AI Summer Camp

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Calculus Agenda

- Functions
- Differentiation
- Maxima and Minima
- Line of Best Fit
- Multivariate Functions

Relation to AI?

In a reductionist view AI boils down to optimizing a loss function.

Loss Function: The loss function is a measure for how good the model performs. A Loss function is either designed from collected data or is made specific to the problem at hand.

Optimizing Algorithm: Many optimizing algorithms utilize derivatives of the loss function. Optimizing means minimizing in most contexts

Learning: The terminology learning can be thought of as the process of applying an iterative optimizing algorithm to a minimize a given loss function.

Suppose we have data of the form (X, Y). We may be interested in approximating a function f that satisfies $Y = f(X)$.

Neural Networks: Consecutive composition of linear maps and a non-linear map.

Functions

Definition

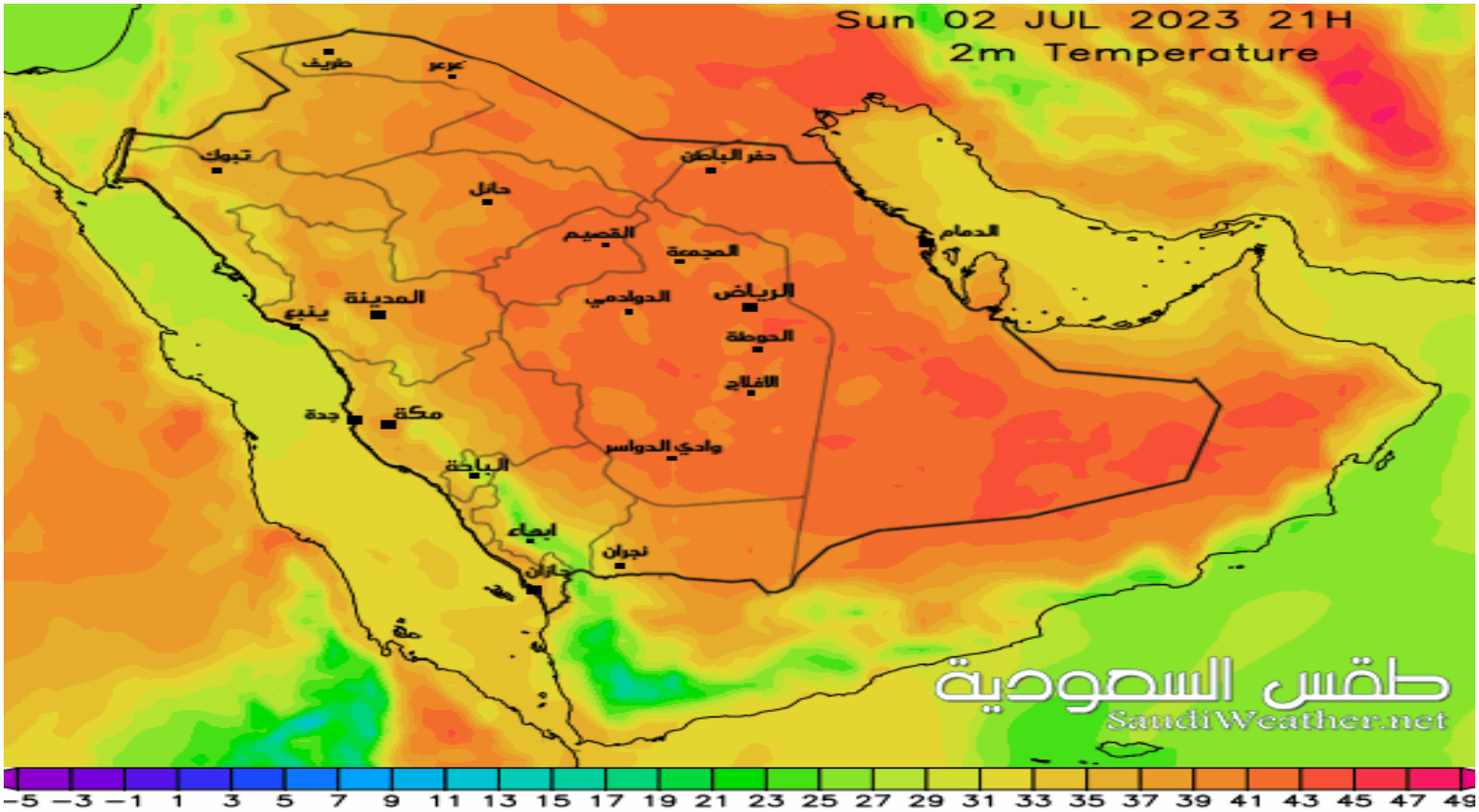
A function (mapping) is a relation, law or rule from a set of inputs into a set of possible outputs where each input has a unique output.

