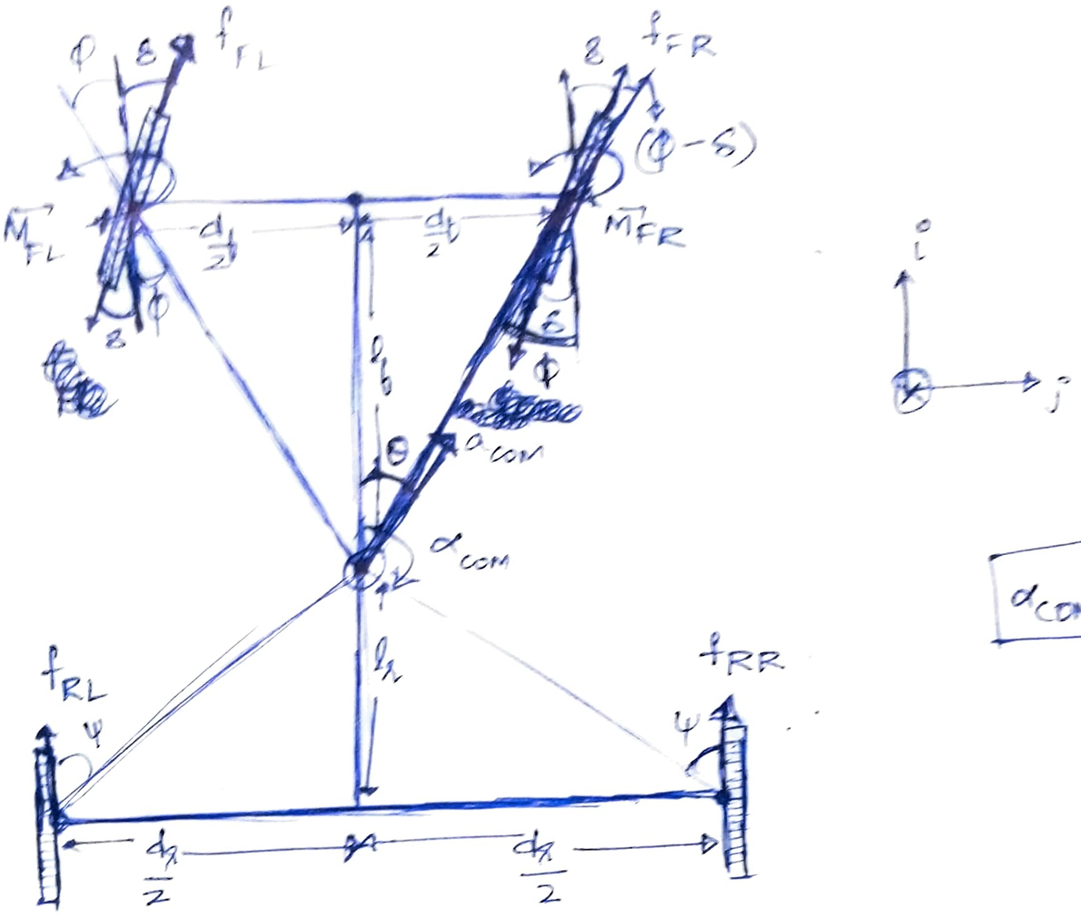


$$\boxed{\alpha_{com} = \ddot{\theta}}$$

Torques :-

$$\Rightarrow \vec{M} = \underbrace{(\vec{M}_{FL} + \vec{M}_{FR})}_{\text{correcting inputs}} + \underbrace{\left[f_{FL}\sin(\phi+\delta)\frac{d_f}{2\sin\phi} - f_{FR}\sin(\phi-\delta)\cdot\frac{d_f}{2\sin\phi}\right]\hat{k}}_{\text{Front wheel}}$$

$$+ \underbrace{\left[f_{RL}\cancel{\sin\psi}\left(\frac{d_r}{2}\right) - f_{RR}\left(\frac{d_r}{2}\right)\right]\hat{k}}_{\text{Rear wheel}}$$

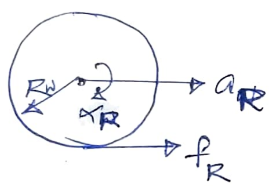
$$\Rightarrow \boxed{\vec{M} = I_c \alpha_{com} = I_c \ddot{\theta} = (\vec{M}_{correcting}) + \frac{[f_{FL}\sin(\phi+\delta) - f_{FR}\sin(\phi-\delta)]\, d_b}{2\sin\phi}\hat{k} + \frac{(f_{RL} - f_{RR})\, d_R}{2}\hat{k}}$$

Forces :-

$$\Rightarrow \vec{F} = (f_{FL}\cos\delta + f_{FR}\cos\delta)\hat{i} + (f_{FL} + f_{FR})\sin\delta\,\hat{j} + (f_{RL} + f_{RR})\hat{i}$$

$$\Rightarrow \boxed{\vec{F} = (f_F\cos\delta + f_R)\hat{i} + (f_F\sin\delta)\hat{j}}$$

Rear wheel :-



$\Rightarrow \boxed{R_W \alpha_R = a_R} \rightarrow$ constraint for no slipping.

