Lab 0

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Part 1 - Setting up the problem

1) Which teams have been eliminated from getting the Dunkin Donuts prize? Which teams have not been eliminated? Why or why not?

To identify if a team has been eliminated from the competition, we can verify if their total potential wins for the entire game are incapable of matching the current number of wins held by the leader (Emily). The total potential wins for each player is equivalent to $w_{name} + r_{name}$ (using the notation of question 2), and the number of wins to beat is w_{Emily}.

1. Emily is the current leader with $w_{Emily} = 83$, and cannot be eliminated yet.
2. Prava, with total potential wins of $w_{Prava} + r_{Prava} = 83$, has announced she will quit as soon as she can no longer win the most games (and at best she will be able to match Emily), so Prava quits and is effectively eliminated.
3. Shashank, with total potential wins of $w_{Shashank} + r_{Shashank} = 84$, can still win, as $84 > w_{Emily}$. Shashank is not eliminated yet.
4. Vicky, with total potential wins of $w_{Vicky} + r_{Vicky} = 80$, cannot possibly win, as $80 < w_{Emily}$. Furthermore, if Prava does quit, we can expect r_{Vicky} to go down, as 2 of Vicky's remaining games are against Prava. Vicky is eliminated.

2) The capacities in Emily's network flow can be represented in the following manner:

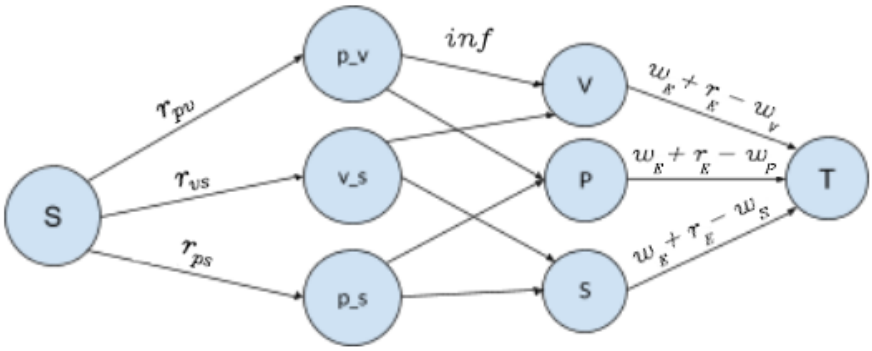

Figure 1: Competition network flow (Names abbreviated to first initial)

As in question 1, we identified the criterion for elimination by using the distance between a particular player's total potential wins and the current leader's wins, so for considering whether Emily (the leader) is eliminated, we can structure the capacities of edges connecting players with the sink vertex of Emily's

network flow using the difference between $w_{Emily} + r_{Emily}$ and w_{name}. With this constraint, we know Emily is not eliminated if the max source to sink flow saturates all the paths leaving the source. This is because if an arc leaving the source is not saturated, it is bottlenecked by an arc between a player to the sink, telling us a player's remaining matches is more than enough to close the gap with Emily.

In the network flow above, let:

1. $r_{pv} = 2$, representing the number of matches remaining between Prava and Vicky.
2. $r_{vs} = 0$, representing the number of matches remaining between Vicky and Shashank.
3. $r_{ps} = 0$, representing the number of matches remaining between Prava and Shashank.
4. $w_{Emily} + r_{Emily} - w_{Vicky} = 14$, representing the difference between Emily's total potential wins and Vicky's current wins.
5. $w_{Emily} + r_{Emily} - w_{Prava} = 11$, representing the difference between Emily's total potential wins and Prava's current wins.
6. $w_{Emily} + r_{Emily} - w_{Shashank} = 13$, representing the difference between Emily's total potential wins and Shashank's current wins.
7. $inf = \infty$, There is an infinite capacity between player pairs and players as we should allow any number of remaining matches to award wins to players, and a constraint here would not be useful.

3) The linear program for the elimination problem for Emily, using variables for edges connecting vertex A to vertex B in Figure 1 defined as (vertex A)2(vertexB), would have the following constraints:

1. $S2pv = pv2p + pv2v$
2. $S2vs = vs2v + vs2s$
3. $S2ps = ps2p + ps2s$
4. $vs2v + pv2v = v2t$
5. $ps2p + pv2p = p2t$
6. $vs2s + ps2s = s2t$
7. $0 < S2pv <= r_{pv}$, $0 < S2vs <= r_{vs}$, $0 < S2ps <= r_{ps}$
8. $0 < v2t <= w_{Emily} + r_{Emily} - w_{Vicky}$
9. $0 < p2t <= w_{Emily} + r_{Emily} - w_{Prava}$
10. $0 < s2t <= w_{Emily} + r_{Emily} - w_{Shashank}$

It is illogical to expect negative matches to remain between players, so all arcs from the source must be positive (Eqs. 7 defines this). It is unnecessary to constrain edges between match pairs and players, as any number of remaining matches should translate into winning points for players. However, as each match can only have one winner, the number of wins any given player can have should add up to their remaining matches against all other players (Eqs. 1-6 define this). Furthermore, as we are trying to use the network to determine whether a player's remaining matches are enough to remain in the running or insufficient to take the lead, we constrain arcs leaving the source by the number of remaining matches for each player pair, and constrain arcs into the sink by the difference between Emily's total potential wins and a specific player's current wins (Eqs. 7-10 define this).

Part 2 - Implementation

See submitted Github URL, or go to https://github.com/sidgarimella/Lab0.