Example

The polynomial $p : \mathbb{R} \to \mathbb{R}$ defined by $p(x) = 5x^3 - 2x^2 + 17$.

Exercise: evaluate $p(0)$ and $p(-1)$.

Temperature measurements can be thought of as a function that takes a point in land as input (ignoring factors such as wind and humidity).



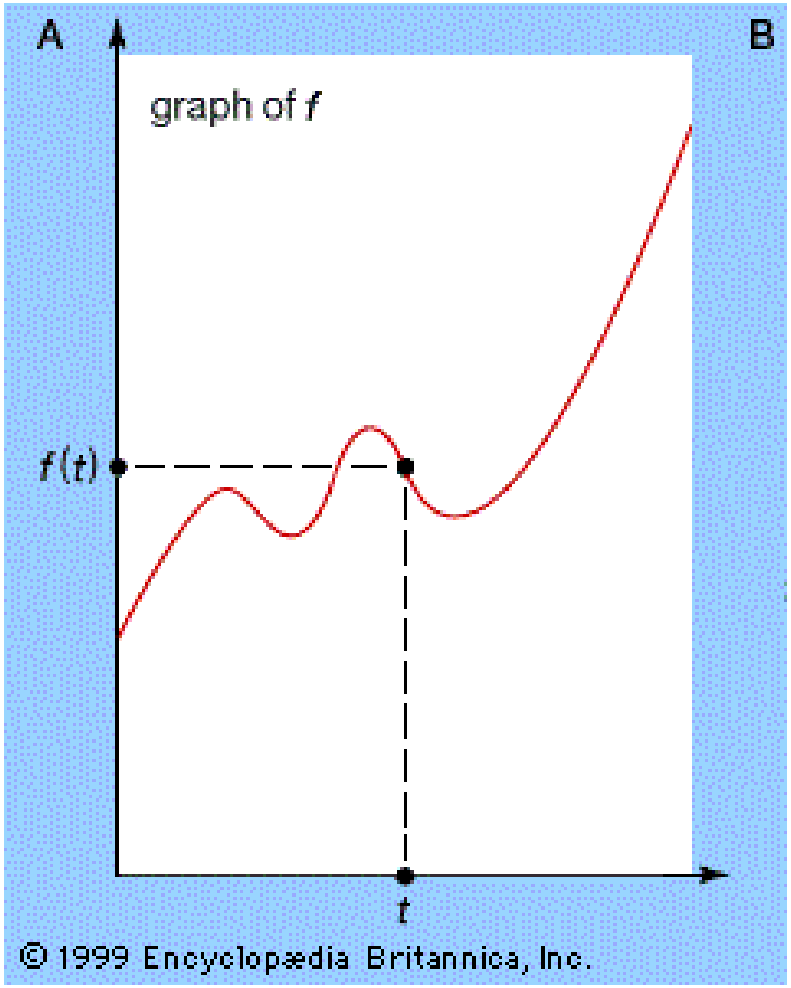
Functions are objects which return the same value whenever given the same input.

Functions can also take "several" inputs.

Example

Area of rectangle. Height and width are input and area$=$ height$\times$width is the output.