$M_e \rightarrow$ Effective Mass.

$\Rightarrow \boxed{u_R - f_R R_W = I_W \cdot \alpha_R} \rightarrow$ Torque

$\Rightarrow \boxed{f_R = M_e a_R} \rightarrow$ Force.

$$\Rightarrow \boxed{\alpha_R = \frac{u_R}{I_W + M_e R_W^2}}$$

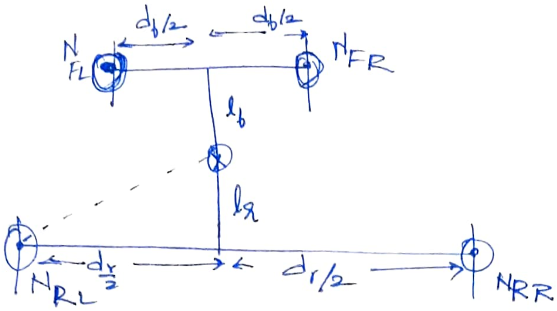
WRT COM :-

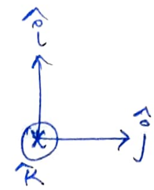
$$\Rightarrow \boxed{\begin{aligned} a_{RL} &= a_{COM}\cos\theta + \frac{\ddot{\theta} d_R}{2} \\ a_{RR} &= a_{COM}\cos\theta - \frac{\ddot{\theta} d_R}{2} \end{aligned}}$$

$$\Rightarrow 2a_{COM}\cos\theta = 2\ddot{x} = a_{RL} + a_{RR} = \frac{(u_{RL} + u_{RR}) R_W}{I_W + M_e R_W^2}$$

$$\Rightarrow \boxed{\ddot{x} = \frac{(u_{RL} + u_{RR}) R_W}{2[I_W + M_e R_W^2]}} \qquad \Rightarrow \boxed{\ddot{\theta} = \frac{(u_{RL} - u_{RR}) R_W}{(I_W + M_e R_W^2) d_R}}$$

Normal Forces :- [Used to understand distribution of frictional forces].



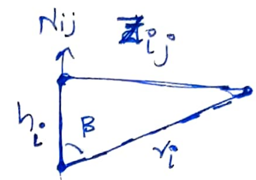


$$\vec{Z}_{FL} = l_f\hat{i} - \frac{d_f}{2}\hat{j}$$

$$\vec{Z}_{FR} = l_f\hat{i} + \frac{d_f}{2}\hat{j}$$

$$\vec{Z}_{RL} = (-l_r)\hat{i} + \left(-\frac{d_r}{2}\right)\hat{j}$$

$$\vec{Z}_{RR} = (-l_r)\hat{i} + \left(\frac{d_r}{2}\right)\hat{j}$$



$N_{ij} \sin\beta\, r_i$

Z_{ij}

$$\Rightarrow \hat{N}_{ij} = -\hat{k} \qquad \text{let } \vec{\tilde{N}}_{ij} = -\vec{N}_{ij}$$

$$\Rightarrow \vec{\tau}_N = -\sum \vec{N}_{ij} \times \vec{Z}_{ij} = \sum \vec{\tilde{N}}_{ij} \times \vec{Z}_{ij}$$

$$\Rightarrow \vec{\tau}_N = N_{FL} l_f \hat{j} + N_{FL}\frac{d_f}{2}\hat{i} + N_{FR} l_f \hat{j} + \left(-N_{FR}\frac{d_f}{2}\right)\hat{i} - N_{RL} l_r \hat{j} + N_{RL}\frac{d_r}{2}\hat{i} + (-N_{RR} l_r)\hat{j} + \left(-N_{RR}\frac{d_r}{2}\right)\hat{i}$$

$$\Rightarrow \vec{\tau}_N = \left[N_{FL} l_f + N_{FR} l_f - (N_{RL} l_r + N_{RR} l_r)\right]\hat{j} + \left[N_{FL}\frac{d_f}{2} - N_{FR}\frac{d_f}{2} + N_{RL}\frac{d_r}{2} - N_{RR}\frac{d_r}{2}\right]\hat{i}$$

$$\Rightarrow \vec{\tau}_N = \left[N_F l_f - N_R l_r\right]\hat{j} + \left[\Delta N_F \frac{d_f}{2} - \Delta N_R \frac{d_r}{2}\right]\hat{i}$$

Since ~~there's~~ no roll, pitch $\vec{\tau}_N = \vec{0}$

$$\Rightarrow \boxed{N_F l_f = N_R l_r}$$

$$\Rightarrow \boxed{\Delta N_F d_f = \Delta N_R d_r}$$

$\rightarrow \quad \sum \vec{N}_{ij} = Mg.$

$\Rightarrow N_F + N_R = Mg.$

$N_F \left[\frac{l_b + l_f}{l_b} \right] = Mg$

$$\boxed{\begin{aligned} N_F &= \frac{Mg\, l_b}{l_f + l_b} = \frac{Mg\, l_b}{l}. \\ N_R &= \frac{Mg\, l_b}{l}. \end{aligned}}$$

