4.1 Q 2, 9, 11 & 12

4.2 Q 3, 4, 14 & 16

4.3 Q 15, 16 & 17

4.4 Q 11 & 14

Ex 4.1

Question 2

$u+v = (u_1+v_1+1, u_2+v_2+1)$,

$ku = (ku_1, ku_2)$

a)

$(u+v) = (u_1+v_1+1, u_2+v_2+1)$

$(0,4)+(1,-3) = (0+1+1, 4-3+1)$

$= (2,2)$.

b) ~~$u+(-u)=0$~~

$u+0 = u$

$0 = u+(-u)$

$0 = (u_1,u_2)+(-u_1,-u_2)$

$= (\cancel{u_1}-\cancel{u_1}+1, \cancel{u_2}-\cancel{u_2}+1)$

$0 = (1,1)$

$0 \neq (0,0)$

Proved!

c) Proved above that $0 = (-1,-1)$.

d) Axiom 5:

$$u + (-u) = 0$$
$$-u = 0 - u$$
$$-u = (1,1) - (u_1, u_2)$$
$$-u = (1 - u_1 - 1, 1 - u_2 - 1)$$
$$-u = (-u_1, -u_2) \in \text{vector space}$$

∴ Axiom 5 holds.

e) Axiom 7:

$$k(u+v) = ku + kv$$
$$k((u_1,u_2) + (v_1,v_2)) = k(u_1,u_2) + k(v_1,v_2)$$
$$k(u_1+v_1+1, u_2+v_2+1) = (ku_1, ku_2) + (kv_1, kv_2)$$
$$(k(u_1+v_1+1), k(u_2+v_2+1)) = (ku_1, ku_2) + (kv_1, kv_2)$$
$$(ku_1 + kv_1 + k, ku_2 + kv_2 + k) \neq (ku_1 + kv_1 + 1, ku_2 + kv_2 + 1)$$

Axiom 7 doesn't hold.

Axiom 8:

$(k+m)u = ku + mu$

$(k+m)(u_1, u_2) = k(u_1, u_2) + m(u_1, u_2)$

$((k+m)(u_1), (k+m)(u_2)) = (ku_1, ku_2) + (mu_1, mu_2)$

$\Rightarrow (ku_1 + mu_1, ku_2 + mu_2) \neq (ku_1 + mu_1 + 1, ku_2 + mu_2 + 1)$

Axiom 8 doesn't hold.

Question #09:

$$\begin{bmatrix} \cancel{0} & \cancel{0} \\ 0 & \cancel{0} \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

let $u = \begin{bmatrix} u_1 & 0 \\ 0 & u_2 \end{bmatrix}, v = \begin{bmatrix} v_1 & 0 \\ 0 & v_2 \end{bmatrix}$

$w = \begin{bmatrix} w_1 & 0 \\ 0 & w_2 \end{bmatrix}$

Axiom 2:

$u + v = v + u$

$$\begin{bmatrix} u_1 + v_1 & 0 \\ 0 & u_2 + v_2 \end{bmatrix} = \begin{bmatrix} v_1 + u_1 & 0 \\ 0 & u_2 + v_2 \end{bmatrix}$$

Axiom 2 holds.

Axiom 1.

$u+v \in$ Vector Space

$$u+v = \begin{bmatrix} u_1 & 0 \\ 0 & u_2 \end{bmatrix} + \begin{bmatrix} v_1 & 0 \\ 0 & v_2 \end{bmatrix}$$

$$u+v = \begin{bmatrix} u_1+v_1 & 0 \\ 0 & u_2+v_2 \end{bmatrix} \in \text{Vector space}$$

Axiom 1 holds.

