20K-
Anwar Siddiqui

# Assignment # 01

Date: 01/10/21

## Question # 01

a) Proposition, True
b) Proposition, False
c) Proposition, True
d) Proposition, false
e) Not a proposition
f) Not a proposition, because imperative sentence not a declarative

## Question # 02

let the RAM be Ra
" " ROM be Ro
" " camera be Cam

a) $BRa > ARa \wedge BRa > CRa$, True

b) $CRo > BRo \vee CCam > BCam$, False

c) $BRa > ARa \wedge BRo > ARo \wedge BCam > ACam$
False

d) $(BRa > CRa \wedge BRo > CRo) \rightarrow BCam > CCam$, False

e) $ARa > BRa \leftrightarrow BRa > ARa$, False

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Question # 03

let
$A_p$ = Acme computer
$N_p$ = Nadir software
$Q_p$ = Quasar Media

therefore, their annual revenue will be
$A_{ar}$, $N_{ar}$, $Q_{ar}$ & profit will be
$A_p$, $N_p$, $Q_p$.

a) $Q_p > A_p \wedge Q_p > N_p$
True.

b) $(N_p < A_p \wedge N_p < Q_p) \wedge (A_{ar} > N_{ar} \wedge A_{ar} > Q_{ar})$
True.

c) $(A_p > N_p \wedge A_p > Q_p) \vee (Q_p > A_p \wedge Q_p > N_p)$
True.

d) $(Q_p < A_p \wedge Q_p < N_p) \rightarrow (A_{ar} > N_{ar} \wedge A_{ar} > Q_{ar})$ ~~$A_p > N_p \wedge A_p > Q_p$~~
True.

e) $(N_p < A_p \wedge N_p < Q_p) \leftrightarrow (A_{ar} > N_{ar} \wedge A_{ar} > Q_{ar})$
True.

Question

p: You
q: You
r: You

a) p →
If you
examin

b) ¬q
If yo
then

c) q
If
you

d) p
Yo
exam

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Question #04

p: You have the flu
q: You miss the final examination
r: You pass the course.

a) $p \rightarrow q$

If you have flu then you miss the final examination

b) $\neg q \leftrightarrow r$

If you dond' miss the final examination, then you pass this course and conversely.

c) $q \rightarrow \neg r$

If you miss the final examination then you dond' pass this course

d) $p \vee q \vee r$

You have flu or you miss final examination or you pass the course.

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e) $(p \to \neg r) \vee (q \to \neg r)$

If you have flu then you don't pass the course if you miss the final examination

f) $(p \wedge q) \vee (\neg q \wedge r)$

You have flu and you miss final examination or you don't miss the final examination and you pass the course.

Question # 05.

p: You get an A on the final exam
q: You do every exercise in this book
r: You get an A in the class

a) $p \wedge \neg q$

b) $p \wedge q \wedge r$

c) $r \to q$

d) $(p \wedge \neg q) \wedge r)$

e) $(p \wedge q) \longrightarrow r$

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f) $p \leftrightarrow (q \vee r)$

Question # 06:

~~a) p = You send me e-mail m~~

a) If you send me email address, then I will remember to send you the address.

b) p → You were born in United States
q → To be a citizen of this country

If you were born in United States then you can be a citizen of this country.

c) If you keep your textbook, then it will be a useful reference in your future courses

d) If their goalies plays well, then the Red wings will win the Stanley Cup.

e) If you get the job, then you had the best credentials.

f) If there is a storm, the the beach erodes.

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g) $q \rightarrow$ to have a valid password
$p \rightarrow$ to log on to the server

If you log onto the server, then you have a valid password.

h) $q$ unless $\neg p$
$\neg p \rightarrow$ you begin your climb too late
$p \rightarrow$ you did not begin your climb too late
$q \rightarrow$ you will reach the summit

If you didn't begin your climb too late, then you will reach the summit.

Question # 07

$p \rightarrow$ it is sunny tomorrow
$q \rightarrow$ I will go for walk in the woods.

a) Describe at least five different ways to write the conditional statement $p \rightarrow q$ in English.

i) it is sunny tomorrow implies that I will go for a walk in the woods

ii) I will go for a walk in the woods if it sunny tomorrow.

iii) I will go for a walk in the woods unless it is not sunny tomorrow.

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ii) I will go for a walk ~~tomorrow~~ in the woods when it is sunny tomorrow

i) It is sunny tomorrow only if I go for a walk in the woods.

b) Converse:-

If I will go for a walk in the woods, then it sunny tomorrow.

Inverse:-

I will not go for a walk in the woods if it is not sunny tomorrow.

