FINAL PROJECT REPORT: Linear Feedback Controls (EECS 565)

PROJECT TITLE: CONTROL SYSTEM DESIGN FOR REACTIVE ION ETCHING (RIE) PROCESS

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1. **Multivariable Feedback Controller Design**

a)

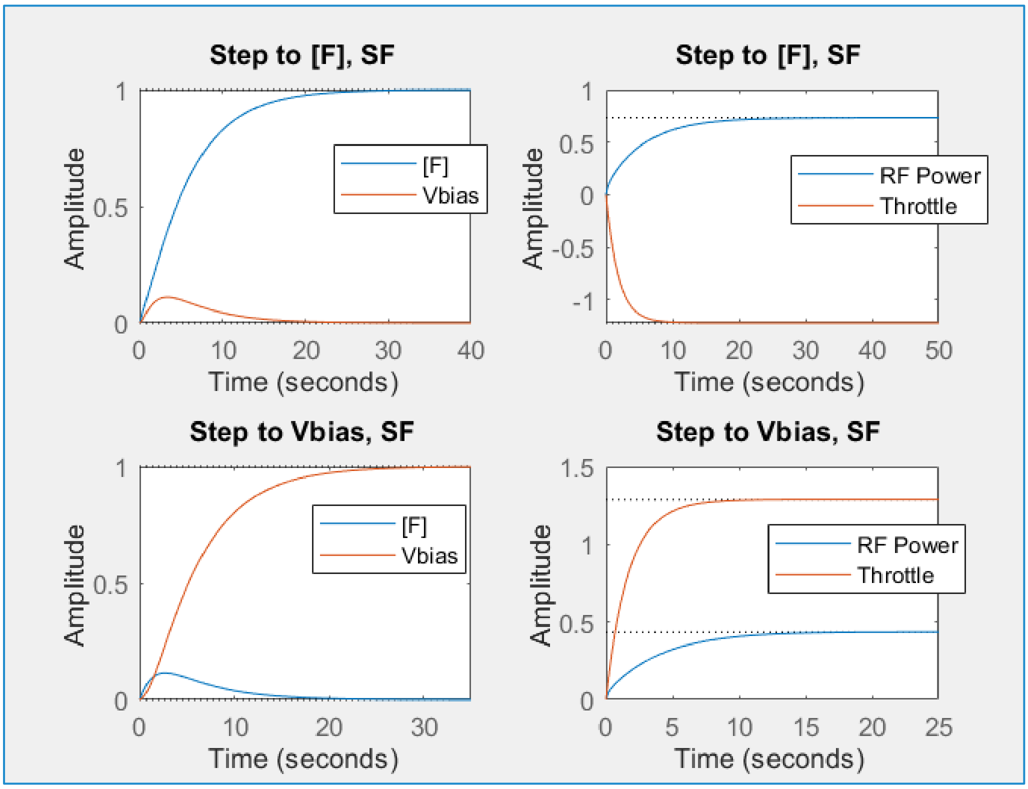


Figure: Step Responses with Multivariable Controller

- Yes, it is possible for us to achieve better compromise among the design goals using a multivariable controller than it was possible for a decentralized controller as shown above.
- We have achieved better step responses than those shown in the Figure(1) of Problem PDF uploaded in Canvas. Our design results in much better settling time, faster response and desired steady state response behavior followed by fulfilling our other design performance specifications.

```
%% Part 1(A): Linear Quadratic Regulator with Integrators

% Augment state equations so that you can do integral control
Aaug = [AP zeros(nx,ny);
    CP zeros(ny,ny)];
Baug = [BP;
    zeros(ny,nu)];
Caug = [CP zeros(ny,ny)];
%%
% LQR Weighting Matrices
Q1 = diag([2.5,2.5]);
Qi = diag([0.1,0.1]);
Q = CP'*Q1*CP;
R = diag([0.1,0.1]);
% LQ state feedback gain
K = lqr(Aaug,Baug,[Q zeros(nx,nu); zeros(nu,nx) Qi],R);
%%
% Closed loop state equations with state feedback and integrators.
sys = ss(Aaug-Baug*K,[zeros(nx,nu); -eye(2)],Caug,0);
sys1 = ss(Aaug-Baug*K,[zeros(nx,nu); -eye(2)],-K,0);
% Verify that closed-loop is stable (Check to verify no bugs in code)
isstable(sys);
% Time vector
Tf = 50;
Nt = 500;
t = linspace(0,Tf,Nt);

% Step responses
trans = tf(sys);
trans_1 = tf(sys1);

figure(1)
% set(findall(gcf,'type','line'),'linewidth',3);
subplot(221)
step(trans(1,1))
hold on
step(trans(2,1))
hold on
title('Step to [F], SF')
legend('[F]','Vbias')

subplot(222)
step(trans_1(1,1))
hold on
step(trans_1(2,1))
hold on
title('Step to [F], SF')
legend('RF Power','Throttle')

subplot(223)
step(trans(1,2))
hold on
step(trans(2,2))
hold on
title('Step to Vbias, SF')
legend('[F]','Vbias')

subplot(224)
step(trans_1(1,2))
hold on
step(trans_1(2,2))
hold on
title('Step to Vbias, SF')
legend('RF Power','Throttle')
```

b)

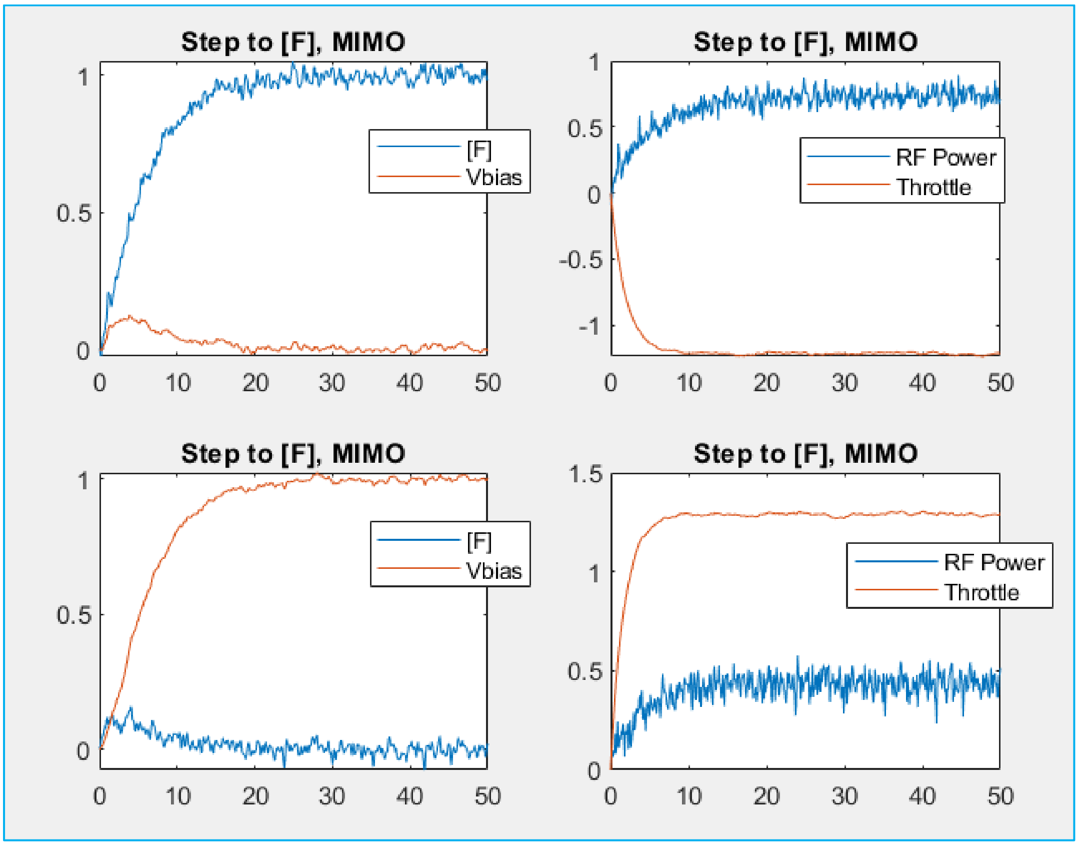


Figure : Step Responses with Multivariable Controller and [F] Sensor Noise.

