Programming Assignment-1
Optimization Methods (Spring 2023)
Submission Deadline: 14th March, 2023 (11.59pm)
Total Marks: 40

Instructions

1. Attempting all questions is mandatory.
2. Marks for each of the question are mentioned at the question itself.
3. You are expected to solve all the questions using python programming language.
4. Use of any in-built libraries to solve the problem directly is not allowed.
5. Submission Format: Check assignment description or announcement post for more details.
6. Plagiarism is a strict No. We will pass all codes through the plagiarism checking tool to verify if the code is copied from somewhere. In that case, you get **F** grade in the course.
7. If any two students codes are found exactly same (if they copy from each other), both will get **F** grade.

Question 1 - Simplex Algorithm (25 marks)

Implement simplex algorithm to solve the given optimization problem. Make sure you handle the cases of unbounded solutions and no solutions. Use two phase method for initialization of simplex algorithm if initial basic feasible solution is not easy to find. Use Bland's rule **only if cycling is detected**.

$$\begin{aligned} \min \quad & \mathbf{c}^T \mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned} \tag{1}$$

Input Format

The input will be given from the Stdin.

The test cases will be of the form:
The first line contains three integers: n, u, and v.
The second line contains n numbers, denoting the vector $\mathbf{c}$.
The next $u + v$ lines contain n numbers each.
The first u lines denote the coefficients of the decision variables in inequalities of the form $\leq$.
The next v lines denote the coefficients of the decision variables in inequalities of the form $\geq$.
The final line contains $u + v$ numbers, which denote the R.H.S values of the $u + v$ constraints given, or in other terms, the vector $\mathbf{b}$.

You have to convert the given input to the standard form mentioned in (1) and then solve it. Remember that all the inputs except u, v and n are real numbers, not integers.

Output Format

All output is to be written to Stdout.

- **If the optimal solution exists and no cycling is detected:** Print the optimal value on line 1. Print the vector of optimal values of the variables on line 2.
- **If the optimal solution exists and cycling is detected:** Print "Cycling detected" on line 1. Print the optimal value on line 2. Print the vector of optimal values of the variables on line 3.
- **If the problem is unbounded:** Print "Unbounded".
- **If the problem is infeasible:** Print "Infeasible".

The output is case-sensitive, “infeasible” instead of “Infeasible” will not be accepted. Do not print the quotes around the words.
The output will be considered correct if your output is within 10^{-6} of the true optimal value (In other words, print output with 7 digit after the decimal point).

Example test cases

Example input 1

2 2 1
5 7
2 -4
-1 7
2 3
4 7 10
The given input corresponds to the following (non-standard LP):

$$\begin{aligned} \min \quad & 5x_1 + 7x_2 \\ \text{subject to} \quad & 2x_1 - 4x_2 \leq 4 \\ & -x_1 + 7x_2 \leq 7 \\ & 2x_1 + 3x_2 \geq 10 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Example output 1

24.2941176
2.8823529 1.4117647

Example input 2

3 2 1
5 10 8
3 5 2
4 4 4
2 4 5
60 72 100
The given input corresponds to the following (non-standard LP):

$$
\begin{aligned}
\min \quad & 5x_1 + 10x_2 + 8x_3 \\
\text{subject to} \quad & 3x_1 + 5x_2 + 2x_3 \leq 60 \\
& 4x_1 + 4x_2 + 4x_3 \leq 72 \\
& 2x_1 + 4x_2 + 5x_3 \geq 100 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

Example output 2

Infeasible

Question 2 - Cutting Plane Method (15 marks)

Implement Gomory's cutting plane method discussed in the class to solve given integer programming problem. Use the simplex algorithm implemented in Q1 to solve the LP-relaxation problems.
Make sure you handle the cases of no solutions.

$$\begin{aligned} \min \quad & \mathbf{c}^T \mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0}, \mathbf{x} \in \mathbb{Z}^n \end{aligned} \tag{2}$$

Input Format

The input will be given from the Stdin.

The test cases will be of the form:
The first line contains three integers: n, u, and v.
The second line contains n numbers, denoting the vector $\mathbf{c}$.
The next $u + v$ lines contain n numbers each.
The first u lines denote the coefficients of the decision variables in inequalities of the form $\leq$.
The next v lines denote the coefficients of the decision variables in inequalities of the form $\geq$.
The final line contains $u + v$ numbers, which denote the R.H.S values of the $u + v$ constraints given, or in other terms, the vector $\mathbf{b}$.

Remember that all the inputs except u, v and n are real numbers, not integers.

Output Format

- **If the optimal solution exists:** Print the optimal value on line 1. Print the n **integers**, the optimal value of the variables, on line 2.
- **If the problem is unbounded:** Print "Unbounded".
- **If the problem is infeasible:** Print "Infeasible".

The output is case-sensitive, "infeasible" instead of "Infeasible" will not be accepted. Do not print the quotes around the words.
The output will be considered correct if your output is within 10^{-6} of the true optimal value (In other words, print output with 7 digit after the decimal point).

Example test cases

Example input 1

2 2 0
-1 -1
1 2
2 -1
2 1
The given input corresponds to the following (non-standard LP):

$$\begin{aligned} \min \quad & -x_1 - x_2 \\ \text{subject to} \quad & x_1 + 2x_2 \leq 2 \\ & 2x_1 - x_2 \leq 1 \\ & x_1, x_2 \geq 0,\ x_1, x_2 \in \mathbb{Z} \end{aligned}$$

Example output 1

-1
0 1

Example input 2

2 1 1
-1 -1
-2 1
1 2
1 2
The given input corresponds to the following (non-standard LP):

$$\begin{aligned} \min \quad & -x_1 - x_2 \\ \text{subject to} \quad & -2x_1 + x_2 \leq 1 \\ & x_1 + 2x_2 \geq 2 \\ & x_1, x_2 \geq 0,\ x_1, x_2 \in \mathbb{Z} \end{aligned}$$

Example output 2

Unbounded