Contrapositive:-

If I will not go for a walk in the woods, then it is not sunny tomorrow.

c) Inverse of its inverse:-

If it is sunny tomorrow, then I will go for a walk in the woods

Nw

the inverse of its converse:-

If I will not go for a walk in the woods then it is not sunny tomorrow

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the inverse of it's contrapositive

If I will go for a walk in the woods then it is sunny tomorrow

Question# 08

a) Jan is rich and happy

Jan is not rich or not happy

b) Laila will bicycle or run tomorrow

Laila will not bicycle and will not run tomorrow

c) the fan is slow or it is very hot

the fan is not slow and it is not very hot

d) Akram is unfit and Saleem is injured

Akram is fit or Saleem is not injured

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Question # 09:

a) exclusive

b) inclusive

c) inclusive

d) inclusive

Question # 10

a) $(p \wedge (\neg(\neg p \vee q))) \vee (p \wedge q) \equiv p$

Taking L.H.S

$(p \wedge (\neg(\neg p \vee q))) \vee (p \wedge q)$

$\Rightarrow (p \wedge (p \wedge \neg q)) \vee (p \wedge q)$ Demorgan's law

$\Rightarrow \cancel{p \wedge p \wedge p \wedge \neg q}$ $(p \wedge p \wedge (q \vee \neg q))$

$(p \wedge \neg q) \wedge p \vee (p \wedge q)$ Commutative law

$(p \wedge \neg q) \wedge p$ Absorption law

$p \wedge (p \wedge \neg q) \vee (p \wedge q)$

$\Rightarrow p \wedge (p \wedge (q \vee \neg q))$ Distributive law

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$\Rightarrow p \wedge (p \wedge (t))$ Negation law

Identity

~~Idempotent law~~

$\Rightarrow p \wedge p$

Idempotent

$\Rightarrow p$

b) $\neg(p \leftrightarrow q) \equiv (p \leftrightarrow \neg q)$

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Exo 10

b) $\neg(p \leftrightarrow q) \equiv (p \leftrightarrow \neg q)$

Sol$^n$

Taking LHS

$= \neg(p \leftrightarrow q)$

$\Rightarrow \neg((p \to q) \wedge (q \to p))$ By definition

$\Rightarrow \neg(p \to q) \vee \neg(q \to p)$ Demorgan's law

$\Rightarrow \neg(\neg p \vee q) \vee \neg(\neg q \vee p)$

$\Rightarrow (p \wedge \neg q) \vee (q \wedge \neg p)$

$\Rightarrow ((p \wedge \neg q) \vee q) \wedge ((p \wedge \neg q) \vee \neg p))$

Distributive law

$\Rightarrow ((q \vee p) \wedge (q \vee \neg q)) \wedge ((\neg q \vee p) \wedge (\neg q \vee \neg p))$

Distributive

$\Rightarrow ((q \vee p) \wedge T) \wedge ((\neg p \vee \neg q) \wedge T)$

~~Identity~~ Negation laws

$\Rightarrow q \vee p \; (p \vee q) \wedge (\neg p \vee \neg q)$

Identity

$\Rightarrow (q \vee p) \wedge (\neg p \vee \neg q)$ Commutative

$\Rightarrow (\neg(\neg q) \vee p) \wedge (p \to \neg q)$ By definition

$\Rightarrow (\neg q \to p) \wedge (p \to \neg q)$ By definition

$\Rightarrow p \leftrightarrow \neg q$ By definition

$\Rightarrow$ RHS

Proved

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c) $\neg p \leftrightarrow q \equiv p \leftrightarrow \neg q$

Taking L.H.S

$\neg p \leftrightarrow q$

$(\neg p \rightarrow q) \wedge (q \rightarrow \neg p)$ From definition

$\Rightarrow (p \vee q) \wedge (\neg q \vee \neg p)$ From definition

$\Rightarrow (\neg q \vee \neg p) \wedge (p \vee q)$ Commutative law

$\Rightarrow (\neg p \vee \neg q) \wedge (q \vee p)$ Commutative law

$\Rightarrow (p \rightarrow \neg q) \wedge (\neg(\neg q) \vee p))$ From definition Negation

$\Rightarrow (p \rightarrow \neg q) \wedge \cancel{(\neg q \vee p)}\ (\neg q \rightarrow p)$

$p \rightarrow q = \neg p \vee q$ (From definition)

$\Rightarrow p \leftrightarrow \neg q$

$\therefore (p \rightarrow q) \wedge (q \rightarrow p) = p \leftrightarrow q$

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d) $(p \wedge q) \rightarrow (p \rightarrow q) \equiv T$

Taking LHS

$= (p \wedge q) \rightarrow (p \rightarrow q)$ — From definition

$\Rightarrow (p \wedge q) \rightarrow (\neg p \vee q)$ — From definition

$\Rightarrow \neg(p \wedge q) \vee (\neg p \vee q)$ — DeMorgan's La

$\Rightarrow (\neg p \vee \neg q) \vee (\neg p \vee q)$

$\Rightarrow \cancel{\neg p \vee (\neg p \vee q) \vee \neg q}$

$\Rightarrow (\neg q \vee (\neg p \vee q)) \vee \neg p$ — Commutative

$\Rightarrow (\neg q \vee (q \vee \neg p)) \vee \neg p$ — Commutative

$\Rightarrow (\neg p \vee (\neg q \vee q)) \vee \neg p$ — Associative

$\Rightarrow (\neg p \vee (T)) \vee \neg p$ — Negation

$\Rightarrow T \vee \neg p$ — Identity

$\Rightarrow T$ — ~~Negation~~ Identity

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e) $\neg(p \vee \neg(p \wedge q)) = F$

Taking L.H.S

$\neg(p \vee \neg(p \wedge q))$

$\Rightarrow \neg(p \vee (\neg p \vee \neg q))$ — De Morgan's law

$\Rightarrow \neg(p \vee (\neg p \vee \neg q))$

$\Rightarrow \neg((p \vee \neg p) \vee \neg q))$ — Associative law

$\Rightarrow \neg(T \vee \neg q)$ — Negation law

$\Rightarrow \neg(T)$ — Identity

$\Rightarrow F$ — Negation law

$\Rightarrow$ R.H.S

Proved!