Graph of a function



(a) label 1

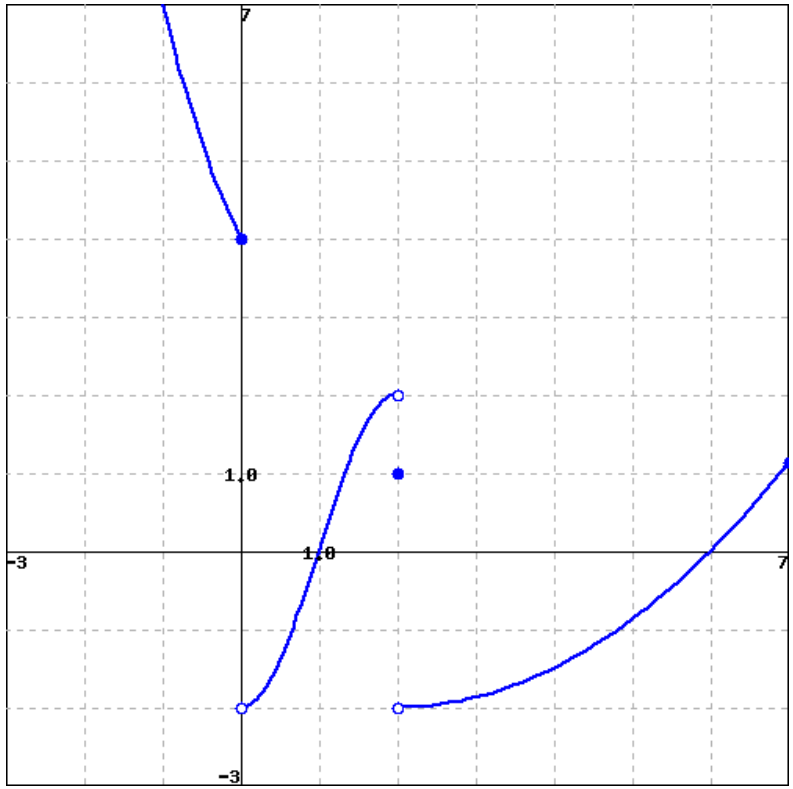


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Figure: Examples of functions

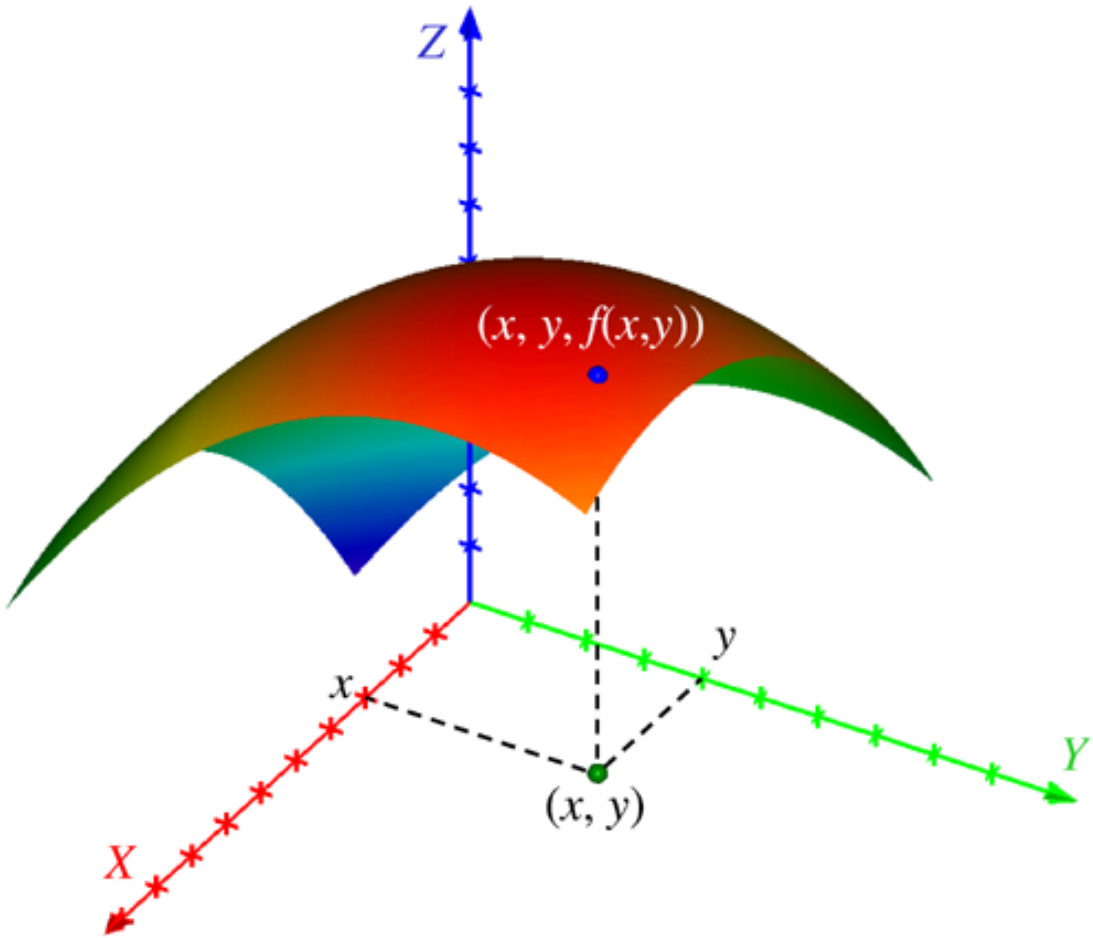
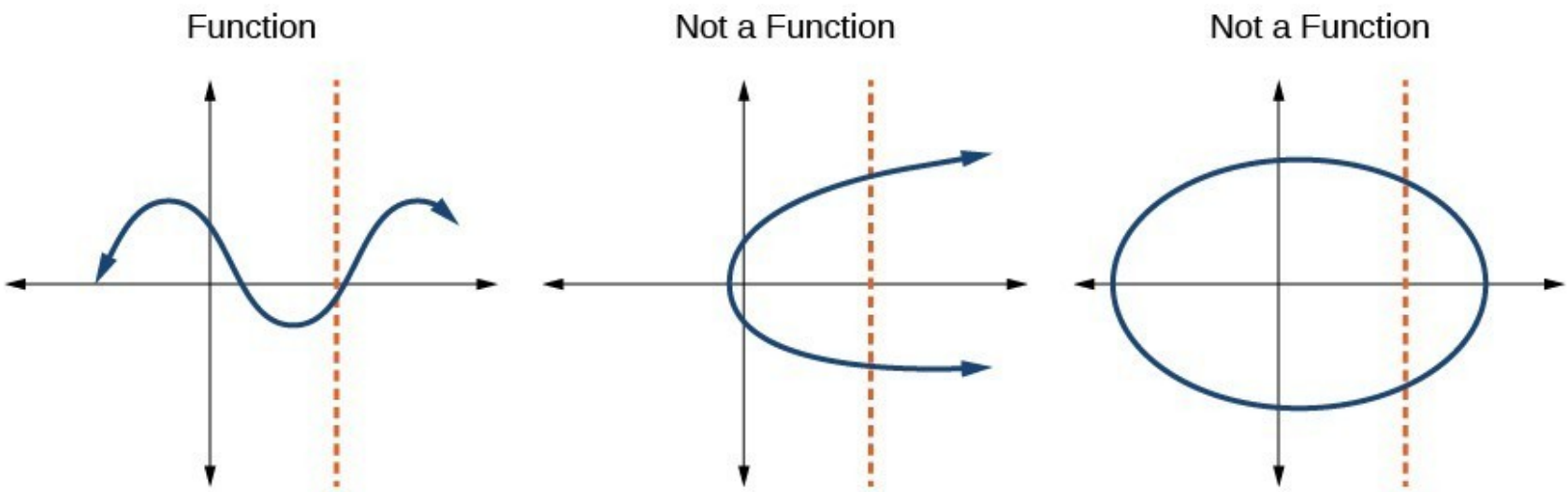