Axiom 3:

$$(u+v)+w = u+(v+w)$$

$$u+v = \begin{bmatrix} u_1+v_1 & 0 \\ 0 & u_2+v_2 \end{bmatrix} = v+w = \begin{bmatrix} v_1+w_1 & 0 \\ 0 & v_2+w_2 \end{bmatrix}$$

$$(u+v)+w = \begin{bmatrix} u_1+v_1+w_1 & 0 \\ 0 & u_2+v_2+w_2 \end{bmatrix}$$

$$(v+w) = \begin{bmatrix} u_1+v_1+w_1 & 0 \\ 0 & u_2+v_2+w_2 \end{bmatrix}$$

Axiom 3 holds

(4) $u + 0 = u$

$0 = u + (-u)$

$$0 = \begin{bmatrix} u_1 & 0 \\ 0 & u_2 \end{bmatrix} + \begin{bmatrix} -u_1 & 0 \\ 0 & -u_2 \end{bmatrix}$$

$$= \begin{bmatrix} u_1 - u_1 & 0 \\ 0 & u_2 - u_2 \end{bmatrix}$$

$$0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in \text{Vector Space}$$

Axiom (4) holds

Axiom 5:

$u + (-u) = 0$

$-u = 0 + (-u)$

$$-u = 0 + \begin{bmatrix} -u_1 & 0 \\ 0 & -u_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -u_1 & 0 \\ 0 & -u_2 \end{bmatrix}$$

$$-u = \begin{bmatrix} -u_1 & 0 \\ 0 & -u_2 \end{bmatrix} \in \text{Vector Space}$$

Axiom 6:

$ku \in$ vector space

$$k\begin{bmatrix} u_1 & 0 \\ 0 & u_2 \end{bmatrix}$$

$$\begin{bmatrix} ku_1 & 0 \\ 0 & ku_2 \end{bmatrix} \in \text{vector space}$$

Axiom 6 holds.

Axiom 7:

$k(u+v) = ku + kv$

$$k\begin{bmatrix} u_1+v_1 & 0 \\ 0 & u_2+v_2 \end{bmatrix} = \begin{bmatrix} ku_1 & 0 \\ 0 & ku_2 \end{bmatrix} + \begin{bmatrix} kv_1 & 0 \\ 0 & kv_2 \end{bmatrix}$$

$$\begin{bmatrix} k(u_1+v_1) & 0 \\ 0 & k(u_2+v_2) \end{bmatrix} = \begin{bmatrix} k(u_1+v_1) & 0 \\ 0 & k(u_2+v_2) \end{bmatrix}$$

Axiom 7 holds.

Axiom 8:

$(k+m)u = ku + mu$

$$(k+m)\begin{bmatrix} u_1 & 0 \\ 0 & u_2 \end{bmatrix} = \begin{bmatrix} ku_1 & 0 \\ 0 & ku_2 \end{bmatrix} + \begin{bmatrix} mu_1 & 0 \\ 0 & mu_2 \end{bmatrix}$$

$$\begin{bmatrix} (k+m)u_1 & 0 \\ 0 & (k+m)u_2 \end{bmatrix} = \begin{bmatrix} (k+m)u_1 & 0 \\ 0 & (k+m)u_2 \end{bmatrix}$$

Axiom 8 holds.

Axiom 9:

$(km)u = k(mu)$

$$km\begin{bmatrix} u_1 & 0 \\ 0 & u_2 \end{bmatrix} = k\begin{bmatrix} mu_1 & 0 \\ 0 & mu_2 \end{bmatrix}$$

$$\begin{bmatrix} kmu_1 & 0 \\ 0 & kmu_2 \end{bmatrix} = \begin{bmatrix} kmu_1 & 0 \\ 0 & kmu_2 \end{bmatrix}$$

Axiom 9 holds

Axiom 10

$1u = u$

$$1\begin{bmatrix} u_1 & 0 \\ 0 & u_2 \end{bmatrix} = \begin{bmatrix} u_1 & 0 \\ 0 & u_2 \end{bmatrix}$$

$$\begin{bmatrix} u_1 & 0 \\ 0 & u_2 \end{bmatrix} = \begin{bmatrix} u_1 & 0 \\ 0 & u_2 \end{bmatrix}$$

<u>Proved!</u>

Axiom 10 holds

$\therefore$ The given space is a vector space

Question #11:

$u=(1,u),\ v=(1,v),\ w=(1,w)$

Axiom 1

$u+v \in$ vector space

$u+v=(1,u)+(1,v)$

$=(1,u+v)$

$u+v=(1,u+v) \in$ vector space.