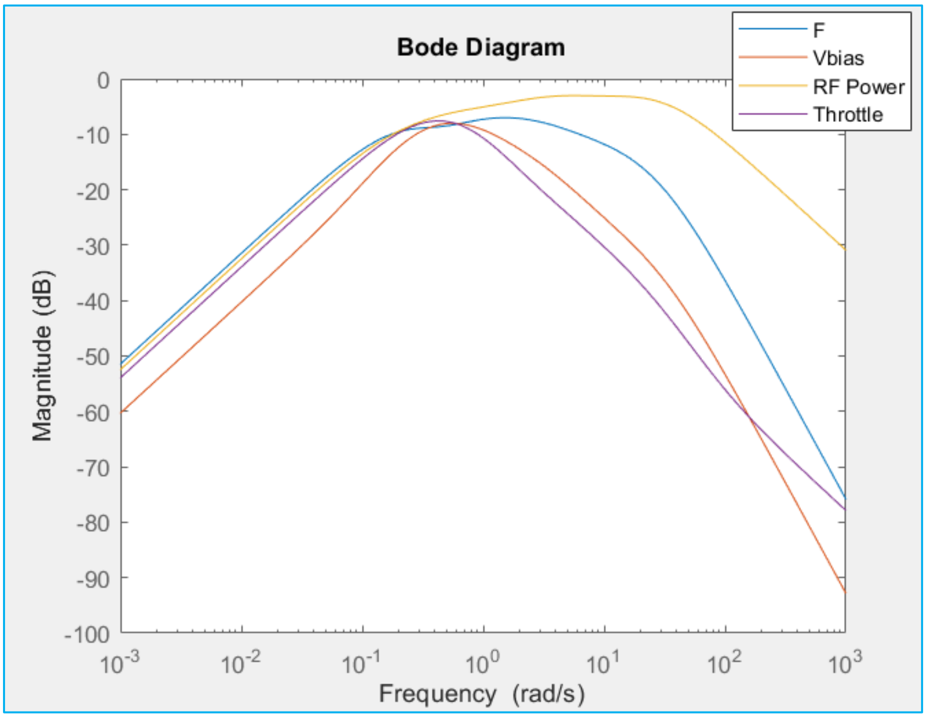


Figure : Bode Plots of the Closed Loop Transfer Functions from [F] Noise to [F], Vbias, Power, and Throttle

- Yes, as ‘q’ is increasing, there is an increased response to sensor noise. For our design, the largest value of ‘q’ that is consistent with satisfying the required specification imposed on the response to [F] noise is 2.5 .
- Our design had better step response and Step to [F] for F response, the settling time is around 15.9 secs (corresponding to 1.a graph subplot 1).

NOTE:

One important thing to note is since our step response is faster than that of graph given in the Problem PDF uploaded in Canvas, We have achieved a ‘q’ value of around 2.5. But if we slow down our step responses then the value of ‘q’ increases. When it is settling for step to [F] for F response is around 22 secs, the max value of q was determined to be around 9.5.

c)

- No, it is not possible to achieve arbitrarily good recovery of the state feedback loop by tuning the observer only. For our design problem, the plant has a “time delay” similar to the presence of RHP Zeros. Due to the time delay in the plant, the loop bandwidth will be restricted to a certain upper limit.
- So, if the upper limit for bandwidth (loop cross-over frequency) corresponding to the time delay is exceeded, tuning the observer gain will not help to recover the state feedback margins and the loop.
- To achieve the loop transfer recovery of the state feedback loop, we can attempt to increase the penalty on the control input matrix, ‘R’, This results in smaller feedback gains which corresponds to a small bandwidth.
- So, while designing the state feedback loop, if the loop bandwidth is chosen to be at least reasonably low compared to the time delay, the LTR can converge to the state feedback loop.
- Hence, when there is a time delay (RHP Zeros), it is possible to apply LTR to recover the margins even though the theorem will not hold technically, as long as we design the loop crossover that is consistent with the fundamental limits of Time Delay (RHP Zeroes) in the plant.

d)

Stability Margins:

(i) Loop-at-a-time margins at the plant input:

For state feedback:

Variables - AM_sf

AM_sf

2x1 struct with 7 fields

Fields	GainMargin	GMFrequency	PhaseMargin	PMFrequency	DelayMargin	DMFrequency	Stable
1	3.2716e-17	0	115.4029	17.2177	0.1170	17.2177	1
2	4.4428e-16	0	90.0588	0.5718	2.7489	0.5718	1

For Recovered with observer:

Variables - AM

AM

2x1 struct with 7 fields

Fields	GainMargin	GMFrequency	PhaseMargin	PMFrequency	DelayMargin	DMFrequency	Stable
1	[3.6864e-15,Inf]	[0,Inf]	127.5615	1.5875	1.4025	1.5875	1
2	[3.9723e-17,10.5003,Inf]	[0,1.2931,Inf]	74.1737	0.1911	6.7728	0.1911	1

(ii) Multi-Loop Disk margins at the plant input:

For Symmetric or Regular disk margin:

For State Feedback:

```
MMI_sfs =

  struct with fields:

           GainMargin: [0.0044 228.4483]
          PhaseMargin: [-89.4984 89.4984]
           DiskMargin: 1.9826
           LowerBound: 1.9826
           UpperBound: 1.9867
            Frequency: 1.8336
    WorstPerturbation: [2×2 ss]
```

For Recovered with observer:

```
MMI_s =

  struct with fields:

           GainMargin: [0.2159 4.6323]
          PhaseMargin: [-65.6364 65.6364]
           DiskMargin: 1.2898
           LowerBound: 1.2898
           UpperBound: 1.2924
            Frequency: 0.4328
    WorstPerturbation: [2×2 ss]
```

For T-based disk margin: ("T-based" disk, i.e. a disk centered at 1)

For State Feedback:

```
>> MMI_sf

MMI_sf =

  struct with fields:

            GainMargin: [0.0020 1.9980]
           PhaseMargin: [-59.8650 59.8650]
            DiskMargin: 0.9980
            LowerBound: 0.9980
            UpperBound: 1
             Frequency: 0
    WorstPerturbation: [2×2 ss]
```

For Recovered with observer:

```
>> MMI

MMI =

  struct with fields:

            GainMargin: [0.0020 1.9980]
           PhaseMargin: [-59.8650 59.8650]
            DiskMargin: 0.9980
            LowerBound: 0.9980
            UpperBound: 1.0000
             Frequency: 0
    WorstPerturbation: [2×2 ss]
```

(iii) Unstructured (fully-coupled) stability margin (USM) at the plant input:

For state feedback:

```
>> StabMarg_Tsf

StabMarg_Tsf =

    1.0000
```

For Recovered with observer:

```
>> StabMarg_TI

StabMarg_TI =

    0.9994
```

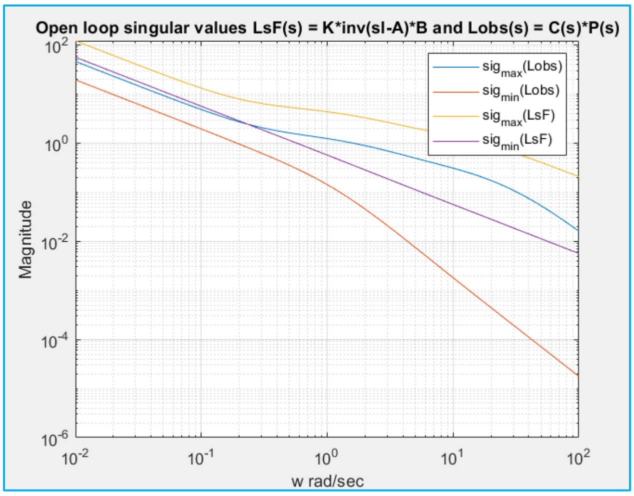


Figure: The State Feedback Loop Transfer Function, Lsf and the Input Loop Transfer Function, LI

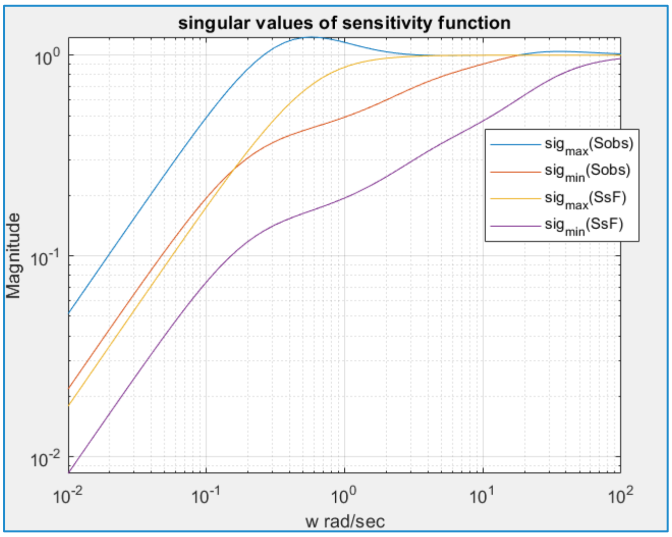


Figure: The State Feedback Sensitivity Function, Ssf and the Input Sensitivity Function, SI

