AI MATH Exercises

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1 Functions

Example: Find the domain of the function

$$f(x) = x + \frac{\sqrt{1 - x^2}}{\log(1 + x)}$$

Solution: We need to consider the following restrictions

- Square root argument must be non-negative.
- log argument must be positive.
- $\log(1 + x)$ must be non-zero since it is in the denominator.

The first condition is $1 - x^2 \geq 0$ which can be manipulated further as

$$1 - x^2 \geq 0$$
$$\Leftrightarrow 1 \geq x^2$$
$$\Leftrightarrow 1 \geq |x|$$
$$\Leftrightarrow 1 \geq x \geq -1$$

The second condition is simply $1 + x > 0$ which is equivalent to $x > -1$.

The third condition is $\log(1 + x) \neq 0$. Since log equals 0 only when its input is 1 the condition becomes $1 + x \neq 1$ which is simplified to $x \neq 0$.

Putting all these conditions together the domain is $(-1, 0) \cap (0, 1]$. This domain can be represented on the real line as shown in figure 1.
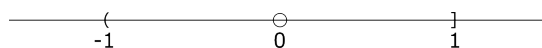

Figure 1: real line representation

Exercise 1: Graph the following functions.

- $f(x) = 2x + 3$

- $f(x) = 13$

- $sgn(x) = \begin{cases} 1 & ; x > 0 \\ 0 & ; x = 0 \\ -1 & ; x < -1 \end{cases}$

- $f(x) = \log(x - 1)$

- $f(x) = 3\sin(2x)$

- $f(x) = 2\sin(3x)$

- $\text{ReLU}(x) = \max(x, 0)$

- $\sigma(x) = \frac{1}{1 + e^{-x}}$

Exercise 2: Find the domain of the following functions.

- $f(x) = \frac{x}{x-1}$

- $f(x) = \frac{x-2}{x^2 - 3x + 2} + \frac{1}{x+5}$

- $f(x) = \log\left(\frac{1}{x-7}\right)$

Exercise 3: Let $f(x + 1) = x^2 - x + 1$. Express $f(x)$ and $f(x - 1)$.

Exercise 4: Let $h \neq 0$. Express and simplify the expression $\frac{f(x+h)-f(x)}{h}$ for the following functions

- $f(x) = 2x + 3$.

- $f(x) = 5x^2$.

- $f(x) = 5x^2 - 2x - 3$.

Exercise 5: Write the equation of the line that passes through the points $(3, 4)$ and $(0, 5)$.