Figure: function graph in 3d

Vertical Line Test

A curve is a graph of function if every vertical line cuts the curve once at most.



Domain and Co-Domain

Definition

The domain of a function is the set of allowed inputs for that function.
The co-domain is the set of possible outputs.

The function $f(x) = \sqrt{x-1}$ has domain $[1, \infty)$.
The polynomial p defined previously works in all real numbers $\mathbb{R}$.

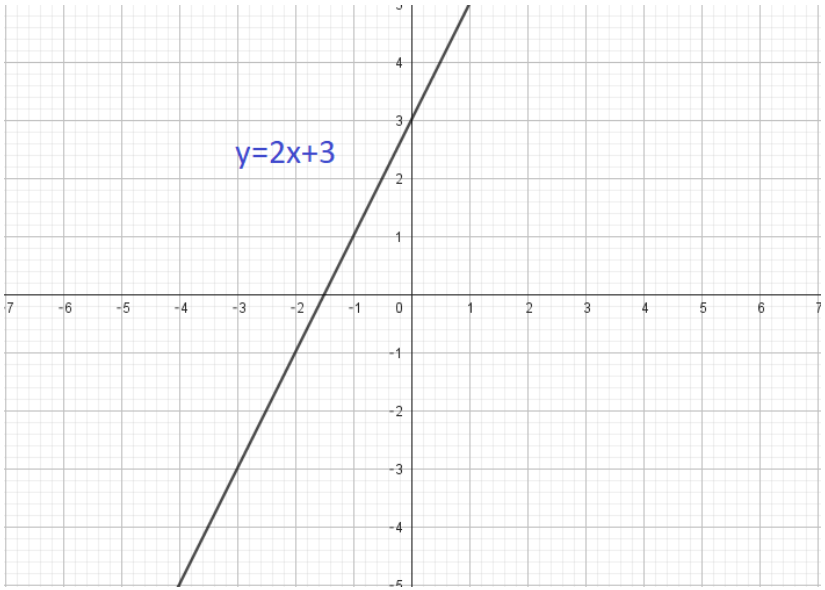
The area of rectangle function has pairs of positive real numbers as inputs (written $\mathbb{R}^+ \times \mathbb{R}^+$).

