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Experiment 7

Objective:

Demonstrate Simple Random Walk. WAP to find the probability that in case of an unrestricted simple random walk the particle is at the kth position at time n using

(a) Central Limit Theorem (b) Generating Function

Theory:

Suppose that the random walks start at origin and the particle is free to move indefinitely in either direction.

- By central limit theorem, we can say that χ n will be normally distributed. Therefore,
  prob(χ_n = k) = $\Phi((k + 1/2 + c - n\mu)/(\sigma\sqrt{n}))$ - $\Phi((k - 1/2 - c - n\mu)/(\sigma\sqrt{n}))$
  where ϕ(y) is the standard normal distribution function.

- The probability generation function of χ n is given by,
  G(z, s) = $(z/(-spz^2 + z\{1 - s(1 - p - q)\} - sp))$
  where prob(χ n = k) = coeff. of z k s n in G(z, s)

For this experiment consider:
A random walk in which the particle starts at origin. Finding the probability that after 10 steps, the particle will be at position 4. Given that probability of positive jump is 0.42 and that of negative jump is 0.40.

So,

p = 0.42, q = 0.40
j = 3.5, k = 4.5

Code:

Central Limit Theorem:

```
import math from scipy.integrate
import quad as integrate
import sympy as sp
x = sp.Symbol('x')
def expression(x):
    return(math.exp(-((x-(n*mu))**2)/(2*(var)*n)))
def findNorm(j, k):
    i = integrate(expression, j, k)
    norm = (1/math.sqrt(2*math.pi*n*(var)))*i[0]
    print("The probability of finding x at k: "+str(norm))
    mu = 0.02
    var = 0.8196
    n = 10
    findNorm(3.5, 4.5)
```

Output: The probability of finding x at k: 0.1285

Generating Function:

```
from sympy import symbols, diff, N
import math
z, s, p, q = symbols('z s p q', real=True)
f = (z) / ((-s * (z ** 2) * p) + (1 - (s * (1 - p - q))) * z - q * s)
n = 10
k = 4
```

```
f = f.subs(p, 0.49)
f = f.subs(q, 0.44)
for _ in range(n):
        f = diff(f, s)
for _ in range(k):
        f = diff(f, z)

f = f.coeff(s * z)
f = f / (math.factorial(10) * math.factorial(4))
f = f.subs(z, 1)
f = f.subs(s, 1)
print("The Probability of Particle being at K=4 after n steps is: ",
float(f))
```

Output: The Probability of Particle being at K=4 after n steps is: 0.1285

Result: used central limit theorem and generating function to calculate the probabilities.

Discussion: we use continuity correction c for better approximation. The value of c = 1 if p + q = 1, and the value of c = ½ if p + q < 1.