Axiom 2:

$u+v=v+u$

$(1,u)+(1,v)=(1,v)+(1,u)$

$(1,u+v)=(1,v+u)$

$(1,u+v)=(1,u+v)$

Axiom 2 holds

Axiom 3:

$(u+v)+w=u+(v+w)$

$\{(1,u)+(1,v)\}+(1,w)=(1,u)+\{(1,v)+(1,w)\}$

$(1,u+v)+(1,w)=(1,u)+(1,v+w)$

$(1,u+v+w)=(1,u+v+w)$

Axiom 3 holds.

Axiom 4:

$u+0=u$

$0=u+(-u)$

$0=u+(1,-u)$

$0=(1,u)+(1,-u)$

$0=(1,u-u)$

$0=(1,0) \in$ vector space

Axiom 4 holds

Axiom 5:

$u+(-u)=0$

$\cancel{(1,u)+(1,u)=}$

$-u=0+(-u)$

$-u=(\cancel{0},(1,0)+(1,-u)$

$-u=(1,-u) \in$ vector space.

Axiom 5 holds

Axiom 6:

$ku \in V$

$k(1,u)=(1,ku) \in$ vector space

Axiom 6 holds.

Axiom 7.

$k(u+v) = ku + kv$

$k\{(1,u)+(1,v)\} = k(1,u) + k(1,v)$

$k\{(1,u+v)\} = (1,ku) + (1,kv)$

$(1, k(u+v)) = (1, ku+kv)$

$(1, k(u+v)) = (1, k(u+v))$

Axiom 7 holds.

Axiom 8:

$(k+m)u = ku + mu$

$(k+m)(1,u) = k(1,u) + m(1,u)$

$(1, (k+m)u) = (1,ku) + (1,mu)$

$(1, (k+m)u) = (1, ku+mu)$

$(1, (k+m)u) = (1, (k+m)u)$

Axiom 8 holds.

Axiom 9.

$k(mu) = (km)u$

$k\{m(1,u)\} = \cancel{k(1,} \; km(1,u)$

$k(1,mu) = (1, kmu)$

$(1, kmu) = (1, kmu)$ Axiom 9 holds.

Axiom 10:

$1u = u$

$1(1, u) = (1, u)$

$(1, u) = (1, u)$

Axiom 10 holds

The given space is a vector space

Question # 12

$a_0 + a_1x$

Sol:

let $u = u_0 + u_1x$

$v = v_0 + v_1x$

$w = w_0 + w_1x$

Axiom 1:

$u + v \in$ vector space

$u + v = (u_0 + u_1x) + (v_0 + v_1x)$

$u + v = u_0 + v_0 + (u_1 + v_1)x$

Axiom 2:

$u + v = v + u$

$(u_0 + u_1x) + (v_0 + v_1x) = (v_0 + v_1x) + (u_0 + u_1x)$

$u_0 + v_0 + (u_1 + v_1)x = v_0 + u_0 + (u_1 + v_1)x$

Axiom 2 holds

Axiom 4:

$u + 0 = u$

$0 = u + (-u)$

$0 = u_0 + u_1 x + (-u_0) + (-u_1) x$

$0 = 0 \in$ vector space

Axiom 4 holds.

~~Axiom 5~~

Axiom 3:

$(u + v) + w = u + (v + w)$

$\{u_0 + v_0 + (u_1 + v_1) x\} + w_0 + w_1 x =$

$u_0 + u_1 x + \{v_0 + w_0 + (v_1 + w_1) x\}$

$u_0 + v_0 + w_0 + (u_1 + v_1 + w_1) x =$

$u_0 + v_0 + w_0 + (u_1 + v_1 + w_1) x$

Axiom 3 holds

Axiom 5.