Exercise: What is the domain of the function $f(x) = \log(x^3 + 1)$?

Important Examples of Functions

Linear Maps

$$f(x) = mx + b$$

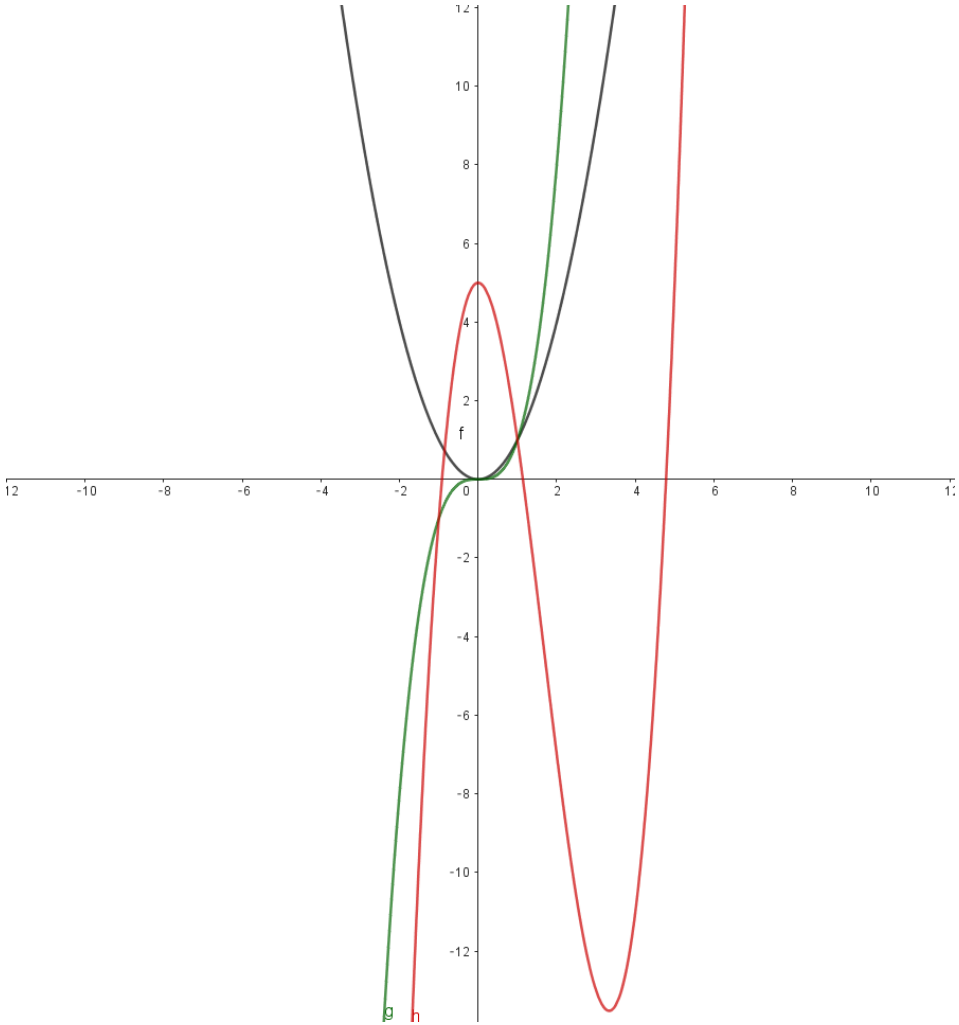


m is called the slope and b is called the intercept.

Exercise: plot the line $y = \frac{1}{2}x + 3$.

Polynomials

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$



Exponential Map

$$f(x) = e^x$$
$$f(x) = 2^x$$

Trigonometric Functions

$$f(x) = \sin(x)$$
$$f(x) = \cos(x)$$

Logarithmic Functions

$$f(x) = \log_b(x)$$

Composition

Definition

Let $f : A \to B$ and $g : B \to C$ be two functions. We define the composition $g \circ f : A \to C$ as

$$(g \circ f)(x) = g(f(x))$$

Example: $f(x) = x^2$ and $g(x) = x - 1$. Then

$$(g \circ f)(x) = (x - 1)^2$$

while

$$(g \circ f)(x) = x^2 - 1$$

Loss Function

Which line?