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Experiment 11

Objective:

Demonstrate Markov Chain Process. WAP to find Fundamental Matrix from Absorbing Markov chain. Demonstrate application of Fundamental Matrix by considering a suitable example.

Theory:

An absorbing Markov chain is a Markov chain in which every state can reach an absorbing state. An absorbing state is a state that, once entered, cannot be left.

Like general Markov chains, there can be continuous-time absorbing Markov chains with an infinite state space. However, this article concentrates on the discrete-time discrete-state-space case.

A basic property about an absorbing Markov chain is the expected number of visits to a transient state *j* starting from a transient state *i* (before being absorbed). The probability of transitioning from *i* to *j* in exactly *k* steps is the (i,j)-entry of Q^k. Summing this for all *k* (from 0 to ∞) yields the fundamental matrix, denoted by *N*. It can be proven that

$$N = \sum_{k=0}^{\infty} Q^k = (I_t - Q)^{-1},$$

where I_t is the *t*-by-*t* identity matrix. The (i, j) entry of matrix *N* is the expected number of times the chain is in state *j*, given that the chain started in state *i*. With the matrix *N* in hand, other properties of the Markov chain are easy to obtain.

Code:

```
P = [0.2 0.0 0.0 0.8 0.0 ;
     0.1 0.1 0.6 0.2 0.0 ;
     0.0 0.7 0.0 0.3 0.0 ;
     0.2 0.0 0.2 0.1 0.5 ;
     0.0 1.0 0.0 0.0 0.0];
Q = [0.1 0.6 0.0; 0.4 0.0 0.5; 0.0 0.2 0.0];
R = [0.1 0.0; 0.0 0.0; 0.2 0.6];
I = [1 0 0; 0 1 0; 0 0 1];
fprintf('Fundamental Matrix: \n\n');
N = (I - Q)^-1;
disp(N);

U=N*R;
fprintf('\n\nAbsorbed Probabilities: \n\n');
disp(U);
```

Output:

```
Fundamental Matrix:

   1.5789   1.0526   0.5263
   0.7018   1.5789   0.7895
   0.1404   0.3158   1.1579


Absorbed Probabilities:

   0.2632   0.3158
   0.2281   0.4737
   0.2456   0.6947
```

Result: fundamental matrix is obtained for absorbing Markov chain.

Discussion: A state i is an absorbing state if once the system reaches state i, it stays in that state, that is $P_{ii} = 1$. The sum of the entries of a row of the fundamental matrix gives us the expected number of steps before absorption for the non-absorbing state associated with that row.