CSCI E-124 Data Structures and Algorithms — Spring 2015

Problem Set 2

Due: 11:59pm, Wednesday, February 18th

See homework submission instructions at
`http://sites.fas.harvard.edu/~cs124/e124/problem_sets.html`

Problem 5 is worth 40% of this problem set, and problems 1-4 constitute the remaining 60%.

1 Problem 1

A directed graph is *nice* if for every pair of distinct vertices u, v, there is at most 1 non-cycling path from u to v (a path is "non-cycling" if it doesn't contain a cycle). Give an efficient algorithm to determine whether an input graph is nice, and prove its correctness.

2 Problem 2

You are given an array A of length n containing distinct integer elements. You are also given the promise that, for each index $1 \leq i \leq n$, $A[i]$ is within t positions of where it should be if A had been sorted. Show that A can be sorted in time $O(n \log t)$.

3 Problem 3

In a directed graph $G = (V, E)$ a *termination* point is a vertex $v \in V$ such that for all $w \in V$, if there exists a path in G from v to w, then there also exists a path from w to v. Describe and analyze an efficient algorithm which receives G in adjacency list representation and outputs a list of all termination points.

4 Problem 4

You are a merchant, and you want to walk from your home to your shop. You figure that you might as well get a good workout while doing so. Your city contains various roads and intersections, with various lengths of road between each intersection. Each intersection is also elevated at some height above sea level. Your primary goal is to get to your shop using the shortest route possible. However, you want a path in which all road segments you take are initially all inclined upward (the exercise) followed by road segments which are all inclined downward (to cool down). The number of road segments you take inclined upward before you switch to going downward is up to you (you could also choose a route that is

entirely upward or downward). Give efficient algorithms to find tshe shortest such paths in the following two cases:

(a) All elevations are distinct.

(b) Some elevations may be the same. You may walk on flat roads during both the uphill and downhill portions of your walk.

5 Problem 5

Solve "Problem A - Snake" on the programming server; see the "Problem Sets" part of the course web page for the link.