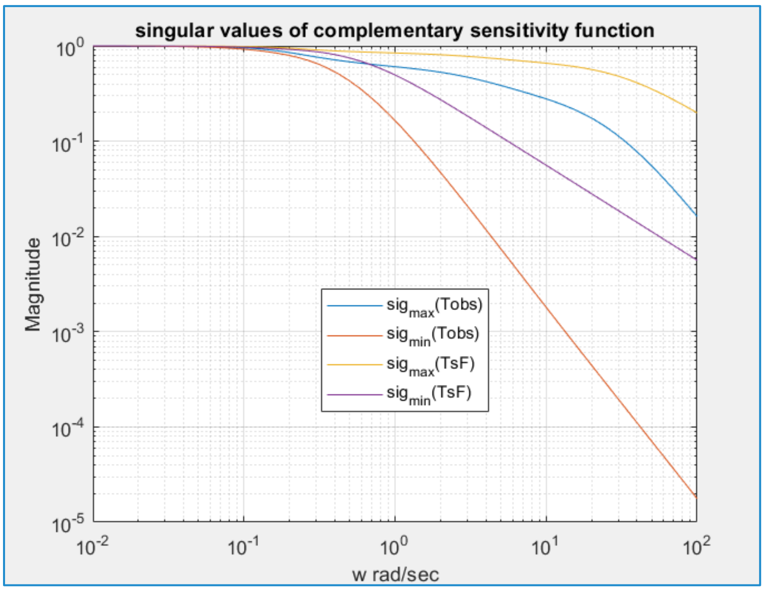


Figure: The State Feedback Complementary Sensitivity Function, Tsf and the Input Complementary Sensitivity Function, TI

```matlab
%% Part 1(D): Stability Margins and Comparison With State Feedback

% Loop-at-a-time margins at the plant input

% for state feedback
Lsf = ss(Aaug,Baug,K,[])
AM_sf = allmargin(Lsf)

AM_sf(1)%For the first channel
AM_sf(2)%For the second channel

%recovered
LI = (Cobs * PN);

AM = allmargin (LI)
AM(1)%For the first channel
AM(2)%For the second channel


% Multi-Loop Disk margins at the plant input

% Regular Disk Margin (symmetric)
% for state feedback
[DMI_sfs,MMI_sfs] = diskmargin(Lsf);
MMI_sfs

% for recovered
[DMI_s,MMI_s] = diskmargin(Cobs*PN);  % T-based disk margin
MMI_s

% For T-based Disk margin ("T-based" disk, i.e. a disk centered at 1 )
% for state feedback
[DMI_sf,MMI_sf] = diskmargin(Lsf,-1);  % T-based disk margin
MMI_sf

% for recovered
[DMI,MMI] = diskmargin(Cobs*PN,-1);  % T-based disk margin
MMI


% Unstructured (fully-coupled) stability margin (USM) at the plant input

%For State Feedback
Tsf = feedback(Lsf,eye(size(Lsf)));
[nsf,wsf] = hinfnorm(Tsf);
StabMarg_Tsf = 1/nsf % unstructured stability margin

%For Recovered
TI = feedback(LI,eye(size(LI)));
[np,wp] = hinfnorm(TI);
StabMarg_TI = 1/np % unstructured stability margin

% Input loop transfer function: Compare Lsf to LI
Lsf = ss(Aaug,Baug,K,[])
Tsf = feedback(Lsf,eye(size(Lsf)));
TI = feedback(LI,eye(size(LI)));
```

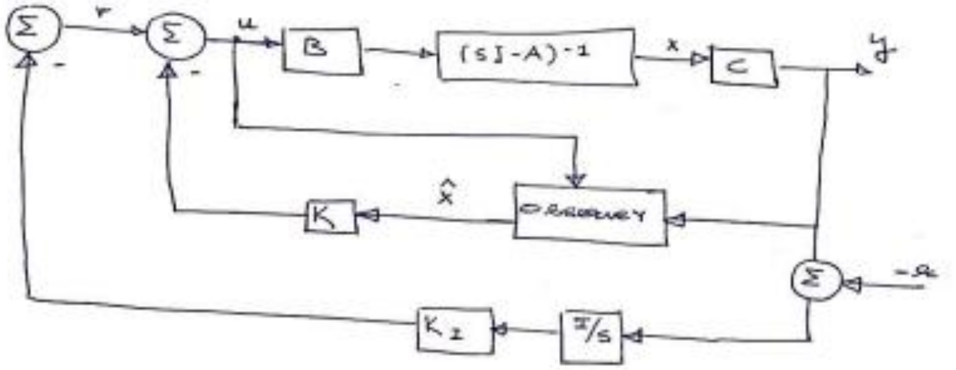
```
figure (5) % for Loop Gain
[sv,wout]=sigma(LI,{1e-2, 1e2});
loglog(wout,sv(1,:),wout,sv(2,:))
grid on
hold on
[sv,wout]=sigma(Lsf,{1e-2, 1e2});
loglog(wout,sv(1,:),wout,sv(2,:))
hold off
title('Open loop singular values LsF(s) = K*inv(sI-A)*B and Lobs(s) = C(s)*P(s)')
legend('sig_{max}(Lobs)','sig_{min}(Lobs)','sig_{max}(LsF)','sig_{min}(LsF)')
xlabel('w rad/sec')
ylabel('Magnitude')

figure(6) % for Complimentary sensitivity
[sv,wout]=sigma(TI,{1e-2, 1e2});
loglog(wout,sv(1,:),wout,sv(2,:))
grid on
hold on
[sv,wout]=sigma(Tsf,{1e-2, 1e2});
loglog(wout,sv(1,:),wout,sv(2,:))
hold off
title('singular values of complementary sensitivity function')
legend('sig_{max}(Tobs)','sig_{min}(Tobs)','sig_{max}(TsF)','sig_{min}(TsF)')
xlabel('w rad/sec')
ylabel('Magnitude')
Ssf = feedback(eye(size(Lsf)),Lsf);
SI = feedback(eye(size(LI)),LI);

figure (7) % for Sensitivity
[sv,wout]=sigma(SI,{1e-2, 1e2});
loglog(wout,sv(1,:),wout,sv(2,:))
grid on
hold on
[sv,wout]=sigma(Ssf,{1e-2, 1e2});
loglog(wout,sv(1,:),wout,sv(2,:))
hold off
title('singular values of sensitivity function')
legend('sig_{max}(Sobs)','sig_{min}(Sobs)','sig_{max}(SsF)','sig_{min}(SsF)')
xlabel('w rad/sec')
ylabel('Magnitude')
```

- Based on the above values for stability margins for loop-at-a-time, Multi-loop disk margins (symmetric and T-based), and Unstructured margins for both feedback design and recovered design with an observer, we see that the margins for the state feedback design are good, and we are able to recover the margins sufficiently well with an observer as desired.

2) **<u>Reverse Engineering the Multivariable Controller</u>**

a)

2]
a]



→ From the above figure, we observe that there is 'no' disturbance being added to the plant system.

→ Also, as per linear systems theory, the command response of the system is unaffected by the presence of an observer.

From the above conclusions we can conclude that $\boxed{x = \hat{x}}$. → ②

Given,

$$P(s) = C(sI - A)^{-1}B$$

For the system, $\dot{x} = Ax + Bu$, $y = Cx$

$$u = -K\hat{x} - K_I w$$

~~$\rightarrow \dot{w}(s) = y - r$~~

$\rightarrow \quad \dot{w} = y - r$

Taking the Laplace Transform,

$$sW(s) = Y(s) - R(s)$$

$$\Rightarrow \boxed{W(s) = \frac{1}{s}\left[Y(s) - R(s)\right]}$$

$\rightarrow \quad \dot{x} = Ax + Bu$

Taking the Laplace Transform,

$$sX(s) = AX(s) + BU(s)$$

$$\therefore \boxed{X(s) = (sI - A)^{-1} BU(s)}$$

From $U(s) = -KX(s) - K_I W(s)$

$$U(s) = -K(sI - A)^{-1} B \left\{U(s)\right\} - \frac{K_I}{s}\left[Y(s) - R(s)\right]$$

$$U(s)\left[I + K(sI - A)^{-1} B\right] = -\frac{K_I}{s}\left[Y(s) - R(s)\right]$$

$$\therefore \boxed{U(s) = -\frac{K_I}{s}\left[Y(s) - R(s)\right]\left[I + K(sI - A)^{-1} B\right]^{-1}}$$

$$Y(s) = P(s).U(s)$$

$$Y(s) = P(s).\left\{-\frac{K_I}{s} Y(s).\left[I + K(sI - A)^{-1} B\right]^{-1} + \frac{K_I}{s} R(s) \cancel{[I + K(sI - A)^{-1}} \left[I + K(sI - A)^{-1} B\right]^{-1}\right\}$$

$$Y(s)\left\{I + P(s).\frac{K_I}{s}\left[I + K(sI - A)^{-1} B\right]^{-1}\right\} = \frac{K_I}{s}.R(s).P(s)\left[I + K(sI - A)^{-1} B\right]^{-1}$$

$$\therefore \frac{Y(s)}{R(s)} = T_{yr}(s) = \left(I + P(s)\left(I + K(sI - A)^{-1} B\right)^{-1}\frac{K_I}{s}\right)^{-1}.P(s).\left(I + K(sI - A)^{-1} B\right)^{-1}\frac{K_I}{s}$$

