Learning-Based Compressive MRI [1]

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October 15, 2020

[1] Gözcü, B., Mahabadi, R.K., Li, Y.H., Ilıcak, E., Çukur, T., Scarlett, J. and Cevher, V., 2018. Learning-based compressive MRI. IEEE transactions on medical imaging, 37(6), pp.1394-1406.

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Introduction

- A learning-based framework for optimizing MRI subsampling patterns for a specific reconstruction rule and anatomy
- Algorithm searches for a sampling pattern that performs well on average for the signals in the training set
- A novel parameter-free greedy mask selection method
- Can find good sampling patterns for various performance metrics such as PSNR, SSIM, etc.
- Works for broad range of decoders, e.g. basis pursuit[2], total variation, neural network, BM3D[3]

[2] D. L. Donoho, "Compressed sensing," IEEE Trans. Inf. Theory, Apr. 2006.

[3] E. M. Eksioglu, "Decoupled algorithm for MRI reconstruction using nonlocal block matching model: BM3D-MRI," J. Math. Imag. Vision 2016.

Background

Compressive Sensing Problem

$$\mathbf{b} = \mathbf{P}_\Omega \boldsymbol{\Psi} \mathbf{x} + \mathbf{w}$$

where

- $\mathbf{b}$ is the sparse measurement
- $\mathbf{P}_\Omega : \mathbb{C}^p \to \mathbb{C}^n$ is the subsampling operator for sampling pattern/mask Ω
- $\boldsymbol{\Psi} \in \mathbb{C}^{p\times p}$ is the Fourier Transform operator
- $\mathbf{w} \in \mathbb{C}^n$ is the additive noise

Uses a reconstruction algorithm g to estimate $\hat{\mathbf{x}}$ from measurements $\mathbf{b}$

$$\hat{\mathbf{x}} = g(\Omega, \mathbf{b})$$

Learning Based Framework

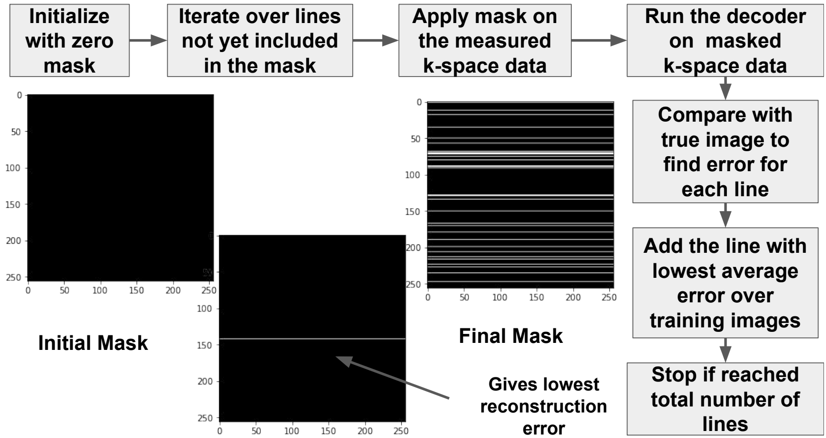
- Access to fully sampled signals $\mathbf{x}_1, \cdots, \mathbf{x}_m$
- Assuming that decoder g is given (can be arbitrary).
- For a subsampling pattern Ω, consider its empirical average performance on the training signals:

$$\frac{1}{m}\sum_{j=1}^{m}\eta_\Omega(\mathbf{x}_j) = \frac{1}{m}\sum_{j=1}^{m}\eta(\mathbf{x}_j, \hat{\mathbf{x}}_j)$$

where η is the performance measure (e.g. PSNR, SSIM).

- Use greedy algorithm to seek an appropriate maximizer by solving $\mathbf{b} = \mathbf{P}_\Omega \boldsymbol{\Psi} \mathbf{x}$

Greedy Algorithm: Flow Chart



Greedy Algorithm: Pseudo Code

Input: Training data $x_1, \cdots, x_m$, reconstruction rule g, sampling set $\mathcal{S}$, cost function c, maximum cost Γ