$u + (-u) = 0$

$-u = 0 + (-u)$

$-u = 0 + (-u_0) + (-u_0)x$

$-u = -u_0 - u_0 x \in$ vector space

Axiom 5 holds

Axiom 6.

$ku \in V$

$k(u_0 + u_1 x) = ku_0 + (ku_1)x \in$ vector space

Axiom 6 holds

Axiom 7:

$k(u + v) = ku + kv$

$k\{(u_0 + u_1 x) + (v_0 + v_1 x)\} \stackrel{?}{=} (ku_0 + ku_1 x) + (kv_0 + kv_1 x)$

$k(u_0 + v_0 + (u_1 + v_1)x) = ku_0 + kv_0 + (ku_1 + kv_1)x$

$ku_0 + kv_0 + (ku_1 + kv_1)x = ku_0 + kv_0 + (ku_1 + kv_1)x$

Axiom 7 holds.

Axiom 8:

$(k+m)u = ku + mu$

$(k+m)(u_0 + u_1x) = k(u_0 + u_1x) + m(u_0 + u_1x)$

$((k+m)u_0 + (k+m)u_1x) = (ku_0 + (ku_1)x) + (mu_0 + (mu_1)x)$

$(ku_0 + mu_0 + (k+m)u_1x) = ku_0 + mu_0 + (ku_1 + mu_1)x$

Axiom 8 holds

Axiom 9:

$(km)u = k(mu)$

~~km ()~~

$km(u_0 + u_1x) = k\{m(u_0 + u_1x)\}$

$kmu_0 + (kmu_1)x = kmu_0 + (kmu_1)x$

Axiom 9 holds

Axiom 10:

$1u = u$

$1(u_0 + u_1 x) = u_0 + u_1 x$

$u_0 + u_1 x = u_0 + u_1 x$

Axiom 10 holds

The given space is a vector space

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Ex 4.2

Question # 3

a) The set of all diagonal $n \times n$ matrices

Axiom 1:

$u+v \in$ vector space

Let $u = \begin{bmatrix} u_1 & 0 \\ 0 & u_2 \end{bmatrix}$, $v = \begin{bmatrix} v_1 & 0 \\ 0 & v_2 \end{bmatrix}$

$u+v = \begin{bmatrix} u_1+v_1 & 0 \\ 0 & u_2+v_2 \end{bmatrix} \in$ vector space

Axiom 6:

$ku \in$ vector space

$k \begin{bmatrix} u_1 & 0 \\ 0 & u_2 \end{bmatrix} = \begin{bmatrix} ku_1 & 0 \\ 0 & ku_2 \end{bmatrix}$

b) The set of all $n \times n$ matrices A such that $\det(A) = 0$

let $u = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $v = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$u+v = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$u+v = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \notin$ vector space

Therefore it's not a subspace of M_{nn}.

c) The set of all $n \times n$ matrices A such that $\mathrm{tr}(A) = 0$

let $u = \begin{bmatrix} u_1 & 0 \\ 0 & -u_1 \end{bmatrix}$, $v = \begin{bmatrix} v_1 & 0 \\ 0 & -v_1 \end{bmatrix}$

Axiom I.

~~u_1+v_1~~ $u+v = \begin{bmatrix} u_1+v_1 & 0 \\ 0 & -u_1-v_1 \end{bmatrix} \in$ vector space

Axiom 1 holds

Axiom 6:

$$ku = k\begin{bmatrix} u_1 & 0 \\ 0 & -u_1 \end{bmatrix}$$

$$\begin{bmatrix} ku_1 & 0 \\ 0 & -ku_1 \end{bmatrix} \in \text{vector space}$$

So the given space is a subspace of M_{nn}.