- From the above formulation derived for the transfer function from the reference input r to the system output y, we can observe that the transfer function does not depend on the observer.
- The primary reason is the absence of any disturbance to the system which may require us to use the observer to estimate the states. We know from linear systems theory, that the command response for the system is unaffected by the presence of an observer, hence we do not need to estimate the states for this case.

b)

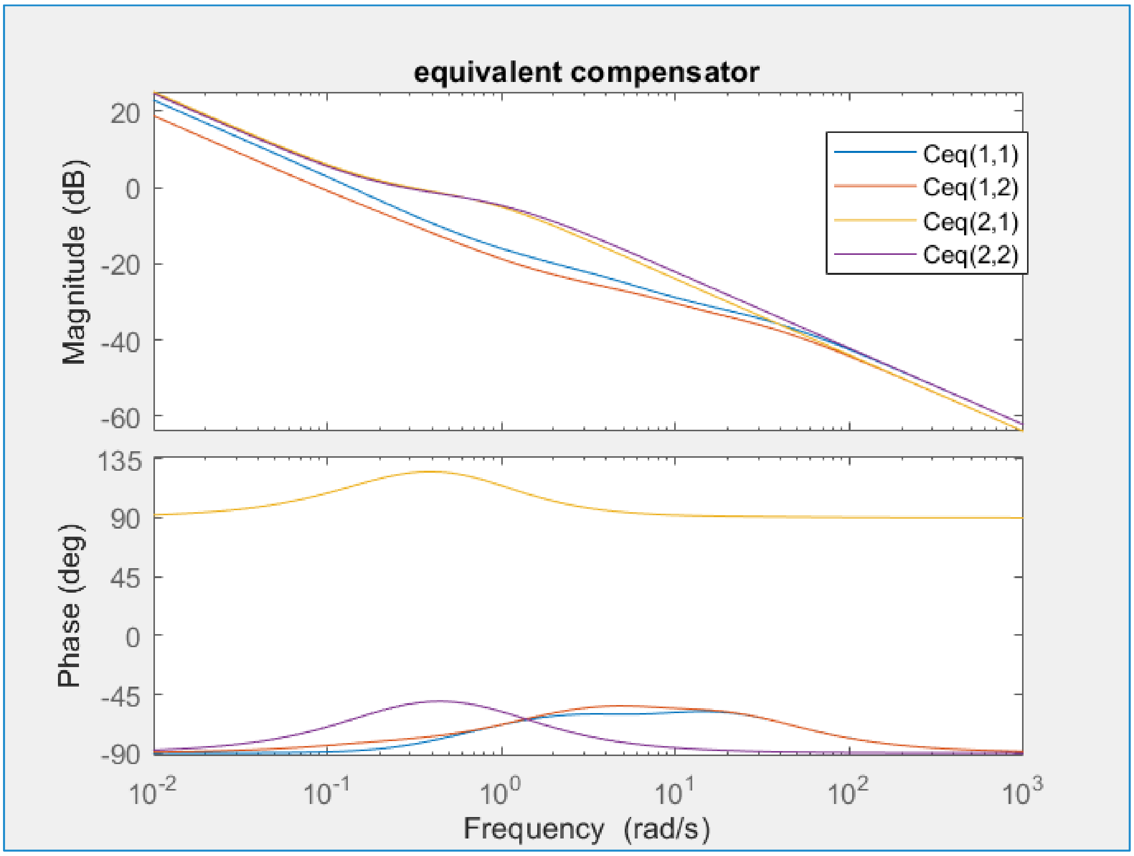


Figure: Equivalent Compensator

- From the above bode plot we can conclude that Ceq(1,1) and Ceq(1,2) are more or less the same (similar ss / tf) with slight differences as the Bode magnitude and phase plot are very similar to each other.
- Also, for Ceq(2,1) and Ceq(2,2) 's bode magnitude are similar and from the corresponding phase plot, we can see that Ceq(2,1)= - Ceq(2,2) has a phase of 180 degrees meaning negative complement to each other.

```
%% Part 2(B): Equivalent Controller

% Equivalent controller
%   Ceq = inv[ I+K1 inv(sI-A) B] (KI/s)
sys = ss(AP,BP,K(:,1:8),0);
tr_ = tf(1,[1 0]);
% Ceq = tf(inv( eye(2) + K(:,1:8)*inv(s*eye(8) - AP)*BP)*K(:,9:10)/s);
Ceq = inv(eye(2) + sys)*K(:,9:10)*tr_;
figure(8)
bode(Ceq(1,1))
hold on
bode(Ceq(1,2))
hold on
bode(Ceq(2,1))
hold on
bode(Ceq(2,2))
hold on
legend('Ceq(1,1)','Ceq(1,2)','Ceq(2,1)','Ceq(2,2)')
title('equivalent compensator')
```

c)