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Question# 11

a) $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$

| $p$ | $q$ | $r$ | $p \to r$ | $q \to r$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | T | F | F | F |
| T | F | T | T | T |
| T | F | F | F | T |
| F | T | T | T | T |
| F | T | F | T | F |
| F | F | T | T | T |
| F | F | F | T | T |

| $(p \to r) \wedge (q \to r)$ | $p \vee q$ | $(p \vee q) \to r$ |
|---|---|---|
| T | T | T |
| F | T | F |
| T | T | T |
| F | T | F |
| T | T | T |
| F | T | F |
| T | F | T |
| T | F | T |

equal

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b) $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$

| $p$ | $q$ | $r$ | $p \to q$ | $p \to r$ | $(p \to q) \vee (p \to r)$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | T | F | T | F | T |
| T | F | T | F | T | T |
| T | F | F | F | F | F |
| F | T | T | T | T | T |
| F | T | F | T | T | T |
| F | F | T | T | T | T |
| F | F | F | T | T | T |

| $q \vee r$ | $p \to (q \vee r)$ |
|---|---|
| T | T |
| T | T |
| T | T |
| F | F |
| T | T |
| T | T |
| T | T |
| F | T |

equal

Dm

c) $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$

Sol:

| p | q | r | $p \to q$ |
|---|---|---|---|
| T | T | T | |
| T | T | F | |
| T | T | T | |
| T | F | T | |
| T | F | F | |
| T | T | T | |
| F | T | F | |
| F | T | T | |
| F | F | T | |
| F | F | F | |

| p | q | r | s | $p \to q$ | $r \to s$ | $(p \to q) \to (r \to$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | T | F | T | F | F |
| T | T | F | T | T | T | T |
| T | T | F | F | T | T | T |
| T | F | T | T | F | T | T |
| T | F | T | F | F | F | T |
| T | F | F | T | F | T | T |
| T | F | F | F | F | T | T |
| F | T | T | T | T | T | T |
| F | T | T | F | T | F | F |
| F | T | F | T | T | T | T |
| F | T | F | F | T | T | T |
| F | F | T | T | T | T | T |
| F | F | T | F | T | F | F |
| F | F | F | T | T | T | T |
| F | F | F | F | T | T | T |

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| $p \to r$ | $q \to s$ | $(p \to r) \to (q \to s)$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |
| T | T | T |
| T | T | T |
| T | T | T |
| F | T | T |
| T | T | T |
| T | F | F |
| T | T | T |
| T | F | F |
| T | T | T |
| T | T | T |
| T | T | T |
| T | T | T |

Not equal!

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Question # 12:

a) $P(4,5)$

$\frac{5}{4} = k$

$k = 1.8$

False because k is an integer while here it is not.

$(k \notin Z^+)$

b) $P(2,4)$

$\frac{4}{2} = k$

$k = 2$

$(k \in Z^+)$, True.

c) $\forall m \forall n \; P(m,n)$

False.

d) $\exists m \forall n \; P(m,n)$

True.

e) $\exists n \forall m \; P(m,n)$

True

f) $\forall n \; P(1,n)$

True.

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Question #13

$U = \mathbb{R}$

a) $\exists x\,(x^2 = 2)$

True.

b) $\exists x\,(x^2 = -1)$

False

c) $\forall x\,(x^2 + 2 \geq 1)$

True.

d) $\exists x\,(x^2 = x)$

True, for $\boxed{x = 1}$.

Question # 14

$F(x, y) =$ "$x$ can fool $y$"
$U =$ all people in the world.

a) Everybody can fool Bob

$\forall x\, F(x, \text{Bob})$.

b) Alice can fool everybody.
$\forall y\, F(\text{Alice}, y)$.

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c) Everybody can fool somebody

$\exists y \forall x F(x,y)$

d) There is no one who can fool everybody.

$\cancel{\neg \exists x \forall y F(x,y)}$ $\neg \exists x \forall y F(x,y)$

$\cancel{\forall x \neg \exists y F(x,y)}$

e) Everybody can be fooled by somebody

$\exists x \forall y F(x,y)$

Question # 15:

$P(x)$ : $x$ can speak Russian

$Q(x)$ : $x$ knows the computer language C++

$U$ : all students of school

a) There is a student at your school who can speak Russian and who know C++

$\exists x (P(x) \wedge Q(x))$

b) There is a student at your school who can speak russian but who does know C++

b)

$\exists x (P(x) \wedge \neg Q(x))$

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c) Every student at your school either can speak Russian or knows C++

$\forall x\,(P(x) \oplus Q(x))$

d) No student at your school can speak Russian or knows C++

$\neg \exists x\,(P(x) \vee Q(x))$

Question # 16

$Q(x,y)$ = " $x$ has sent an email-message to $y$"

U = All students of class

a) $\exists x \exists y\, Q(x,y)$

Some students have sent an email message to some students of class

b) $\exists x \forall y\, Q(x,y)$

There exists a students who has sent email message to all the students of class.