Ex d) The set of all symmetric $n\times n$ matrices

let $u = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$v = \begin{bmatrix} c & d \\ d & c \end{bmatrix}$

(i) $u+v = \begin{bmatrix} a+c & d+b \\ d+b & a+c \end{bmatrix} \in$ vector space

(ii) $ku = k\begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} ka & kb \\ kb & ka \end{bmatrix} \in$ vector space

The given space is a subspace of M_{nn}.

Question #4

a) let $A^T = -A$ & $B^T = -B$

Axiom 1:

$(A+B)^T = A^T + B^T = -A - B = -(A+B)$

$\therefore$ Axiom 1 holds.

Axiom 6:

$k(A^T) = k(-A) = -kA$

$k(A^T) = -kA$

$\therefore$ Axiom 6 holds.

Hence $A^T = -A$ is a subspace of M_{nn}

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b) The set of all $n \times n$ matrices A such that for which $AX = 0$ has only trivial solution.

let $u = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$v = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$u+v = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \in$ vector space

$ku = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} \in$ vector space

So it is a subspace of M_{nn}.

c) The set of all $n \times n$ matrices A such that AB = BA for some fixed $n \times n$ matrix B.

if AB = BA then A & B are symmetric

therefore

let $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$B = \begin{bmatrix} c & d \\ d & a \end{bmatrix}$

$A+B = \begin{bmatrix} a+c & b+d \\ b+d & a+c \end{bmatrix} \in$ vector space

$k(A) = \begin{bmatrix} ka & kb \\ kb & ka \end{bmatrix} \in$ vector space

$\therefore$ The given space is a subspace of M_{nn}

d) The set of all invertible $n \times n$ matrices

let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

let $B = \begin{bmatrix} -1 & 0 \\ 0 & +2 \end{bmatrix}$

$A + B = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix} \notin$ vector space

$\therefore$ the given space is not a subspace of M_{nn}.

Question #14

a)

$$Ax = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Sol:

let there be two vectors u & v in R^4 such that

$$Au = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ \& } Av = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A(u+v) = Au + Av$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 2 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\therefore$ it is not a subspace of R^4.

b) All vectors in R^4 such that $Ax = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, where A is as in

Part (a)

let u & v be two vectors in R^4 such that $Au = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ & $Av = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

~~u+v~~

$A(u+v) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$Au + Av = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \in$ vector space

$A(ku) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$k \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \in$ vector space

The given space is a subspace of R^4

Question # 16

a) All polynomials with even coeff

let $u = 2k + 2kx$

let $v = 2l + 2lx$

$u + v = 2k + 2kx + 2l + 2lx$

$= 2k + 2l + 2kx + 2lx$

$= 2(k+l) + x(2(k+l)) \in$ vector space

$\Rightarrow$

~~$ku = k(2k + 2kx)$~~

$hu = h(2k + 2kx)$

If $k = 1$ & $h = 2$

$ku = 1 + x \notin$ vector space

$\therefore$ the given space is not a subspace.

b) All polynomials whose coefficients sum to 0.

Let $u = u_0 - u_0 x$
$v = v_0 - v_0 x$

$u + v = (u_0 - u_0 x) + (v_0 - v_0 x)$
$= (u_0 + v_0) + (-u_0 x - v_0 x)$
$= (u_0 + v_0) + x(-u_0 - v_0) \in$ vector space

$ku = k(u_0 - u_0 x)$
$= ku_0 + (-ku_0)x \in$ vector space

So it is subspace of P_∞.

c)

let $u = 1 + 4x + 6x^2$
let $v = 2 + 3x \cancel{+} - 6x^2$

$u + v = 3 + 7x + \cancel{6x^2} - 6x^2$
$u + v = 3 + 7x \notin$ vector space

So it is not a subspace of P_2.