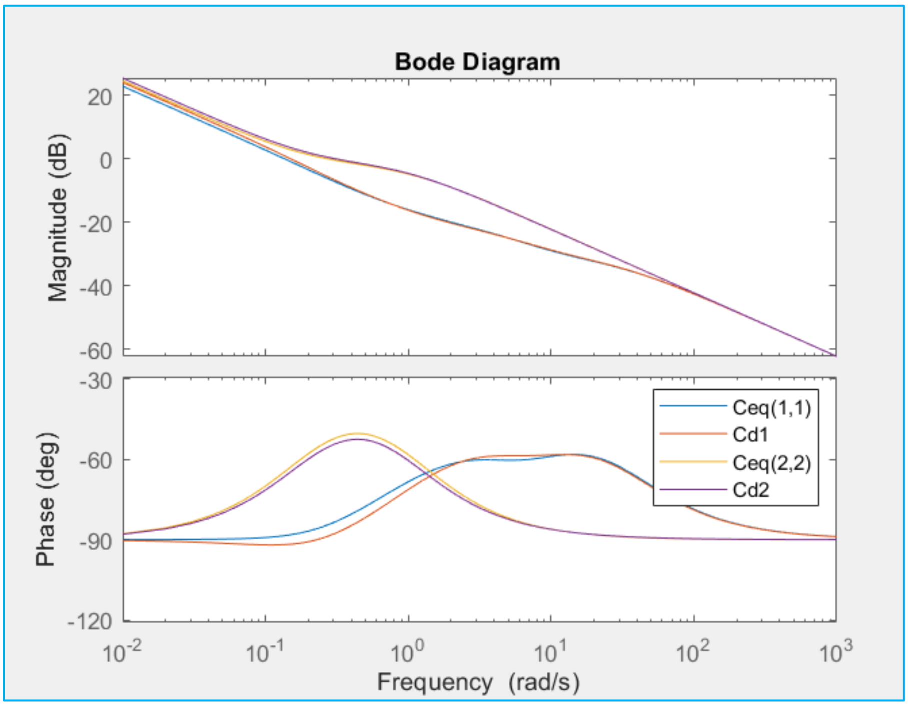


Figure: Bode plots of diagonal elements of Ceq and approximations Cd1 and Cd2

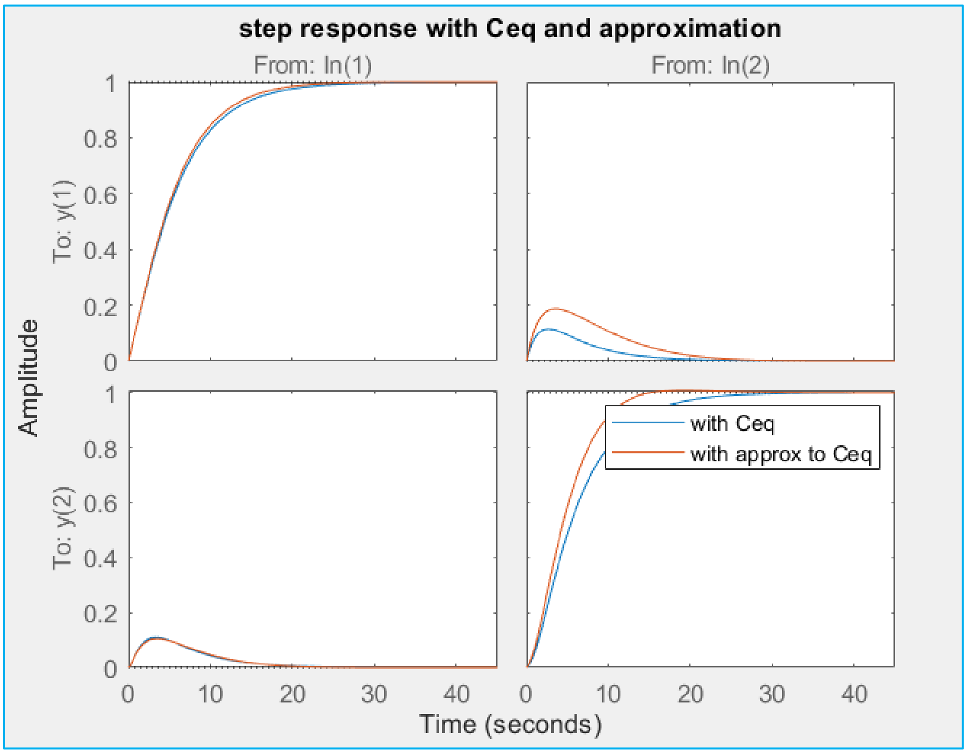
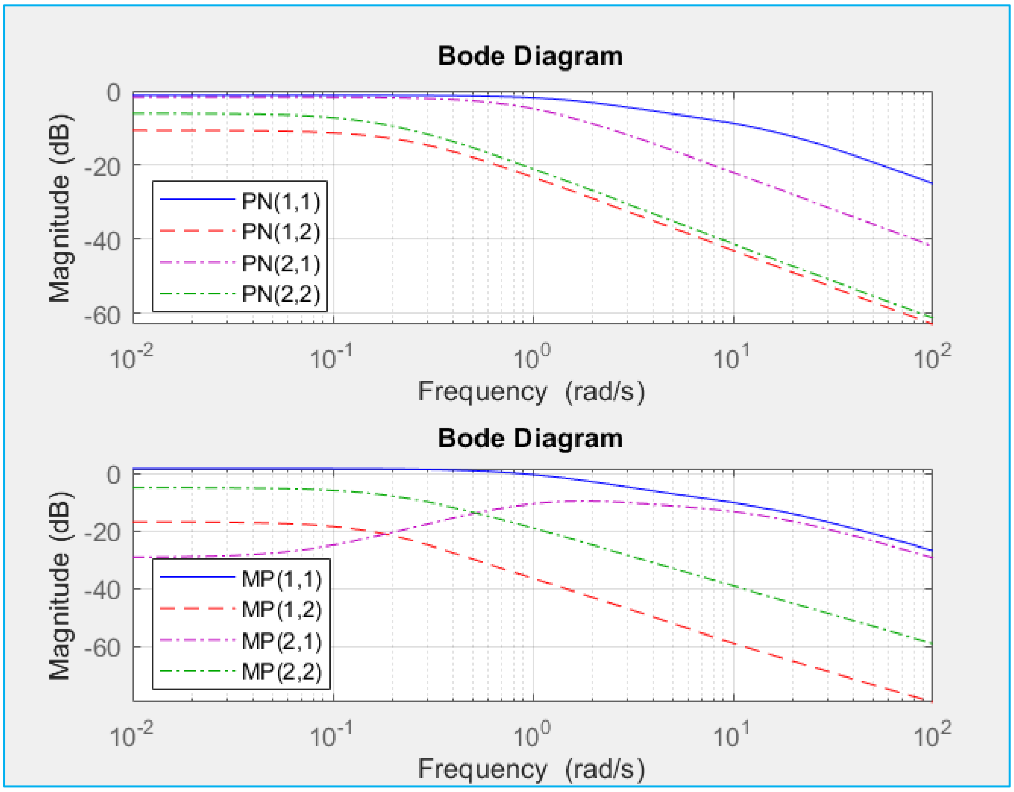


Figure: Step response with Ceq and $\hat{C}$ eq.

- Based on our design, the Ceq approximation is much better than that shown in the Problem PDF shown in Canvas.
- For our Ceq(1,1) approximation, we have considered a 5th order system (5 poles and 4 zeros).
- For our Ceq(2,2) approximation, we have considered a 3rd order system. This results in a response closer to the actual Ceq.

```
%% Part 2(C): Decentralized Approximation of Equivalent Controller
s = tf('s');
Chateq1 = 0.775*((s+13)*(s+0.25)*(s+0.97)*(s+2)/((s+31)*(s+4.1)*(s+1.2)*s*(s+0.2)));
% Chateq2 = 0.632*((s+13)*(s+2)*(s+0.96)*(s+0.97)*(s+0.18)/((s+31)*(s)*(s+4.1)*(s+0.9)*(s+0.21)*(s+1.17)));
Chateq2 = 0.775*((s+0.24)*(s+0.19)/((s+0.9)*(s)*(s+0.21)))
% Chateq2 = tf([1],[1 0])*tf(0.38*[1 .203],0.203*[1 0.38])*tf([0.1275]);
Chateq = [Chateq1 Chateq1; -Chateq2 Chateq2];
figure(9)
bode(Ceq(1,1),10^-2:10^-2:10^3)
hold on
bode(Chateq(1,1),10^-2:10^-3:10^3)
hold on
bode(Ceq(2,2),10^-2:10^-2:10^3)
bode(Chateq(2,2),10^-2:10^-2:10^3)
hold on
legend('Ceq(1,1)','Cd1','Ceq(2,2)','Cd2')

figure(10)
step(feedback(PN*Ceq,eye(2)))
hold on
step(feedback(PN*Chateq,eye(2)))
hold on
legend('with Ceq','with approx to Ceq')
title('step response with Ceq and approximation')
```

Comment on the Plant Transformation



3) <u>**Oxygen as an Additional Actuator**</u>

a) The conditional number of the DC gain matrix of the scaled plant came around **5.5844**, whereas the conditional number for the plant considered in 1. a came around **2.1595**. The conditional number of the DC gain matrix for the unscaled plant considering three inputs came around **235.9343**. If the conditional number is large, then the plant will be susceptible to disturbances.

```
>> num

num =

    5.5844
```

Since the conditional number is small, as compared to other, it is feasible to use 3 x 3 Integral controller.

```
%% Part 3(A) -- DC Analysis With Oxygen Sensor

% Model
% Inputs: [Power; Throttle; %O2]
% Outputs: [|F|; Vbias; Pressure]
P1 = [zpk(-0.067,[-0.095 -19.69],0.49); ...
    zpk(-0.27,[-0.19 -62.42],12.23); ...
    zpk(0.006,[-0.19 -2.33],-0.011)];

P2 = [zpk(0.73,[-0.11; -39.76],4.85); tf(1.65,[1 0.16]); ...
      zpk([],[-0.18; -3],-0.97)];
P2.InputDelay = 0.42;

P3 = [tf(0.33,[1 0.17]); tf(0.25,[1 0.41]); tf(0.024,[1 0.4])];
P3.InputDelay = 0.77;

Pox = [P1 P2 P3];

% Use second-order Pade for plant
Pox = pade(Pox, 2);

% Condition number of DC gain for not scaled plant
Pox_0 = freqresp(Pox,0);
num  = cond(Pox_0,2)

% Input and output scalings based on equilibrium values
DO = diag([16.52 340 17.83]);
DI = diag([1000 12.5 5]);

% Normalize Plant
PoxN = inv(DO)*Pox*DI;
PoxN = ss(PoxN);
% Condition number of DC gain
PoxN_0 = freqresp(PoxN,0);
num  = cond(PoxN_0,2)
```

b)