Output: Sampling pattern Ω

1. $\Omega \leftarrow \emptyset$
2. **while** $c(\Omega) \leq \Gamma$ **do**
3. **for** $S \in \mathcal{S}$ such that $c(\Omega \cup S) \leq \Gamma$ **do**
4. $\Omega' = \Omega \cup S$
5. For each j, set $\mathbf{b}_j \leftarrow \mathbf{P}_{\Omega'} \boldsymbol{\Psi} \mathbf{x}_j$, $\hat{\mathbf{x}}_j \rightarrow g(\Omega', \mathbf{b}_j)$
6. $\eta(\Omega') \leftarrow \frac{1}{m} \sum_{j=1}^{m} \eta(\mathbf{x}_j, \hat{\mathbf{x}}_j)$
7. $\Omega \leftarrow \Omega \cup S^*$, where

$$S^* = \underset{S : c(\Omega \cup S) \leq \Gamma}{\arg\max} \frac{\eta(\Omega \cup S) - \eta(\Omega)}{c(\Omega \cup S) - c(\Omega)}$$

8. **return** Ω

Noisy Setting

Acquired noisy signal given by

$$\mathbf{b} = \mathbf{P}_\Omega \boldsymbol{\Psi} \mathbf{x} + \mathbf{w}$$

Learning algorithm have access to only noisy training signals

$$\mathbf{z}_j = \mathbf{x}_j + \mathbf{v}_j, \;\; j = 1, \cdots, m$$

Can get reduced noise using a denoiser $\xi(\mathbf{z}) = \mathbf{x}_j + \tilde{\mathbf{v}}_j$

Selection rule now becomes

$$\hat{\Omega} = \arg\max_{\Omega \in \mathcal{A}} \frac{1}{m} \sum_{j=1}^{m} \eta(\mathbf{x}_j + \tilde{\mathbf{v}}_j, \hat{\mathbf{x}}(\mathbf{P}_\Omega \boldsymbol{\Psi}(\mathbf{x}_j + \mathbf{v}_j))$$

with $\hat{\mathbf{x}} = g(\Omega, \mathbf{b})$ from the reconstruction algorithm.

Learning-based approach outperforms the baselines

TABLE I
PSNR AND SSIM PERFORMANCES AVERAGED ON 60 TEST SLICES AT 25% SUBSAMPLING RATE. THE ENTRIES WHERE THE LEARNING IS MATCHED TO THE DECODER AND PERFORMANCE MEASURE ARE SHOWN IN BOLD.

Mask \ Decoder	TV	BP	BM3D	NN
Coherence-based	30.76	31.48	30.04	32.02
Single-image	32.79	33.32	32.42	33.67
TV-greedy	**34.84**	36.08	35.95	36.04
BP-greedy	34.76	**36.16**	36.11	36.17
BM3D-greedy	34.77	36.04	**36.19**	35.92
NN-greedy	34.81	36.05	36.16	**36.36**
Low Pass	31.96	32.41	32.59	32.59

Mask \ Decoder	TV	BP	BM3D	NN
Coherence-based	0.832	0.85	0.822	0.798
Single-image	0.876	0.889	0.879	0.854
TV-greedy	0.907	0.922	0.921	0.869
BP-greedy	0.906	0.923	0.921	0.859
BM3D-greedy	0.906	0.922	0.922	0.909
NN-greedy	0.907	0.923	0.923	0.925
Low Pass	0.876	0.888	0.893	0.893

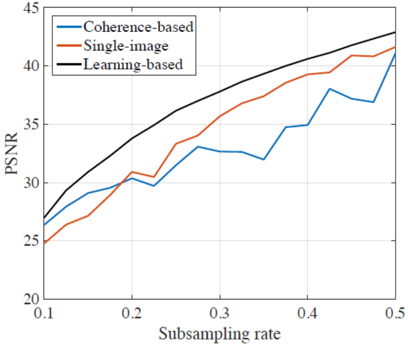
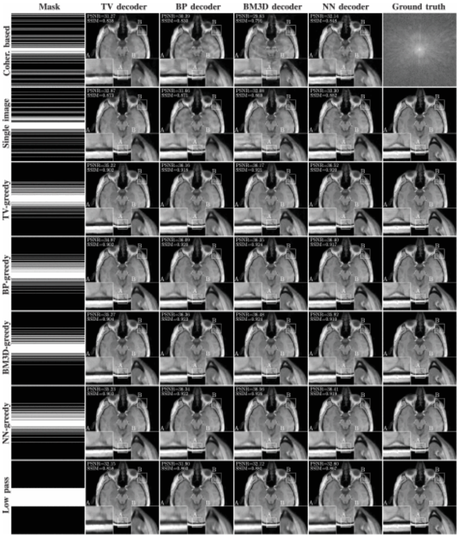


Fig. 1. PSNR as a function of subsampling rates with BP reconstruction.