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c) $\forall x \exists y \, O(x,y)$

All the students have sent an email message to some students of the class.

d) $\exists y \forall x \, O(x,y)$

There exist a student who has received email message from all the students of the class.

e) $\forall y \exists x \, O(x,y)$

All the students have received an email message from some students of the class

f) $\forall x \forall y \, O(x,y)$

All of the students have sent an email message to all the students of the class.

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# Question # 17

$P(x,y)$ = "Student $x$ has taken class $y$"
$U_1$ = All students of class
$U_2$ = All computer science courses at your school

a) $\exists x \exists y \, P(x,y)$

There exist a student who has taken a computer science course at school

b) $\exists x \forall y \, P(x,y)$

There is a student in class who has taken all the computer science courses in school

c) $\cancel{\exists x} \forall x \exists y \, P(x,y)$

All the students have taken a ~~class in~~ computer science course

d) $\exists y \forall x \, P(x,y)$

There exist a course that has been taken by all of the students.

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e) $\forall y \exists x P(x,y)$

All the computer science courses have been taken by some students of the class.

f) $\forall x \forall y P(x,y)$

All the student have taken all computer science courses.

Question # 18:

a) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.

let p = Alice is a mathematics major
q = Alice is a computer science major

$$\frac{p}{\therefore p \vee q}$$

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b)

Let $p$ = Jerry is a mathematics major
$q$ = Jerry is a computer science major

$$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array}$$

Simplification Rule.

c)

Let $p$ = It is raining
$q$ = the pool will be closed

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

Modus Ponens.

d)

Let $p$ = It snows today
$q$ = the university will close

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Modus Tollens

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e) Let $p$ = I go swimming
$q$ = I will stay in the sun too long
$r$ = I will sunburn

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore\ p \to r \end{array}$$

Hypothetical syllogism.

Question #19:

a) Let $p$ = Today is Tuesday
$q$ = I have a test in mathematics
$r$ = I have a test in economics
$s$ = My economics professor is sick

$$\begin{array}{l} p \to q \vee r \\ s \to \neg r \\ p \wedge s \\ \hline \therefore\ q \end{array}$$

$p \to q$
$s \to \neg r$
$p \wedge s$

~~Hypothesis~~
~~$p \to q \vee r$~~
~~$s \to \neg r$~~
~~$p \wedge s$~~
~~Conclusion:~~
~~$q$~~

~~$(p \to q \vee r) \wedge (s \to \neg r) \wedge$~~

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| | | |
|---|---|---|
| 1. | $p \wedge s$ | premise |
| 2. | $p$ | simplification law on 1 |
| 3. | $p \rightarrow q \vee r$ | premise |
| 4. | $q \vee r$ | Modus ponens on (2) & (3) |
| 5. | $s \rightarrow \neg r$ | Premise |
| 6. | $s$ | Simplification on (1) |
| 7. | $\neg r$ | Modus ponens on (5) & (6) |
| 8. | $r \vee q$ | Commutative law on (4) |
| 9. | $q$ | Elimination on (7) & (8). |

b) let $p$ = Ali is a lawyer

$q$ = Ali is ambitious

$r$ = Ali is an early riser

$s$ = Ali does not like chocolates

~~t = A~~

Hypothesis

$p \rightarrow q$

$r \rightarrow s$

Hypothesis

$p \rightarrow q$

$r \rightarrow s$

$q \rightarrow r$

Conclusion

$p \rightarrow s$

Hypothesis

$p \rightarrow s$

$\underbrace{(p \rightarrow q) \wedge (q \rightarrow r)} \wedge (r \rightarrow s)$

$\underbrace{(p \rightarrow r) \wedge (r \rightarrow s)}$ By Hypothetical syllogism

$\boxed{p \rightarrow s}$ By hypothetical syllogism

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Question #20.