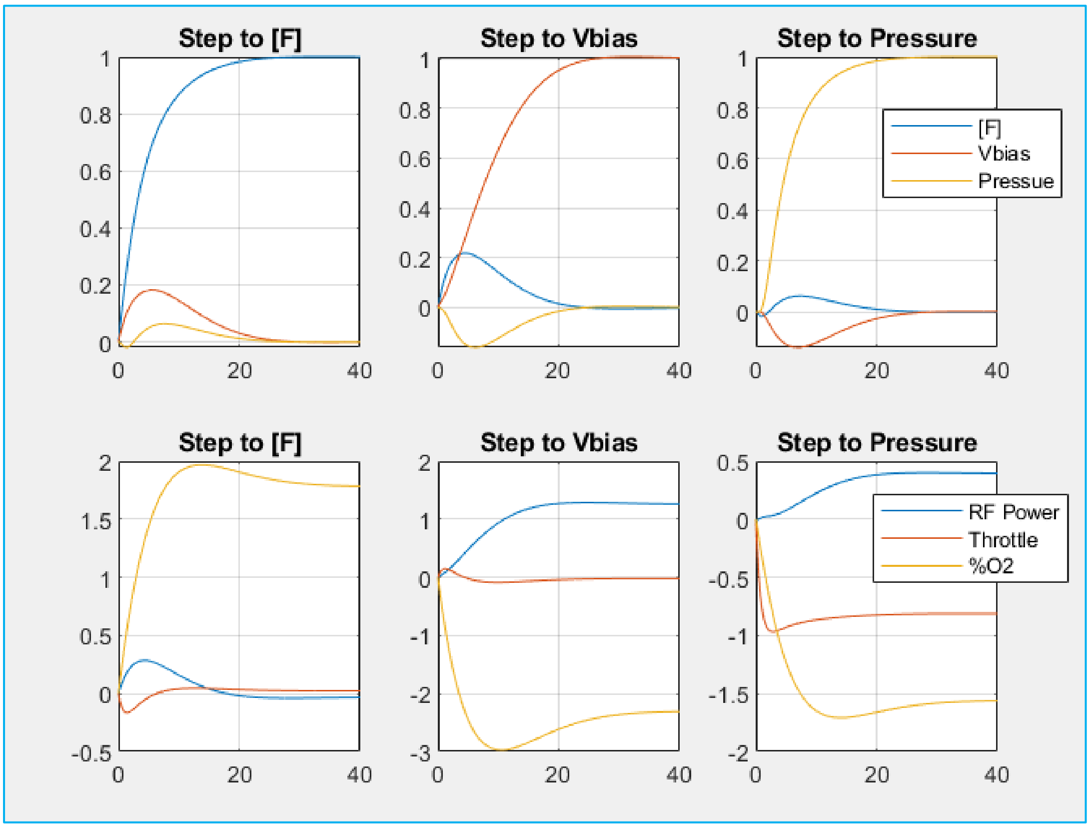


Figure: Step Responses with % O_2 Added as an Actuator

```
%% Part 3(B) -- DC Analysis With Oxygen Sensor

% State-space data for scaled plant
[As,Bs,Cs] = ssdata(PoxN);
[nx,nu] = size(Bs);
ny = size(Cs,1);

% Weighting matrices (Q,R,V,W)
% Assume Q of the form Q = blkdiag(alpha*Cs'*Cs, Qw)

% Q1 = diag([1,1,1]);
% Qi = diag([0.1,0.1,0.1]);
% Q = blkdiag(Cs'*Q1*Cs,Qi);
% R = diag([1,1,1]);

Q1 = diag([2.5,2.5,2.5]);
Qi = diag([0.2,0.2,0.2]);
Q = blkdiag(Cs'*Q1*Cs,Qi);
R = diag([0.1,0.1,0.2]);

% Augmented Plant with integrators
Aaug = [As zeros(nx,ny);
    Cs zeros(ny,ny)];
Baug = [Bs;
    zeros(ny,nu)];
Caug = [Cs zeros(ny,ny)];

% Compute state feedback and observer gains
K = lqr(Aaug,Baug,Q,R);
q = 3;
V =  q^2*Bs*Bs';
W = eye(3);
G = eye(26);
L = lqe(As,G,Cs,V,W);

sys = ss(Aaug-Baug*K,[zeros(nx,nu); -eye(nu)],Caug,0);
sys1 = ss(Aaug-Baug*K,[zeros(nx,nu); -eye(nu)],-K,0);
```

```matlab
%%
% Construct controller:
% This includes the observer, integrators, and feedback gains.
%   Inputs: [|F|Ref; Vbias Ref; Press Ref; |F| Meas; Vbias Meas; Press Meas]
%   Outputs: [Power; Throttle; %O2]
A_obs = [As -Bs*K(:,1:26) -Bs*K(:,27:29);
        L*Cs As-Bs*K(:,1:26)-L*Cs -Bs*K(:,27:29);
        Cs zeros(3,29)];
B_obs = [[zeros(52,3); eye(3)] [zeros(26,1); L(:,1); zeros(3,1)] [zeros(26,1); L(:,1); zeros(3,1)] [zeros(26,1); L(:,1); zeros(3,1)]];
C_obs= [Cs zeros(size(Cs)) zeros(3,3); zeros(3,26) -K];
sys5 = -ss(A_obs,B_obs,C_obs,0);
sys6 = ss(A_obs,B_obs,[zeros(3,26) K],0);

% Form Closed-Loop
%   Inputs: [|F|Ref; Vbias Ref; Press Ref; |F| Noise; Vbias Noise; Press Noise]
%   Outputs: [|F|; Vbias; Press; Power; Throttle; %O2]
trans = tf(sys5);
trans_1 = tf(sys6);


% Verify that closed-loop eigenvalues are the union of the observer
% and state-feedback eigenvalues. This is a useful debugging step
% to verify that you have correctly formed the closed-loop.
eig2 = eig(Aaug-Baug*K);
eig3 = eig(AP-L*CP);
eig1 = eig(A_obs);
isstable(sys5);
```

APPENDIX:

Complete MATLAB Code For The Project:

```
%% Final Project Parts 1 and 2:  Reactive Ion Etching with MIMO Control

% Plant Model from [Power; Throttle] to [|F|; Vbias]
P1 = [tf([0.17 0.7],[1 15 26.7]); tf(0.28,[1 0.97])];
P2 = [tf(-0.17,[1 0.24]); tf([2.41 9.75],[1 4 0.7])];
P2.InputDelay = 0.5;
P = [P1 P2];
P_0 = freqresp(P,0);
num  = cond(P_0,2)
% Normalized System
DO = diag([30 350]);
DI = diag([1000 12.5]);
PN = inv(DO)*P*DI;
PN = ss(PN);

% Condition number of DC gain
PN_0 = freqresp(PN,0);
num  = cond(PN_0,2)

% Use second-order Pade approximation for input delay
PN = pade(PN,2);
PN.InputName = 'u';
PN.OutputName = 'y';

% State-space matrices and dimensions
[AP,BP,CP,DP] = ssdata(PN);
[nx,nu] = size(BP);
ny = size(CP,1);

%% Part 1(A): Linear Quadratic Regulator with Integrators

% Augment state equations so that you can do integral control
Aaug = [AP zeros(nx,ny);
    CP zeros(ny,ny)];
Baug = [BP;
    zeros(ny,nu)];
Caug = [CP zeros(ny,ny)];
%%
% LQR Weighting Matrices
Q1 = diag([2.5,2.5]);
Qi = diag([0.1,0.1]);
Q = CP'*Q1*CP;
R = diag([0.1,0.1]);
% LQ state feedback gain
K = lqr(Aaug,Baug,[Q zeros(nx,nu); zeros(nu,nx) Qi],R);
%%
% Closed loop state equations with state feedback and integrators.
sys = ss(Aaug-Baug*K,[zeros(nx,nu); -eye(2)],Caug,0);
sys1 = ss(Aaug-Baug*K,[zeros(nx,nu); -eye(2)],-K,0);
% Verify that closed-loop is stable (Check to verify no bugs in code)
isstable(sys);
% Time vector
Tf = 50;
Nt = 500;
t = linspace(0,Tf,Nt);

% Step responses
trans = tf(sys);
trans_1 = tf(sys1);
```

```
figure(1)
% set(findall(gcf,'type','line'),'linewidth',3);
subplot(221)
step(trans(1,1))
hold on
step(trans(2,1))
hold on
title('Step to [F], SF')
legend('[F]','Vbias')

subplot(222)
step(trans_1(1,1))
hold on
step(trans_1(2,1))
hold on
title('Step to [F], SF')
legend('RF Power','Throttle')

subplot(223)
step(trans(1,2))
hold on
step(trans(2,2))
hold on
title('Step to Vbias, SF')
legend('[F]','Vbias')

subplot(224)
step(trans_1(1,2))
hold on
step(trans_1(2,2))
hold on
title('Step to Vbias, SF')
legend('RF Power','Throttle')
%% Part 1(B): Linear Quadratic Regulator with Integrators

% Covariance matrices for loop transfer recovery observer
q = 2.5;
V =  q^2*BP*BP';
W = eye(2);

% Observer gain
% Note: We only need to estimate the plant states. We do not need the
% observer to construct an estimate of the integrator states.
G = eye(8);
L = lqe(AP,G,CP,V,W);
Laug = [L;
    eye(2)];
Cobs = ss([AP-BP*K(:,1:8)-L*CP -BP*K(:,9:10); zeros(2,10)],Laug,K,0);
% sys3 = ss([AP-L*CP-BP*K(:,1:8) -BP*K(:,9:10); zeros(2,8) zeros(2,2)],)
loop_sys3 = Cobs*PN;
sys3 = feedback(loop_sys3,eye(2));
sys4 = tf(Cobs)*feedback(eye(2),loop_sys3);
%%
% Verify that observer error is stable (Check to verify no bugs in code)
eig2 = eig(Aaug-Baug*K);
eig3 = eig(AP-L*CP);

% Construct controller:
% This includes the observer, integrators, and feedback gains.
%   Inputs: [|F| Ref.; Vbias Ref; |F| measurement; Vbias measurement]
%   Outputs: [Power; Throttle]
A_obs = [AP -BP*K(:,1:8) -BP*K(:,9:10);
        L*CP AP-BP*K(:,1:8)-L*CP -BP*K(:,9:10);
        CP zeros(2,10)];
```

```
B_obs = [[zeros(16,2); eye(2)] [zeros(8,1); L(:,1); zeros(2,1)] [zeros(8,1); L(:,1);
zeros(2,1)]];
C_obs= [CP zeros(size(CP)) zeros(2,2); zeros(2,8) -K];

% Form Closed-Loop
%   Inputs are [|F| Ref.; Vbias Ref; |F| noise; Vbias noise]
%   Outputs: [|F|; Vbias; Power; Throttle]
sys5 = -ss(A_obs,B_obs,C_obs,0);
sys6 = ss(A_obs,B_obs,[zeros(2,8) K],0);

% Verify that closed-loop eigenvalues are the union of the observer
% and state-feedback eigenvalues. This is a useful debugging step
% to verify that you have correctly formed the closed-loop.
eig1 = eig(A_obs);
isstable(sys5);

% trans = tf(sys3);
% trans_1 = tf(sys4);
%%
% Step responses with noise on |F|
noise=0.1*randn(501,1);
ref1 = [ones(size(noise)) zeros(size(noise)) noise zeros(size(noise))];
noise=0.1*randn(501,1);
ref2 = [zeros(size(noise)) ones(size(noise)) zeros(size(noise)) noise];
[y1 t]=lsim(sys5,ref1,[0:0.1:50]);

[y2 t]=lsim(sys5,ref2,[0:0.1:50]);

figure(1)
subplot(221)
plot(t,y1(:,1),t,y1(:,2))
hold on
title('Step to [F], MIMO')
legend('[F]','Vbias')
xlim([0 50])

subplot(222)
plot(t,y1(:,3),t,y1(:,4))
hold on
title('Step to [F], MIMO')
legend('RF Power','Throttle')
xlim([0 50])

subplot(223)
plot(t,y2(:,1),t,y2(:,2))
hold on
title('Step to [F], MIMO')
legend('[F]','Vbias')
xlim([0 50])

subplot(224)
plot(t,y2(:,3),t,y2(:,4))
hold on
title('Step to [F], MIMO')
legend('RF Power','Throttle')
xlim([0 50])
%%
% Bode magnitude from |F| noise to [|F|; Vbias; Power; Throttle]
figure(5)
bodemag(sys5(1,3),sys5(2,3),sys5(3,3),sys5(4,3))
legend('F ','Vbias','RF Power','Throttle')
% Sigma magnitude from [|F| Ref.; Vbias Ref] to [Power; Throttle]
figure(6)
sigma(sys5(3,1),sys5(3,2),sys5(4,1),sys5(4,2))
title('Sigma magnitude from [|F| Ref.; Vbias Ref] to [Power; Throttle]')
```

```
legend('|F| Ref. to Power',' Vbias Ref to Power','|F| Ref. to Throttle',' Vbias Ref to
Throttle')
% Sigma magnitude from [|F| Noise; Vbias Noise] to [Power; Throttle]
figure(7)
sigma(sys5(3,3),sys5(3,4),sys5(4,3),sys5(4,4))
title('Sigma magnitude from [|F| Noise; Vbias Noise] to [Power; Throttle]')
legend('|F| Noise to Power',' Vbias Noise to Power','|F| Noise to Throttle',' Vbias Noise to
Throttle')

%% Part 1(D): Stability Margins and Comparison With State Feedback

% Loop-at-a-time margins at the plant input

% for state feedback
Lsf = ss(Aaug,Baug,K,[])
AM_sf = allmargin(Lsf)

AM_sf(1)%For the first channel
AM_sf(2)%For the second channel

%recovered
LI = (Cobs * PN);

AM = allmargin (LI)
AM(1)%For the first channel
AM(2)%For the second channel


% Multi-Loop Disk margins at the plant input

% Regular Disk Margin (symmetric)
% for state feedback
[DMI_sfs,MMI_sfs] = diskmargin(Lsf);
MMI_sfs

% for recovered
[DMI_s,MMI_s] = diskmargin(Cobs*PN);  % T-based disk margin
MMI_s

% For T-based Disk margin ("T-based" disk, i.e. a disk centered at 1 )
% for state feedback
[DMI_sf,MMI_sf] = diskmargin(Lsf,-1);  % T-based disk margin
MMI_sf

% for recovered
[DMI,MMI] = diskmargin(Cobs*PN,-1);  % T-based disk margin
MMI


% Unstructured (fully-coupled) stability margin (USM) at the plant input

%For State Feedback
Tsf = feedback(Lsf,eye(size(Lsf)));
[nsf,wsf] = hinfnorm(Tsf);
StabMarg_Tsf = 1/nsf % unstructured stability margin

%For Recovered
TI = feedback(LI,eye(size(LI)));
[np,wp] = hinfnorm(TI);
StabMarg_TI = 1/np % unstructured stability margin

% Input loop transfer function: Compare Lsf to LI
Lsf = ss(Aaug,Baug,K,[])
Tsf = feedback(Lsf,eye(size(Lsf)));
TI = feedback(LI,eye(size(LI)));
```

```
figure (5) % for Loop Gain
[sv,wout]=sigma(LI,{1e-2, 1e2});
loglog(wout,sv(1,:),wout,sv(2,:))
grid on
hold on
[sv,wout]=sigma(Lsf,{1e-2, 1e2});
loglog(wout,sv(1,:),wout,sv(2,:))
hold off
title('Open loop singular values LsF(s) = K*inv(sI-A)*B and Lobs(s) = C(s)*P(s)')
legend('sig_{max}(Lobs)','sig_{min}(Lobs)','sig_{max}(LsF)','sig_{min}(LsF)')
xlabel('w rad/sec')
ylabel('Magnitude')

figure(6) % for Complimentary sensitivity
[sv,wout]=sigma(TI,{1e-2, 1e2});
loglog(wout,sv(1,:),wout,sv(2,:))
grid on
hold on
[sv,wout]=sigma(Tsf,{1e-2, 1e2});
loglog(wout,sv(1,:),wout,sv(2,:))
hold off
title('singular values of complementary sensitivity function')
legend('sig_{max}(Tobs)','sig_{min}(Tobs)','sig_{max}(TsF)','sig_{min}(TsF)')
xlabel('w rad/sec')
ylabel('Magnitude')
Ssf = feedback(eye(size(Lsf)),Lsf);