Reconstruction Examples



Observation

- Greedy masks outperform the the low pass mask as well, in terms of PSNR, SSIM and also visual quality,
- Offer sharper images with less aliasing artefacts by balancing between low and high frequency components.
- Pure low-pass mask introduces strong blurring
- Baseline masks (coherence-based[4] and single image[5]) cause highly visible aliasing due to suboptimal sampling across low to intermediate frequencies.

[4] M. Lustig, D. Donoho, and J. M. Pauly, "Sparse MRI: The application of compressed sensing for rapid MR imaging," Magn. Reson. Med., 2007.

[5] J. Vellagoundar and R. R. Machireddy, "A robust adaptive sampling method for faster acquisition of MR images," Magnetic resonance imaging, 2015.

Testing on OCMR dataset [6]

Parameter	Value
Data	Cardiac
Image Size	512×208
No. of coils	18
No. of lines in Mask	32
Performance Metric	Mean Squared Error
Decoder	Inverse FFT + Sum of Squares

Table: Simulation Parameters

[6] Chen, Chong, et al. "OCMR (v1. 0)–Open-Access Multi-Coil k-Space Dataset for Cardiovascular Magnetic Resonance Imaging." arXiv e-prints (2020): arXiv-2008.

Performance Metric: L2-Norm of the Error

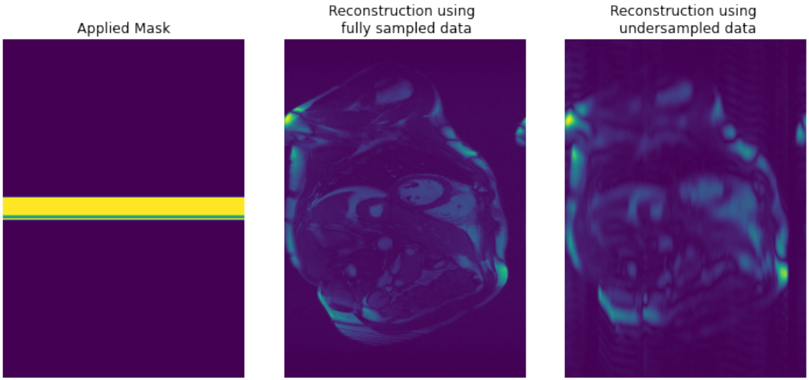


Figure: Reconstruction using using greedy algorithm with mask of 32 lines tested on fully sampled OCMR data

Performance Metric: L1-Norm of the Error

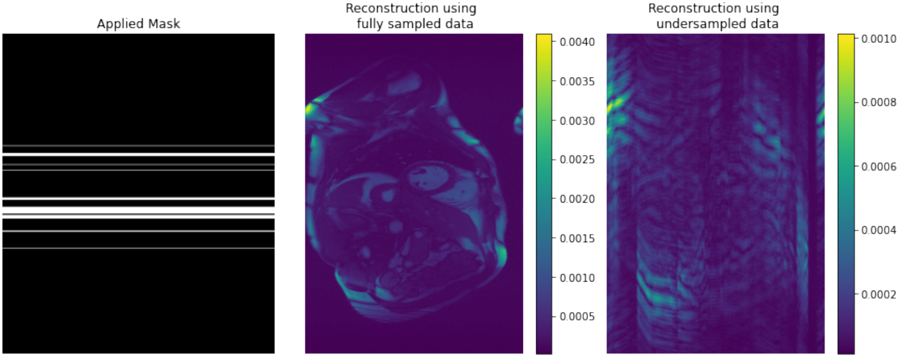


Figure: Reconstruction using using greedy algorithm with mask of 32 lines tested on fully sampled OCMR data

Conclusion

- A versatile learning-based framework for selecting masks for compressive MRI.
- Provides improved performance on realworld data sets for a variety of reconstruction methods (statistical learning theory).
- Framework suited to general decoders, hence can be used for new reconstruction methods that are yet to be discovered.
- Focused on 1D subsampling for 2D MRI, 2D subsampling (via horizontal and vertical lines) for 2D MRI, and 1D subsampling for 3D MRI,
- Challenge with 3D MRI, since the candidate set is large.