$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$A = \{1, 2, 4, 5\}$

$B = \{2, 3, 5, 6\}$

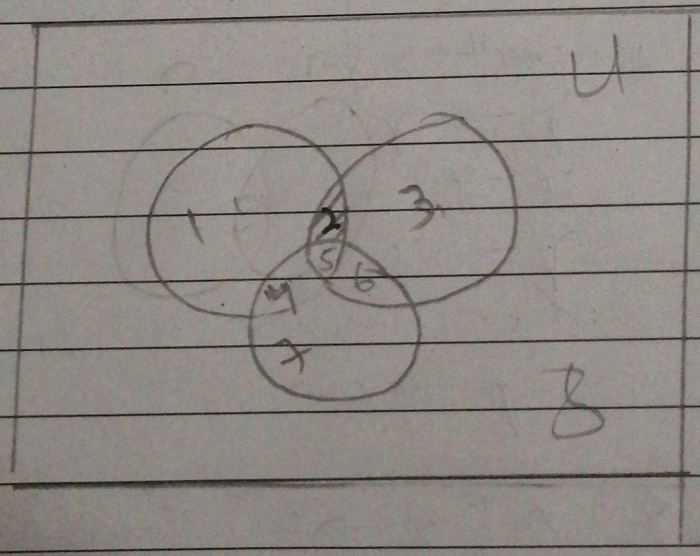
$C = \{4, 5, 6, 7\}$

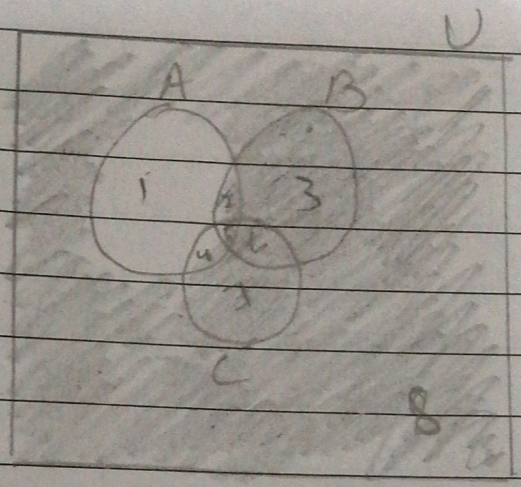
a) $(A \cap B) \cap \bar{C}$

$\bar{C} = \{1, 2, 3, 8\}$

$A \cap B = \{2, 5\}$

$(A \cap B) \cap \bar{C} = \{2\}$

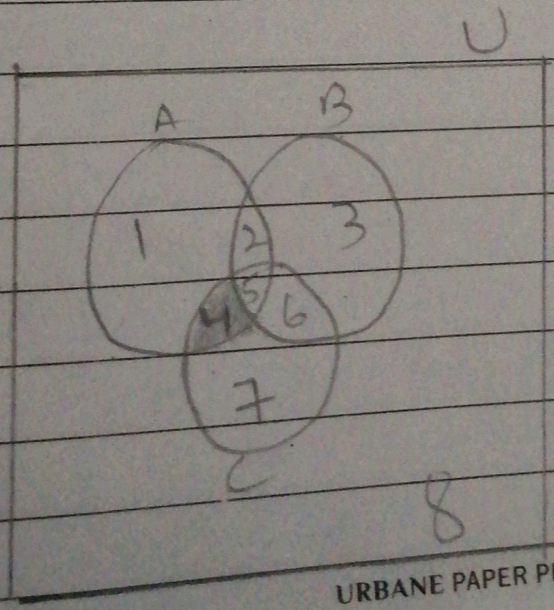





$(A-B)\cap C$

$A-B=\{1,4\}$

$(A-B)\cap C=\{4\}$

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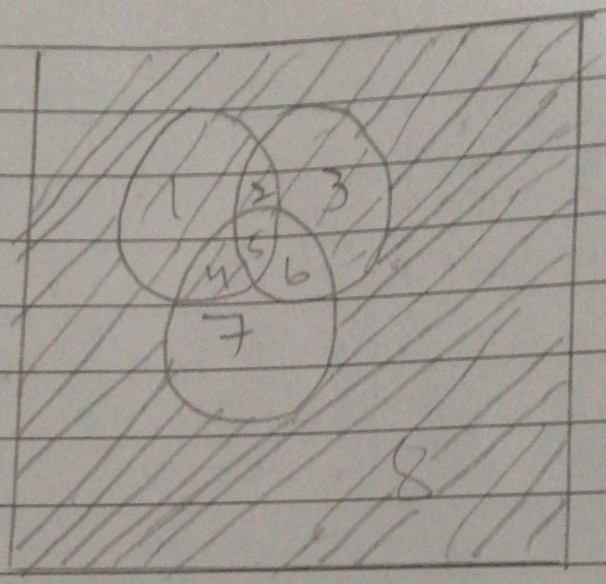
d) $(A \cap B) \cup \bar{C}$

$\bar{B} = \{1, 4, 7, 8\}$
$\bar{C} = \{1, 2, 3, 8\}$

$A \cap \bar{B} = \{1, 4\}$

$(A \cap \bar{B}) \cup \bar{C} = \{1, 2, 3, 4, 8\}$.



Question # 21

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c) $(A - (A \cap B)) \cap (B - (A \cap B)) = \phi$

Taking L.H.S

$= (A - (A \cap B)) \cap (B - (A \cap B))$

$\because A - B = A \cap B'$

(By def 2)

$\Rightarrow (A \cap (\overline{A \cap B})) \cap (B \cap (\overline{A \cap B}))$

$\Rightarrow A \cap (\bar{A} \cup \bar{B}) \cap (B \cap (\bar{A} \cup \bar{B}))$

By De Morgan's law

$\Rightarrow (A \cap \bar{A} \cup A \cap \bar{B}) \cap (B \cap \bar{A} \cup B \cap \bar{B})$

$\Rightarrow$ Distributive property

$\Rightarrow (\phi \cup A \cap \bar{B}) \cap (B \cap \bar{A} \cup \phi)$

$\because A \cap \bar{A} = \phi$

$\Rightarrow (A \cap \bar{B}) \cap (B \cap \bar{A})$

$(A \cap B) \cap ((\overline{A \cap B}) \cap (\overline{A \cap B}))$

Associative law

$(A \cap B) \cap (\overline{A \cap B})$

$\because A \cap A = A$

$\Rightarrow \phi$ = R.H.S

$\because A \cap \bar{A} = \phi$

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b) $(A-B)\cup(A\cap B) = A$

Taking LHS

$\Rightarrow (A\cap \bar{B})\cup(A\cap B)$ $\quad \because A-B = A\cap B'$

$\Rightarrow A\cap(\bar{B}\cup B)$ $\quad$ - Distributive law

$\Rightarrow A\cap(U)$ $\quad$ - $B\cup\bar{B} = U$ complement law

$\Rightarrow A$ $\quad$ - ~~$A\cup U$~~ $A\cap U = A$ Identity law.