Question#15

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$-R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\cancel{R_1+R_2}\ R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 + s = 0 \Rightarrow x_1 = -s$

$x_2 + t = 0 \Rightarrow x_2 = -t$

$x_3 = s$

$x_4 = t$

$$\begin{bmatrix} -s \\ -t \\ s \\ t \end{bmatrix} = \underbrace{s\underbrace{\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{v_1} + t\underbrace{\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}}_{v_2}}_{W}$$

a) $u = (1, 0, -1, 0)$
$v = (0, 1, 0, -1)$

$u = -v_1$
$v = -v_2$

$\therefore$ u, v spans W i.e the sol^n space of A

b) $u = (1, 0, -1, 0)$
$v = (1, 1, -1, -1)$

$u = -v_1$
$v = -(v_1 + v_2)$

$\therefore$ u & v spans W

Question # 10:

$$A = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 2 \\ 0 & 3 & -3 & 3 \end{bmatrix}$$

$R_2 - 2R_1$

$$= \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 3 \end{bmatrix}$$

$R_3 - 3R_1$

$$= \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_2 - s + t = 0 \Rightarrow x_2 = s - t$

$x_3 = s$

$x_4 = t$

Sol^n space: $\begin{bmatrix} 0 \\ s-t \\ s \\ t \end{bmatrix}$

$$= \begin{bmatrix} 0 \\ s \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -t \\ 0 \\ t \end{bmatrix}$$

$$= s \underbrace{\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}}_{v_1} + t \underbrace{\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}}_{v_2}$$

a) $u = (1, 1, 1, 0)$
$v = (0, -1, 0, 1)$

Since u is not a linear combination of v_1 & v_2 therefore (u, v) doesn't span W.

b) $u = (0, 1, 1, 0)$
$v = (1, 0, 1, 1)$

Since v is not a linear combination of v_1 & v_2 therefore (u, v) doesn't span W

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$\begin{bmatrix}1 & 0\end{bmatrix}\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$ $\begin{bmatrix}1+0 \\ 3+0\end{bmatrix}$ $2+0$

Q#17

a) $A = \begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}$

$u_1 = \begin{bmatrix}1\\2\end{bmatrix}, \quad u_2 = \begin{bmatrix}-1\\1\end{bmatrix}$

$T_A(u_1) = \begin{bmatrix}1\\2\end{bmatrix}\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}$

$= \begin{bmatrix}1+0 & -1+4\\ 2+0 & +2-1\end{bmatrix}$

$= \begin{bmatrix} \end{bmatrix}$

$u_1 = \begin{bmatrix}1 & 2\end{bmatrix}, \quad u_2 \begin{bmatrix}-1 & 1\end{bmatrix}$

$T_A(u_1) = \begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}$

$= \begin{bmatrix}1-2\\ 0+4\end{bmatrix}$

$T_A(u_1) = \begin{bmatrix}-1\\ 4\end{bmatrix}$

$T_A(u_2) = \begin{bmatrix}-1 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}$

$T_A(u_2) = \begin{bmatrix}-1+0\\ 1+2\end{bmatrix} \Rightarrow \begin{bmatrix}-1\\ 3\end{bmatrix}$

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$$\begin{pmatrix} \cdots \\ 0 & 2 \end{pmatrix}\begin{pmatrix} \cdots \\ 2 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 0 & -14 \end{pmatrix}$$

$$k_1 [T_a(u_1)] + k_2 [T_a(u_2)] = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$k_1 \begin{bmatrix} -1 \\ 4 \end{bmatrix} + k_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} -k_1 - k_2 \\ 4k_1 + 3k_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$-k_1 - k_2 = b_1$$
$$4k_1 + 3k_2 = b_2$$

$$\begin{bmatrix} -1 & -1 & b_1 \\ 4 & 3 & b_2 \end{bmatrix}$$

$-R_1$

$$\begin{bmatrix} 1 & 1 & -b_1 \\ 4 & 3 & b_2 \end{bmatrix}$$

$R_2 - 4R_1$

$$\begin{bmatrix} 1 & 1 & -b_1 \\ 0 & -1 & b_2 - 4b_1 \end{bmatrix}$$

$-R_2$

$$= \begin{bmatrix} 1 & 1 & -b_1 \\ 0 & 1 & 4b_1 - b_2 \end{bmatrix}$$

$R_2 - R_1$

$$\begin{bmatrix} 1 & 4 & 7 \\ 5 & 6 & 9 \end{bmatrix} \begin{bmatrix} 3 & 6 & 5 \\ 4 & 7 & 1 \\ 5 & 8 & 0 \end{bmatrix}$$

