_o(x) = Im(2x + 1) \quad (5)$$
- Prediction: In this phase, the value of an odd element is predicted using even elements:

$$E(x) = Im_o(x) - P(Im_e(x)) \quad (6)$$

where $P(\cdot)$ denotes the prediction operator. The prediction phase represents the high pass filtering operation and $E(x)$ refers to the high-frequency synthesis which is the error between the original element and its predicted value.

- Updating: This phase is considered as the low-pass filtering operation and $L(x)$ refers to the low-frequency synthesis which shows the coarse approximation to the original image:

$$L(x) = Im_o(x) + U(E(x)) \quad (7)$$

III. METHODOLOGY

In this section, an imperceptible and robust scheme of watermarking is proposed, which relies on FIS and LU Matrix Decomposition using two-level of the LWT. The proposed watermarking scheme can be characterized as follows:

A. Watermark Embedding based on FIS and LU Matrix Decomposition

The process of embedding the watermark into the cover image based on FIS and LU Matrix Decomposition is illustrated in Fig. 5. In the beginning, the low-high (LH) band (mid-frequency) of the 2- level LWT is divided into 2×2 overlapping blocks. This is because the needed capacity to embed the watermark is 64×64 . The size of the original image is 512×512 , so that each band after applying 2-level LWT attains a size of 128×128 . As a result, the LH2 band will consist of 64×64 blocks which are the same size as the watermark .

The use of the matrix U of the algebraic matrix decomposition allows finding a suitable position to embed the watermark without remarkably appearing on the image and the watermark can be extracted correctly itself.

Mamdani FIS is used to generate the weighting factor for embedding the watermark to control balance achieved between robustness and imperceptibility where the values of

the robustness and imperceptibility vary by the value of the weighting factor, and the main steps of this process are detailed as follows:

1) Input (512×512) a grayscale original image and (64×64) binary watermark image.

2) First, divide the original grayscale image into (8×8) blocks, and for each block, apply DCT.

3) For each block, compute the contrast sensitivity and edge sensitivity.

4) Provide these parameters as inputs to the Mamdani FIS built on 9 fuzzy rules to generate the weight factor α .

5) The original image is decomposed by a two-level of the LWT.

6) Divide the band LH2 into 2×2 non-overlapping blocks, the number of blocks is 64×64 .

7) Apply the LU matrix decomposition on each block.

8) Embed the binary watermark bits in the block U_{ij} according to ij th bit:

$$\begin{aligned} U_{ij}(2,2) &= U_{ij}(2,2) - U_{ij}(2,2) \bmod \alpha + T1 \quad \text{if } w(i,j) = 1 \\ U_{ij}(2,2) &= U_{ij}(2,2) - U_{ij}(2,2) \bmod \alpha + T2 \quad \text{if } w(i,j) = 0 \end{aligned} \quad (8)$$

where α represents the weight factor gained from the designed FIS and $T1 = 0.75 * \alpha$, $T2 = 0.25 * \alpha$ and the mod is the modulo operation.

9) Convert block to the matrix and obtain a watermarked image.

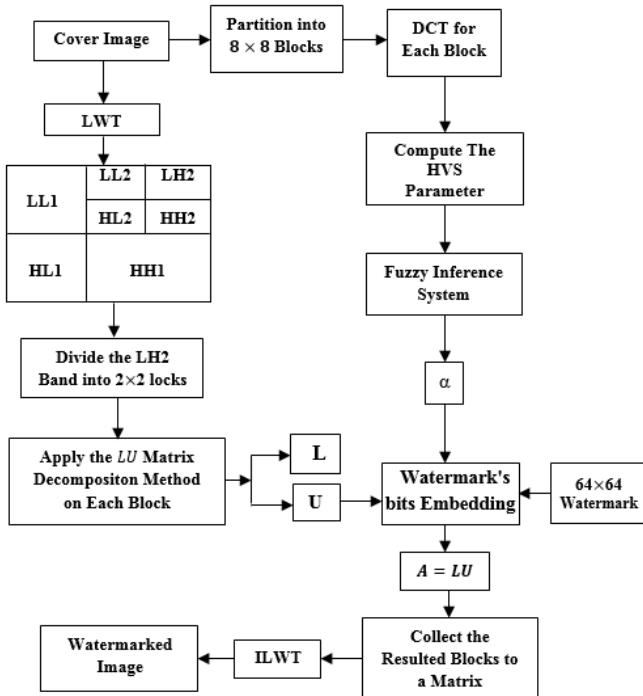


Fig. 5. The Diagram of Embedding Algorithm.