$=$ R.H.S

Proved!

c) $(A-B)-C = (A-C)-B$

$(A\cap\bar{C})-B$

$(A\cap\bar{C})\cap B'$

Taking L.H.S:

$\Rightarrow (A-B)-C$

$\Rightarrow (A\cap B')-C$ $\quad \because A-B = A\cap B'$

$\Rightarrow (A\cap\bar{B})\cap\bar{C}$ $\quad \because A-B = A\cap B'$

$\Rightarrow (\bar{B}\cap A)\cap\bar{C}$ $\quad$ Commutative law

$\Rightarrow \bar{B}\cap(A\cap\bar{C})$ $\quad$ Associative law

$\Rightarrow (A\cap\bar{C})\cap\bar{B}$ $\quad$ Commutative law

$\Rightarrow (A\cap\bar{C})-B$ $\quad \because A-B = A\cap B'$

$\Rightarrow (A-C)-B$ $\quad \because A-B = A\cap B'$

$\Rightarrow$ R.H.S

Proved!

Date:

d) $\overline{(\bar{B} \cup (\bar{B} - A)} = B$

Taking LHS

$\Rightarrow \overline{\bar{B} \cup (\bar{B} - A)}$

$\Rightarrow \overline{\bar{B} \cup (\bar{B} \cap \bar{A})}$ $\quad$ $A - B = A \cap B'$

$\Rightarrow \overline{(\bar{B} \cup \bar{B}) \cap (\bar{A} \cup \bar{B})}$ $\quad$ Distributive law

$\Rightarrow \overline{\bar{B} \cap (\bar{A} \cup \bar{B})}$ $\quad$ Idempotent law

$\Rightarrow (\bar{\bar{B}}) \cup \overline{(\bar{A} \cup \bar{B})}$ $\quad$ De Morgan's law

$\Rightarrow B \cup (A \cap B)$ $\quad$ Double complement)

$\Rightarrow (A \cup B) \cap (B \cup B)$ $\quad$ De Morgan's Distributive law

$\Rightarrow B \cap (A \cup B)$

$\Rightarrow B \cap (B \cup A)$ $\quad$ Commutative law

$\Rightarrow B$ $\quad$ Absorption law

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# Question # 22

a)

Total apples = 100

worms = 20 = $|A|$

bruises = 15 = $|B|$

worms + bruised = 10 $|A \cap B|$

can't ~~are be sold~~

let $x$ be the apples that ~~can be sold~~

$x = |A| + |B| - |A \cap B|$

$x = 20 + 15 - 10$

$x = 35 - 10$

$\boxed{x = 25}$

Total apples = $100 - x$

= 75 apples can be sold

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b) Total students $= P(x) = 1000$
Those who like CS $= C(x) = 350$
Those who like SE $= S(x) = 450$
Those who like both $(C(x) \cap S(x)) = 100$

$$(C(x) \cup S(x)) = 350 + 450 - 100$$
$$= 700$$

700 students like either computer science or software.

$1000 - 700 = 300$ students like neither

c)

Mixed berry $= M(x) = 78$
Iced cream $= I(x) = 32$
Tiramisu $= T(x) = 57$
$(M(x) \cap I(x)) = 13$
$(I(x) \cap T(x)) = 21$
$(T(x) \cap M(x)) = 16$
$(M(x) \cap I(x) \cap T(x)) = 5$

$M(x) \cup T(x) = 78 + 57 - 16 = 119$
$M(x) \cup I(x) = 78 + 32 - 13 = 97$
$I(x) \cup T(x) = 32 + 57 - 21 = 68$

$$M(x) \cup T(x) \cup I(x) = 78 + 32 + 57 - 13 - 21 - 16 - 5$$
$$= 112$$

112 liked either one of the ~~follo~~ gelato
Since 14 liked none of the
$\therefore$ $112 + 14 = 126$ were the total number of

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students surveyed.

d) Use set builder notation to prove the following.

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

Taking LHS

$\Rightarrow \{x, y \mid x \in A \wedge y \in B \cap C\}$ By definition

$\Rightarrow \{x, y \mid x \in A \wedge y \in B \wedge y \in C\}$ $\because B \cap C = B \wedge C$

$\Rightarrow \{x, y \mid x \in A \wedge x \in A \wedge y \in B \wedge y \in C\}$ law of ~~addi~~ identity

$A = A \wedge B$

$A = A \wedge A$

$\Rightarrow \{x, y \mid x \in A \wedge y \in B \wedge x \in A \wedge y \in C\}$ Commutative law

$\Rightarrow \{x, y \mid x \in A \wedge y \in B\} \cap \{x, y \mid x \in A \wedge y \in C\}$

By definition of $\wedge$

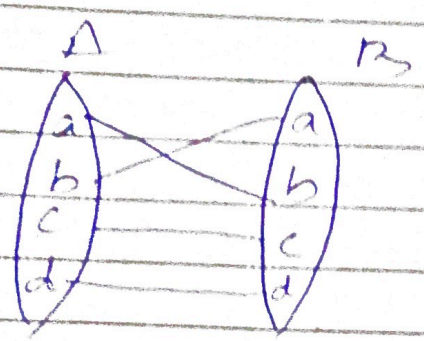
$\Rightarrow (A \times B) \cap (A \times C)$

By definition of Cartesian Product.