$[3+16+35$

$$= \begin{bmatrix} 1 & 0 & -b_1 - 4b_1 + b_2 \\ 0 & 1 & 4b_1 - b_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -5b_1 + b_2 \\ 0 & 1 & 4b_1 - b_2 \end{bmatrix}$$

Since ~~it~~ the system is unconditionally consistent

Therefore

$\{T_A(u_1), T_A(u_2)\}$ will span R^2.

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$$\begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1-0 \\ -2+0 \end{bmatrix}$$

b) $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$

$u_1 = \begin{bmatrix} 1 & 2 \end{bmatrix}$

$u_2 = \begin{bmatrix} -1 & 1 \end{bmatrix}$

~~$T_A(u_1) = \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$~~

~~$= \begin{bmatrix} 1-2 \\ -2+4 \end{bmatrix}$~~

$$T_A(u_1) = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 \\ -2+4 \end{bmatrix}$$

$$T_A(u_1) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$T_A(u_2) = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-1 \\ 2+2 \end{bmatrix}$$

$$T_A(u_2) = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

$$k_1\begin{bmatrix}-1\\2\end{bmatrix}+k_2\begin{bmatrix}-2\\4\end{bmatrix}=\begin{bmatrix}b_1\\b_2\end{bmatrix}$$

$$\begin{bmatrix}-k_1\\2k_1\end{bmatrix}+\begin{bmatrix}-2k_2\\4k_2\end{bmatrix}=\begin{bmatrix}b_1\\b_2\end{bmatrix}$$

$$-k_1-2k_2=b_1.$$
$$2k_1+4k_2=b_2$$

$$\begin{bmatrix}-1 & -2 & b_1\\2 & 4 & b_2\end{bmatrix}$$

$-R_1$

$$\begin{bmatrix}1 & 2 & -b_1\\2 & 4 & b_2\end{bmatrix}$$

R_2-2R_1

$$\begin{bmatrix}1 & 2 & -b_1\\0 & 0 & b_2+2b_1\end{bmatrix}$$

The system is conditionally consistent, therefore $\{T_A(u_1), T_A(u_2)\}$ will not span R^2.

Exercise 4.4:

Question #11

$v_1 = \left(\lambda, \frac{-1}{2}, \frac{-1}{2}\right)$

$v_2 = \left(\frac{-1}{2}, \lambda, \frac{-1}{2}\right)$

$v_3 = \left(\frac{-1}{2}, \frac{-1}{2}, \lambda\right)$

Sol:-

$k_1 v_1 + k_2 v_2 + k_3 v_3 = 0$

$k_1\left(\lambda, \frac{-1}{2}, \frac{-1}{2}\right) + k_2\left(\frac{-1}{2}, \lambda, \frac{-1}{2}\right)$

$+ k_3\left(\frac{-1}{2}, \frac{-1}{2}, \lambda\right) = (0, 0, 0)$

$\left(k_1\lambda, -\frac{k_1}{2}, -\frac{k_1}{2}\right) + \left(\frac{-k_2}{2}, k_2\lambda, -\frac{k_2}{2}\right)$

$\left(\frac{-k_3}{2}, \frac{-k_3}{2}, k_3\lambda\right) = (0, 0, 0)$

$k_1\lambda - \frac{k_2}{2} - \frac{k_3}{2} = 0$

$-\frac{k_1}{2} + k_2\lambda - \frac{k_3}{2} = 0$

$-\frac{k_1}{2} - \frac{k_2}{2} + k_3\lambda = 0$

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The coefficient matrix A is

$$\begin{bmatrix} \lambda & -1/2 & -1/2 \\ -1/2 & \lambda & -1/2 \\ -1/2 & -1/2 & \lambda \end{bmatrix}$$