$\rightarrow$

$$\boxed{\begin{aligned} M_f &= \frac{M l_b}{l}. \\ M_b &= \frac{M l_b}{l}. \\ M_e &= \frac{M_b}{2}. \end{aligned}}$$

$\rightarrow$ Since it's a straight line motion

$\Delta N_F \propto \Delta f_F$ and $\Delta N_F \simeq 0$

$\Rightarrow \Delta N_R \simeq 0$

$$\Rightarrow I_c \ddot{\theta} = \vec{M}_c + \left[\cancel{\Delta f_{F_L}}^{0} \sin\varphi \cos\delta + f_F \cos\varphi \sin\delta\right] \frac{d_b}{2\sin\varphi} + \frac{\Delta f_R d_r}{2}$$

$$\Delta f_R = M_e (a_{RL} - a_{RR}) = M_c (\ddot{\theta} d_r) \quad \text{—(i)}$$

$$f_R = M_c (a_{RL} + a_{RR}) = 2M_e \ddot{x} \quad \text{—(ii)}$$

$$f_F = \frac{M\left(1 - \frac{2M_e}{M}\right)\ddot{x}}{\cos\delta} \quad \text{—(iii)}$$

$$\Rightarrow \quad I_c\ddot{\theta} = \frac{2u_s l_w}{l_s} + M\left(1-\frac{2M_e}{M}\right)\ddot{x}\,\frac{d_b}{2\tan\varphi}\,\frac{\sin\delta}{\cos\delta} + M_e\ddot{\theta}\,\frac{d_r^2}{2}$$

$$\Rightarrow \quad \boxed{\frac{\left[I_c - \frac{M_e d_r^2}{2}\right]\ddot{\theta} - \frac{2u_s l_w}{l_s}}{M\left(1-\frac{2M_e}{M}\right)\frac{d_b}{2\tan\varphi}\,\ddot{x}} = \tan\delta}$$

$$\Rightarrow \quad \frac{\ddot{y}}{\ddot{x}} = \tan\theta = \frac{f_F\sin\delta}{\ddot{x}} = \frac{M\left(1-\frac{2M_e}{M}\right)\ddot{x}}{\cos\delta}\sin\delta.$$

$$\Rightarrow \quad \tan\theta = M\left(1-\frac{2M_e}{M}\right)\ddot{x}\,\tan\delta$$

$$\Rightarrow \quad \tan\theta = \frac{2\left\{\left[I_c - \frac{M_e d_r^2}{2}\right]\ddot{\theta} - \left(\frac{2l_w}{l_s}\right)u_s\right\}\tan\varphi}{d_b}$$

Ideally $\theta \to 0$

$$\Rightarrow \quad \left[I_c - \frac{M_e d_r^2}{2}\right]\frac{\Delta u_R R_w}{I_0 d_r} \simeq \left(\frac{2l_w}{l_s}\right)u_s.$$

$$\Rightarrow \quad \frac{\overbrace{\left[\frac{I_c}{2} - \frac{M_e d_r^2}{4}\right]}^{I_e}}{I_0} \times \frac{R_w}{d_r} \times \frac{l_s}{l_w} \times \Delta u_R \simeq u_s.$$

$$\Rightarrow \quad \boxed{\left[\frac{I_e R_w l_s}{I_0 d_r l_w}\right]\Delta u_R \simeq u_s}$$

//

Control

$\Rightarrow e = \theta_{desired} - \theta_{observed}.$

$\Rightarrow$ Exponentially decreasing error

$$\Rightarrow \ddot{e} + K_d \dot{e} + K_p e + K_I \int_0^t e(\tau)\,d\tau = 0.$$

$$\Rightarrow (\ddot{\theta}_d - \ddot{\theta}_o) + K_d(\dot{\theta}_d - \dot{\theta}_o) + K_p(\theta_d - \theta_o) + K_I \int_0^t (\theta_d - \theta_o)(\tau)\,d\tau = 0$$

$$\underbrace{\theta_d = \dot{\theta}_d}_{\text{set-points}} = \ddot{\theta}_d = 0 \rightarrow \text{No angular motion.}$$

$$\boxed{\ddot{\theta}_o = \frac{\Delta u_R R_w}{I_o d_r} \rightarrow \text{Our Model}}$$

$$\boxed{\begin{bmatrix} \theta_o \\ \dot{\theta}_o \end{bmatrix} \rightarrow \text{From Sensors.}}$$

$$\Rightarrow \boxed{\frac{\Delta u_R R_w}{I_o d_r} = -\left[K_p \theta_o + K_d \dot{\theta} + K_I \int_0^t \theta(\tau)\cdot d\tau\right]}$$