Date: A

# Question # 23

a)




(i)

$D(f(x)) = \{a, b, c, d\}$

Domain $(f(x)) = \{a, b, c, d\}$

$R(f(x)) = \{a, b, c, d\}$

(ii)

The function is bijective.

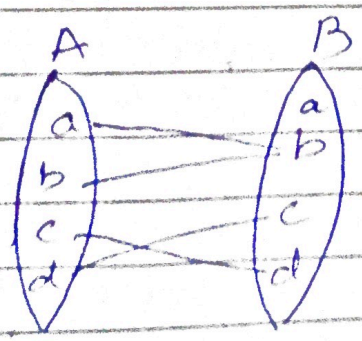
(iii)

The inverse of $f(x)$ exists.

$D(f^{-1}(x)) = B$

$R(f^{-1}(x)) = A$.

b)




(i)

$D(f(x)) = \{a, b, c, d\}$

Domain $= \{a, b, c, d\}$

$R(f(x)) = \{b, c, d\}$

(ii)

The function is not surjective or bijective

The inverse does not exist.

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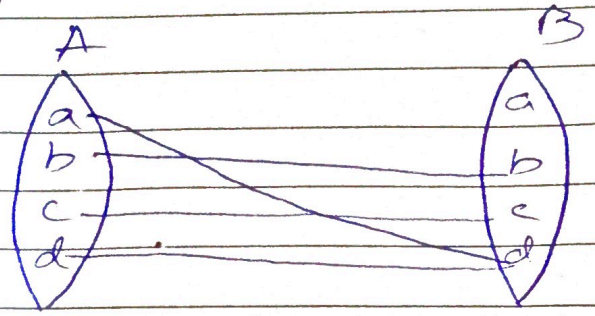
c)




(i)

$D(f(x)) = \{a, b, c, d\} = A$

Co Domain $= B$

$R(f(x)) = \{b, c, d\}$

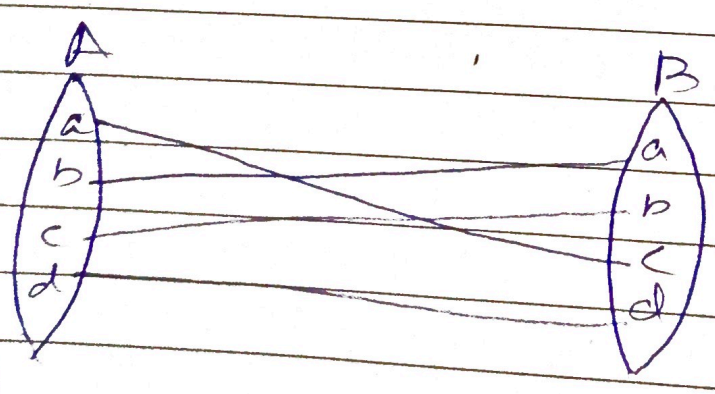
(ii)

Not an injective or surjective function.

(iii)

The inverse is not possible.

d)




(i)

$D(f(x)) = A$

$R(f(x)) = B$

Co Domain $= B$

(ii)

The function is bijective

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(iv)

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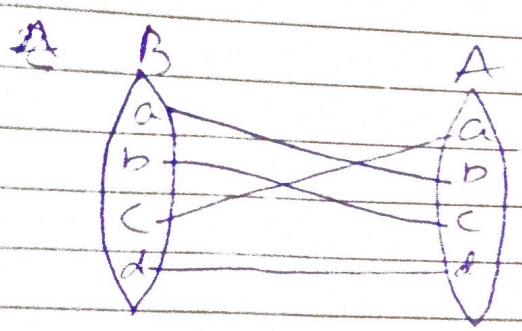
Inverse exist—




Question #24.

a) Let $f(x) = \left\lfloor \frac{x^2}{3} \right\rfloor$, find $f(S)$ if

(i) $S = \{-2, -1, 0, 1, 2, 3\}$

$f(-2) = \left\lfloor \frac{4}{3} \right\rfloor = \lfloor 1.33 \rfloor = 1$

$f(-1) = \left\lfloor \frac{1}{3} \right\rfloor = \lfloor 0.33 \rfloor = 0$

$f(0) = \lfloor 0 \rfloor = 0$

$f(1) = \left\lfloor \frac{1}{3} \right\rfloor = \lfloor 0.33 \rfloor = 0$

$f(2) = \left\lfloor \frac{4}{3} \right\rfloor = \lfloor 1.33 \rfloor = 1$

$f(3) = \left\lfloor \frac{9}{3} \right\rfloor = \lfloor 3 \rfloor = 3$

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(ii) $S = \{0, 1, 2, 3, 4, 5\}$