SI = feedback(eye(size(LI)),LI);

figure (7) % for Sensitivity
[sv,wout]=sigma(SI,{1e-2, 1e2});
loglog(wout,sv(1,:),wout,sv(2,:))
grid on
hold on
[sv,wout]=sigma(Ssf,{1e-2, 1e2});
loglog(wout,sv(1,:),wout,sv(2,:))
hold off
title('singular values of sensitivity function')
legend('sig_{max}(Sobs)','sig_{min}(Sobs)','sig_{max}(SsF)','sig_{min}(SsF)')
xlabel('w rad/sec')
ylabel('Magnitude')
% figure (11)
% y1= (sigma(db2mag(TI,{1e-2, 1e2})))
% y2= (sigma(db2mag(Tsf,{1e-2, 1e2})))

% plot({1e-2, 1e2},y1,{1e-2, 1e2},y2)

% Input sensitivity: Compare Ssf to SI

% Input complementary sensitivity: Compare Tsf to TI


%% Part 2(B): Equivalent Controller

% Equivalent controller
%   Ceq = inv[ I+K1 inv(sI-A) B] (KI/s)
sys = ss(AP,BP,K(:,1:8),0);
tr_ = tf(1,[1 0]);
% Ceq = tf(inv( eye(2) + K(:,1:8)*inv(s*eye(8) - AP)*BP)*K(:,9:10)/s);
Ceq = inv(eye(2) + sys)*K(:,9:10)*tr_;
figure(8)
bode(Ceq(1,1))
hold on
bode(Ceq(1,2))
```

```
hold on
bode(Ceq(2,1))
hold on
bode(Ceq(2,2))
hold on
legend('Ceq(1,1)','Ceq(1,2)','Ceq(2,1)','Ceq(2,2)')
title('equivalent compensator')

%% Part 2(C): Decentralized Approximation of Equivalent Controller
s = tf('s');
Chateq1 = 0.775*((s+13)*(s+0.25)*(s+0.97)*(s+2)/((s+31)*(s+4.1)*(s+1.2)*s*(s+0.2)));
% Chateq2 =
0.632*((s+13)*(s+2)*(s+0.96)*(s+0.97)*(s+0.18)/((s+31)*(s)*(s+4.1)*(s+0.9)*(s+0.21)*(s+1.17)));
Chateq2 = 0.775*((s+0.24)*(s+0.19)/((s+0.9)*(s)*(s+0.21)))
% Chateq2 = tf([1],[1 0])*tf(0.38*[1 .203],0.203*[1 0.38])*tf([0.1275]);
Chateq = [Chateq1 Chateq1; -Chateq2 Chateq2];
figure(9)
bode(Ceq(1,1),10^-2:10^-2:10^3)
hold on
bode(Chateq(1,1),10^-2:10^-3:10^3)
hold on
bode(Ceq(2,2),10^-2:10^-2:10^3)
bode(Chateq(2,2),10^-2:10^-2:10^3)
hold on
legend('Ceq(1,1)','Cd1','Ceq(2,2)','Cd2')

figure(10)
step(feedback(PN*Ceq,eye(2)))
hold on
step(feedback(PN*Chateq,eye(2)))
hold on
legend('with Ceq','with approx to Ceq')
title('step response with Ceq and approximation')
%% Part 2: Comment on Plant Transformation
M = [1 1; -1 1]/sqrt(2);
MP = M*PN;

figure(12)
subplot(2,1,1)
bodemag(PN(1,1),'b',PN(1,2),'r--',PN(2,1),'m-.',PN(2,2),'g-.',{1e-2,1e2});
legend('PN(1,1)','PN(1,2)','PN(2,1)','PN(2,2)','Location','Southwest');
grid on;
if exist('garyfyFigure','file'), garyfyFigure, end

subplot(2,1,2)
bodemag(MP(1,1),'b',MP(1,2),'r--',MP(2,1),'m-.',MP(2,2),'g-.',{1e-2,1e2});
legend('MP(1,1)','MP(1,2)','MP(2,1)','MP(2,2)','Location','Southwest');
grid on;
if exist('garyfyFigure','file'), garyfyFigure, end

%% Final Project Part 3:  Reactive Ion Etching with MIMO Control

%% Part 3(A) -- DC Analysis With Oxygen Sensor

% Model
% Inputs: [Power; Throttle; %O2]
% Outputs: [|F|; Vbias; Pressure]
P1 = [zpk(-0.067,[-0.095 -19.69],0.49); ...
    zpk(-0.27,[-0.19 -62.42],12.23); ...
    zpk(0.006,[-0.19 -2.33],-0.011)];

P2 = [zpk(0.73,[-0.11; -39.76],4.85); tf(1.65,[1 0.16]); ...
      zpk([],[-0.18; -3],-0.97)];
P2.InputDelay = 0.42;
```

```
P3 = [tf(0.33,[1 0.17]); tf(0.25,[1 0.41]); tf(0.024,[1 0.4])];
P3.InputDelay = 0.77;

Pox = [P1 P2 P3];

% Use second-order Pade for plant
Pox = pade(Pox, 2);

% Condition number of DC gain for not scaled plant
Pox_0 = freqresp(Pox,0);
num  = cond(Pox_0,2)

% Input and output scalings based on equilibrium values
DO = diag([16.52 340 17.83]);
DI = diag([1000 12.5 5]);

% Normalize Plant
PoxN = inv(DO)*Pox*DI;
PoxN = ss(PoxN);
% Condition number of DC gain
PoxN_0 = freqresp(PoxN,0);
num  = cond(PoxN_0,2)

%% Part 3(B) -- DC Analysis With Oxygen Sensor

% State-space data for scaled plant
[As,Bs,Cs] = ssdata(PoxN);
[nx,nu] = size(Bs);
ny = size(Cs,1);

% Weighting matrices (Q,R,V,W)
% Assume Q of the form Q = blkdiag(alpha*Cs'*Cs, Qw)

% Q1 = diag([1,1,1]);
% Qi = diag([0.1,0.1,0.1]);
% Q = blkdiag(Cs'*Q1*Cs,Qi);
% R = diag([1,1,1]);

Q1 = diag([2.5,2.5,2.5]);
Qi = diag([0.2,0.2,0.2]);
Q = blkdiag(Cs'*Q1*Cs,Qi);
R = diag([0.1,0.1,0.2]);

% Augmented Plant with integrators
Aaug = [As zeros(nx,ny);
    Cs zeros(ny,ny)];
Baug = [Bs;
    zeros(ny,nu)];
Caug = [Cs zeros(ny,ny)];

% Compute state feedback and observer gains
K = lqr(Aaug,Baug,Q,R);
q = 3;
V =  q^2*Bs*Bs';
W = eye(3);
G = eye(26);
L = lqe(As,G,Cs,V,W);

sys = ss(Aaug-Baug*K,[zeros(nx,nu); -eye(nu)],Caug,0);
sys1 = ss(Aaug-Baug*K,[zeros(nx,nu); -eye(nu)],-K,0);
% %%
% % Time vector
% Tf = 50;
% Nt = 500;
% t = linspace(0,Tf,Nt);
```

```
%
% % Step responses
% trans = tf(sys);
% trans_1 = tf(sys1);
%
% figure(1)
%
% subplot(2,3,1)
% step(trans(1,1))
% hold on
% step(trans(2,1))
% hold on
% step(trans(3,1))
% title('Step to [F], SF')
% grid on
%
% subplot(2,3,2)
% step(trans(1,2))
% hold on
% step(trans(2,2))
% hold on
% step(trans(3,2))
% title('Step to [F], SF')
% grid on
%
% subplot(2,3,3)
% step(trans(1,3))
% hold on
% step(trans(2,3))
% hold on
% step(trans(3,3))
% title('Step to Vbias, SF')
%
% grid on
%
% subplot(2,3,4)
% step(trans_1(1,1))
% hold on
% step(trans_1(2,1))
% hold on
% step(trans_1(3,1))
% title('Step to Vbias, SF')
%
% grid on
%
% subplot(2,3,5)
% step(trans_1(1,2))
% hold on
% step(trans_1(2,2))
% hold on
% step(trans_1(3,2))
%
%
% subplot(2,3,6)
% step(trans_1(1,3))
% hold on
% step(trans_1(2,3))
% hold on
% step(trans_1(3,3))
%%
% Construct controller:
% This includes the observer, integrators, and feedback gains.
%   Inputs: [|F|Ref; Vbias Ref; Press Ref; |F| Meas; Vbias Meas; Press Meas]
%   Outputs: [Power; Throttle; %O2]
A_obs = [As -Bs*K(:,1:26) -Bs*K(:,27:29);
```

```
        L*Cs As-Bs*K(:,1:26)-L*Cs -Bs*K(:,27:29);
        Cs zeros(3,29)];
B_obs = [[zeros(52,3); eye(3)] [zeros(26,1); L(:,1); zeros(3,1)] [zeros(26,1); L(:,1);
zeros(3,1)] [zeros(26,1); L(:,1); zeros(3,1)]];
C_obs= [Cs zeros(size(Cs)) zeros(3,3); zeros(3,26) -K];
sys5 = -ss(A_obs,B_obs,C_obs,0);
sys6 = ss(A_obs,B_obs,[zeros(3,26) K],0);

% Form Closed-Loop
%   Inputs: [|F|Ref; Vbias Ref; Press Ref; |F| Noise; Vbias Noise; Press Noise]
%   Outputs: [|F|; Vbias; Press; Power; Throttle; %O2]
trans = tf(sys5);
trans_1 = tf(sys6);


% Verify that closed-loop eigenvalues are the union of the observer
% and state-feedback eigenvalues. This is a useful debugging step
% to verify that you have correctly formed the closed-loop.
eig2 = eig(Aaug-Baug*K);
eig3 = eig(AP-L*CP);
eig1 = eig(A_obs);
isstable(sys5);

%%
% Time vector
Tf = 40;
Nt = 400;
t = linspace(0,Tf,Nt);

% Step Responses (Without Noise)
noise=0.1*randn(401,1);
ref1 = [ones(size(noise)) zeros(size(noise)) zeros(size(noise)) zeros(size(noise))
zeros(size(noise)) zeros(size(noise))];
noise=0.1*randn(401,1);
ref2 = [zeros(size(noise)) ones(size(noise)) zeros(size(noise)) zeros(size(noise))
zeros(size(noise)) zeros(size(noise))];
ref3 = [zeros(size(noise)) zeros(size(noise)) ones(size(noise)) zeros(size(noise))
zeros(size(noise)) zeros(size(noise))];
[y1 t]=lsim(sys5,ref1,[0:0.1:40]);

[y2 t]=lsim(sys5,ref2,[0:0.1:40]);

[y3 t]=lsim(sys5,ref3,[0:0.1:40]);

figure

subplot(231)
plot(t,y1(:,1),t,y1(:,2),t,y1(:,3))
title('Step to [F]')
grid on

subplot(234)
plot(t,y1(:,4),t,y1(:,5),t,y1(:,6))
title('Step to [F]')
grid on

subplot(232)
plot(t,y2(:,1),t,y2(:,2),t,y2(:,3))
title('Step to Vbias')

grid on

subplot(235)
plot(t,y2(:,4),t,y2(:,5),t,y2(:,6))
```

```
title('Step to Vbias')

grid on

subplot(233)
plot(t,y3(:,1),t,y3(:,2),t,y3(:,3))
title('Step to Pressure')
legend('[F]','Vbias','Pressue')
grid on

subplot(236)
plot(t,y3(:,4),t,y3(:,5),t,y3(:,6))
title('Step to Pressure')
legend('RF Power','Throttle','%O2')
grid on
```