$\Rightarrow$ For a system to be dependent det(A) should be 0

$$\lambda \begin{vmatrix} \lambda & -1/2 \\ -1/2 & \lambda \end{vmatrix} + \frac{1}{2} \begin{vmatrix} -1/2 & -1/2 \\ -1/2 & \lambda \end{vmatrix} - \frac{1}{2} \begin{vmatrix} -1/2 & \lambda \\ -1/2 & -1/2 \end{vmatrix}$$

$$\lambda\left(\lambda^2 - \frac{1}{4}\right) + \frac{1}{2}\left(-\frac{\lambda}{2} - \frac{1}{4}\right) - \frac{1}{2}\left(\frac{1}{2} + \frac{\lambda}{2}\right) = 0$$

$$\lambda^3 \cancel{+\frac{\lambda}{4}} \cancel{-\frac{\lambda}{4}} + \frac{1}{8} - \frac{1}{2} - \frac{1}{4} - \frac{\lambda}{4}$$

$$\frac{4\lambda^3 - \lambda}{4} \quad \frac{+1}{8} = 0 \quad \frac{\lambda}{4} = 0$$

$4\lambda^3 - \lambda + 2 = 0$

$4\lambda^3 - 2\lambda + \lambda + 2 = 0$

$\lambda^3 + \frac{1}{8} - \frac{1}{4} - \frac{\lambda}{4} = 0$

$\frac{8\lambda^3 + 1 - 2 - 2\lambda}{8} = 0$

$8\lambda^3 - 2\lambda - 1 = 0$

$8\lambda^3 - 4\lambda + 2\lambda - 1 = 0$

$4\lambda(2\lambda^2 - 1) + 1$

$\lambda^3 - \frac{\lambda}{4} - \frac{\lambda}{4} - \frac{1}{8} - \frac{1}{4} - \frac{\lambda}{4} = 0$

$\lambda^3 - \frac{3\lambda}{4} - \frac{3}{8} = 0$

$8\lambda^3 - 6\lambda - 3 = 0$

$(\lambda + \frac{1}{2})(\lambda + \frac{1}{2})(\lambda - 1) = 0$

$\therefore \lambda = \{1, -\frac{1}{2}\}$ Ans.

Q#14:

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & 2 & 0 \end{bmatrix}$$

Sol:

$$T_A(u_1) = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$T_A(u_1) = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$T_A(u_2) = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1+2 \\ 2+0-3 \\ 4-2+0 \end{bmatrix}$$

$$T_A(u_2) = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

$$T_a(u_3) = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$T_a(u_3) = \begin{bmatrix} 0+1+2 \\ 0+0-3 \\ 0+2+0 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix}$$

$$k_1 \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + k_2 \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} + k_3 \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$k_1 + 3k_2 + 3k_3 = 0$$
$$k_1 - k_2 - 3k_3 = 0$$
$$2k_1 + 2k_2 + 2k_3 = 0.$$

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & -1 & -3 \\ 2 & 2 & 2 \end{bmatrix}$$

$$\begin{vmatrix} 1 & 3 & 3 \\ 1 & -1 & -3 \\ 2 & 2 & 2 \end{vmatrix} = -8 \neq 0$$

$\therefore$ $\{T_a(u_1), T_a(u_2), T_a(u_3)\}$ is linearly independent.

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$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -3 \\ 2 & 2 & 0 \end{bmatrix}$$

$$T_a(u_1) = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$T_a(u_2) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$T_a(u_2) = \begin{bmatrix} 2-1+1 \\ 2-1-3 \\ 4-2+0 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$$

$$T_a(u_3) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+1 \\ 0+1-3 \\ 0+2+0 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$$

Since we can see that $T_a(u_2)$ is a linear combination of $T_a(u_3)$, then T_a is linearly dependent.

$T_a(u_2) = 1\, T_a(u_3)$