$f(0) = \lfloor 0 \rfloor = 0$

$f(1) = \left\lfloor \frac{1}{3} \right\rfloor = \lfloor 0.33 \rfloor = 0$

$f(2) = \left\lfloor \frac{4}{3} \right\rfloor = \lfloor 1.33 \rfloor = 1$

$f(3) = \left\lfloor \frac{9}{3} \right\rfloor = \lfloor 3 \rfloor = 3$

$f(4) = \left\lfloor \frac{16}{3} \right\rfloor = \lfloor 5.33 \rfloor = 5$

$f(5) = \left\lfloor \frac{25}{3} \right\rfloor = \lfloor 8.33 \rfloor = 8.$

(iii) $S = \{1, 5, 7, 11\}$

$f(1) = 0$

$f(5) = 8$

$f(7) = \left\lfloor \frac{49}{3} \right\rfloor = 16$

$f(11) = \left\lfloor \frac{121}{3} \right\rfloor = \lfloor 40.33 \rfloor = 40$

iv) $S = \{2, 6, 10, 14\}$

$f(2) = 1$

$f(6) = 12$

$f(10) = 33$

$f(14) = \lfloor 65.33 \rfloor = 65.$

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b)

(i) $\left[\frac{3}{4}\right]$

$[0.75] = 1$

(ii) $\left[\frac{7}{8}\right]$

$[0.87] = 0$.

(iii) $\left[\frac{-3}{4}\right]$

$[-0.75] = 0$

iv) $\left[\frac{-7}{8}\right]$

$[-0.87] = -1$

v) $[3]$

$[3] = 3$

vi) $[-1]$

$[-1] = -1$

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(vii) $\left\lfloor \frac{1}{2} + \left\lfloor \frac{3}{2} \right\rfloor \right\rfloor$

$\Rightarrow \left\lfloor \frac{1}{2} + 2 \right\rfloor$

$\Rightarrow \left\lfloor \frac{5}{2} \right\rfloor$

$\Rightarrow \lfloor 2.5 \rfloor$

$\Rightarrow 2$

(viii) $\left\lfloor \frac{1}{2} \cdot \left\lfloor \frac{5}{2} \right\rfloor \right\rfloor$

$\Rightarrow \left\lfloor \frac{1}{2} \cdot \lfloor 2.5 \rfloor \right\rfloor$

$\Rightarrow \left\lfloor \frac{1}{2} \cdot 2 \right\rfloor$

$\Rightarrow \lfloor 1 \rfloor$

$\Rightarrow 1$

Date:

c) Prove or disproof that if $x$ is a real number

$$\lfloor -x \rfloor = -\lceil x \rceil \quad \text{and} \quad \lceil -x \rceil = -\lfloor x \rfloor$$

Sol:

let $x = \frac{1}{2} \in \mathbb{R}$

$\lfloor -x \rfloor = -\lceil x \rceil$

$\lfloor -1/2 \rfloor = -\lceil 1/2 \rceil$

$\lfloor -0.5 \rfloor = -\lceil 0.5 \rceil$

$-1 = -1$

Proved.

$\lceil -x \rceil = -\lfloor x \rfloor$

$\lceil -1/2 \rceil = -\lfloor 1/2 \rfloor$

$\lceil -0.5 \rceil = -\lfloor 0.5 \rfloor$

$0 = -0$

$0 = 0$

Proved.

Date

Question # 25:

$f(g)$ $f(a) = 2a+3$

$g(a) = 3a+2$

a)

Composition of f and g.

$f \circ g = f(g(a)) = 2(g(a)) + 3$

$= 2(3a+2) + 3$

$= 6a+4+3$

$= 6a+7.$

Composition of g and f

$g \circ f = g(f(a)) = 3(f(a)) + 2$

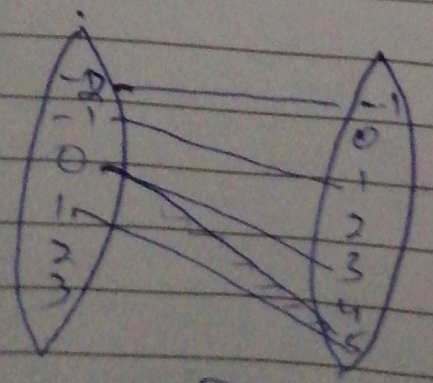
$= 3(2a+3) + 2$

$= 6a+9 + 2$

$g \circ f = 6a+11$

b)

$f(a)$

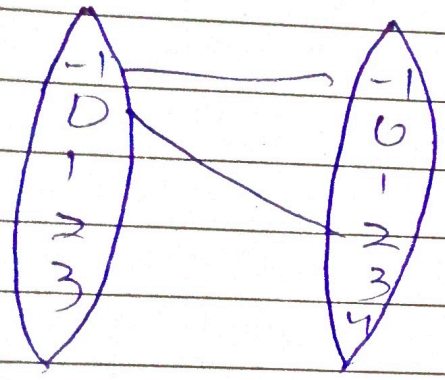



No

One - One function

Date:

g(a)




One - One function / Injective

(c)

f and g are not invertible because they are not bijective.