B. Watermark Extraction based on FIS and LU Matrix Decomposition

The process of extracting the watermark of the proposed scheme is illustrated in Fig. 6. As can be seen, the original

cover image is unrequired in the extraction process of the watermark. The extraction steps are given as follows:

- 1) Input the 512×512 watermarked image.
 - 2) Divide the watermarked image into (8×8) blocks, and for each block, apply the DCT.
 - 3) For each block, compute the contrast sensitivity and edge sensitivity.
 - 4) Supply the obtained parameters as inputs to the FIS to generate the weight factor β .
 - 5) The watermarked image is decomposed by a two-level LWT.
 - 6) Divide the band LH2 into 2×2 non-overlapping blocks, the number of blocks is 64×64 .
 - 7) Apply the LU matrix decomposition method to each block.
 - 8) Extraction binary watermark bits from the blocks U_{ij} according to
- $$\begin{aligned} w(i,j) &= 0 \quad \text{if } U_{ij}(2,2) \bmod \beta < \text{ave} \\ w(i,j) &= 1 \quad \text{if } U_{ij}(2,2) \bmod \beta > \text{ave} \end{aligned} \quad (9)$$

where β represents the weight factor gained from the designed FIS and $\text{ave} = (T1 + T2)/2$ represents the average.

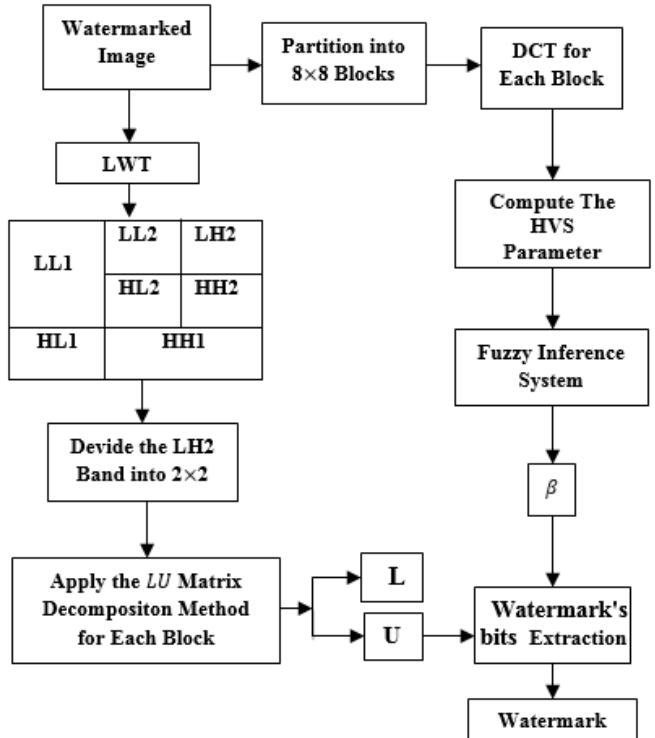


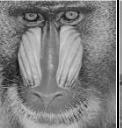
Fig. 6. The Diagram of Extraction Algorithm.

IV. EXPERIMENTAL RESULTS

In this section, some experiments are implemented to assess the robustness and imperceptibility of the proposed watermarking algorithm. The proposed image watermarking technique is examined with different grayscale cover images of size 512×512 . A binary image of size 64×64 is utilized as a

watermark image. Table I shows the watermark and the original images utilized for testing the proposed algorithm.

TABLE I. THE WATERMARK AND THE IMAGES UTILIZED FOR TESTING THE PROPOSED ALGORITHM

Image 1	Image 2	Image 3	Image 4	Watermark Image
				

To evidence the soundness of the proposed algorithm of watermarking, some results are given. Seven sorts of attacks are utilized to test the robustness of the proposed watermarking algorithm. In general, the performances of image watermarking techniques are measured by the robustness, invisibility, computation complexity, etc. Two metrics are utilized in this paper for measuring the robustness and imperceptibility of the presented technique. The PSNR is a good tester for the watermark visibility assess and it is computed by the following equation:

$$\text{PSNR} = 10\log_{10}\left(\frac{\text{MAX}^2}{\text{MSE}}\right) \quad (8)$$

where

$$\text{MSE} = \frac{1}{pq} \sum_{i=0}^{p-1} \sum_{j=0}^{q-1} [I(i,j) - K(i,j)]^2 \quad (9)$$

MAX is the maximum grayscale value which is equal to 255.

The matching between the extracted watermark W' and the authentic watermark W is computed based on NC (the normalized correlation) between W and W' .

$$NC = \frac{\sum_i \sum_j w(i,j) \cdot w'(i,j)}{\sqrt{\sum_i \sum_j w(i,j)} \sqrt{\sum_i \sum_j w'(i,j)}} \quad (10)$$

The values of PSNR and NC to test images with no attack are explained in Table II.

TABLE II. THE VALUES OF PSNR & NC TO TEST IMAGES WITH NO ATTACK.

Images	Image1	Image2	Image3	Image4
PSNR	44.6788	45.8655	47.6877	44.9221
NC	1	1	1	1

The invisibility of the results of the proposed algorithm is shown Table III, in which the average PSNR values of all watermarked images after attack are listed.

