CSCI E-124 Data Structures and Algorithms — Spring 2015

Problem Set 7

Due: 11:59pm, Wednesday, April 15

See homework submission instructions at
`http://sites.fas.harvard.edu/~cs124/e124/problem_sets.html`

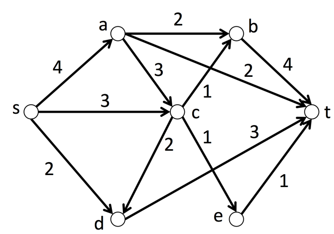
Problem 5 is worth 40% of this problem set, and problems 1-4 constitute the remaining 60%.

Upon completing this problem set, if you have roughly two minutes to spare then please fill out this survey: `http://goo.gl/forms/v8aLB0gYdz`

Problems 1 through 3 and problem 5 concern flow, which you will see in class on Tuesday, April 7th. Problem 4 is related to Karger's algorithm, which you will see in class Tuesday, March 31 and Thursday, April 2.

1 Problem 1

Find the maximum flow from s to t and the minimum cut between s and t in the network below, using the method of augmenting paths discussed in class. (This means give the flow along each edge, along with the final flow value; similarly, give the edges that cross the cut, along with the final cut value.) Show the residual network at the intermediate steps as you build the flow. (If you need more information on the algorithm, it's called the Ford-Fulkerson algorithm.)



2 Problem 2

You have been given a square plot of land that has been divided into n rows and columns, yielding n^2 square subplots. Some of these subplots have rocky ground and cannot support plant growth, while others have soil. You would like to plant palm trees on a subset of the square subplots so that every row and every column has exactly the same number p of palm trees. Furthermore, you would like to do this so that p is as large as possible. Devise an efficient algorithm to determine how to accomplish this.

3 Problem 3

There are n hungry amphibians sitting around a pond, when n insects suddenly fly overhead. Each amphibian looks at the insects and decides on a subset of them that it is willing to eat. Suppose that for any subset U of the amphibians, the collective set of insects they are willing to eat, denoted as $N(U)$, satisfies the condition $|N(U)| \geq |U|$. Prove that there is a way for every amphibian to eat exactly one desired insect without conflicts. **Hint:** Construct a graph and reason about flows.

4 Problem 4

Let $G = (V, E)$ be an unweighted, undirected graph with n vertices and m edges. Suppose that we do not want to find just one minimum cut, but want to count the *number* of minimum cuts (recall in class that we said the number of minimum cuts is never more than $\binom{n}{2}$, which is achieved by the n-cycle, but in general the number of minimum cuts could be any integer between 1 and $\binom{n}{2}$). Any running time of the form $O(n^C)$ for constant $C > 0$ receives full credit, and your algorithm may be Monte Carlo with failure probability at most $1/3$. What would you change if you wanted failure probability P for some given $0 < P < 1/3$, and how would that affect your running time?

5 Problem 5

Solve "Problem A - Caterpillars" on the programming server; see the "Problem Sets" part of the course web page for the link.