TABLE III. THE PSNR VALUES UNDER DIFFERENT ATTACKS TO TEST IMAGES

Types of Attacks	The PSNR Values for Watermarked Images			
	Image1	Image2	Image3	Image4
1% Salt pepper noise	39.1131	38.3837	39.4522	39.5953
5% Salt pepper noise	31.4219	30.1906	32.3879	31.2811

75% Jpeg compression	29.0232	28.9308	28.5726	28.9971
Gaussian noise (mean=0 & variance=0.0004)	38.4736	38.449	37.8585	38.4691
Hist. equalization	19.019	20.5258	16.4809	19.0768
Speckle noise	31.6554	31.7192	31.5054	31.5857
Image adjust.	22.2909	21.3985	19.3864	17.758

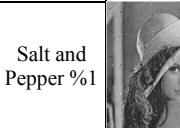
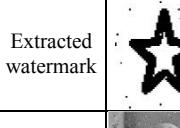
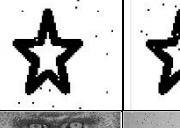
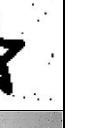
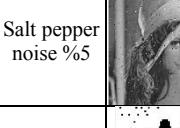
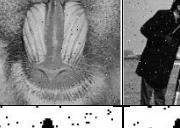
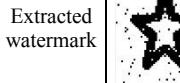
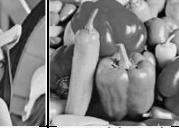
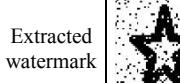
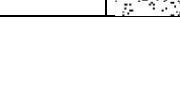
For showing the robustness of the proposal, diverse attacks are implemented on the watermarked image to assess the robustness of the proposed scheme as shown in Table IV.

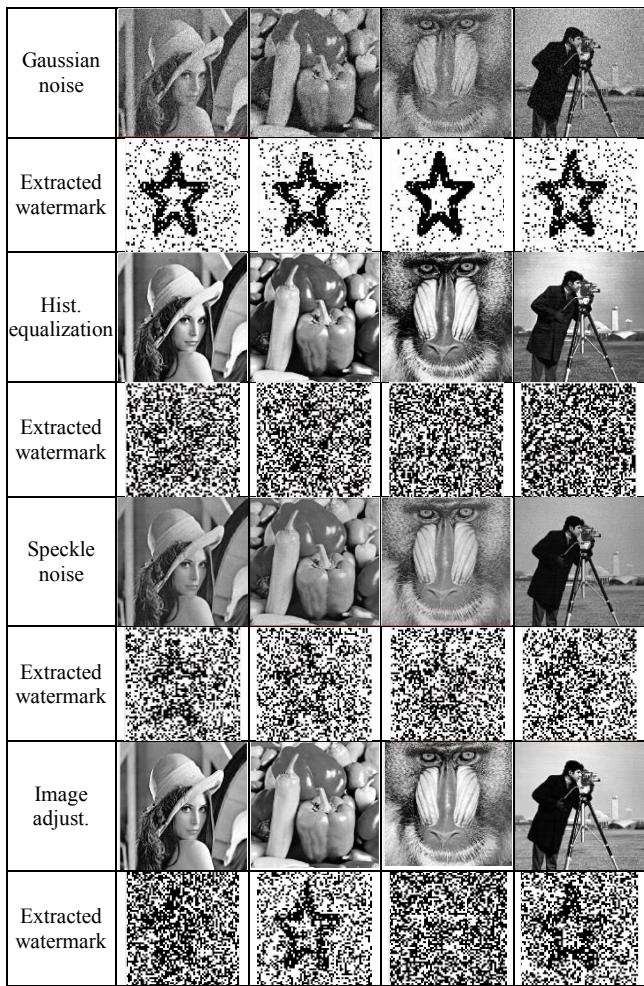
TABLE IV. THE VALUES OF NC UNDER DIFFERENT ATTACKS TO TEST IMAGES

Types of Attacks	The NC Values for Watermarked Images			
	Image1	Image2	Image3	Image4
1% Salt pepper noise	0.9970	0.9971	0.9975	0.9964
5% Salt pepper noise	0.9850	0.9881	0.9856	0.9887
75% Jpeg compression	0.9550	0.9704	0.9883	0.9625
Gaussian noise (mean=0 & variance=0.0004)	0.91149	0.9207	0.9607	0.9183
Hist. equalization	0.70821	0.68352	0.63641	0.62256
Speckle noise	0.75454	0.76079	0.74513	0.75336
Image adjust.	0.71123	0.7916	0.5982	0.7394

Table V explains the watermarked images and the extracted watermark image from each one respectively after attack implementation.

TABLE V. TEST OF THE EXTRACTED WATERMARK AFTER ATTACKS.

Attack	Image1	Image2	Image3	Image4
Extracted watermark				
Salt and Pepper %1				
Extracted watermark				
Salt pepper noise %5				
Extracted watermark				
JPEG compression				
Extracted watermark				



V. CONCLUSION

There are a lot of watermarking techniques that differ in their mechanisms and tools used to complete the embedding process. In the proposed algorithm that depending on the Mamdani fuzzy inference system (FIS) and the algebraic matrix decomposition method (LU) using two-level of the lifting wavelet transform (LWT), seven kinds of attacks are utilized for testing the imperceptibility and robustness, 4 types of these attacks show the resistant of the algorithm. The behavior of the synthesis between the FIS, LU, and the two-level of the LWT is showing the importance of every one of these components.

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