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Objective:

Demonstrating Poisson Process. WAP to find the probability that in case of Poisson process with rate λ, in a length of time t there are exactly k arrivals.

For example: Consider a transistor battery having exponential lifetime with mean as 2 months. In case six such spares batteries are available and if the time to replace a battery is negligible, find the probability that transistor will work for at least one year. In case mean failure time of the successive spare batteries are given by 2/n, then find this probability.

Theory:

The basic form of Poisson process, often referred to simply as "the Poisson process", is a continuous time counting process $\{N(t), t \geq 0\}$ that possesses the following properties:

- Independent increments (the numbers of occurrences counted in disjoint intervals are independent of each other).
- Stationary increments (the probability distribution of the number of occurrences counted in any time interval only depends on the length of the interval).
- The probability distribution of N(t) is a Poisson distribution.
- No counted occurrences are simultaneous.

Consequences of this definition include:

- The probability distribution of the waiting time until the next occurrence is an exponential distribution.
- The occurrences are distributed uniformly on any interval of time. (Note that N(t), the total number of occurrences, has a Poisson distribution over (0, t], whereas the location of an individual occurrence on $t \in (a, b]$ is uniform).

Code and Output:

```
import math
def poisson_homo(parameter, n):
    p=0;
```

```
    for i in range(n+1):
        temp = math.exp(-parameter) * math.pow(parameter, i)
        temp /= math.factorial(i)
        p += temp
    return p

ans = poisson_homo(12, 12)
print(ans)
```

```
0.5759652485730646
```

```
import math
def poisson_non_homo(parameter, n):
    p=0;
    for i in range(1, n+1):
        temp = math.exp(-parameter/i) * math.pow(parameter/i, i)
        temp /= math.factorial(i)
        p += temp
    return p

ans = poisson_non_homo(12, 12)
print(ans)
```

```
0.48202206526406666
```

Result:

With the help of the above program, we have successfully managed to solve a Poisson process, both for homogenous as well as non homogeneous poisson process.

Discussion:

As observed, non homogenous process has uncertainty as can be seen with the help of the low probability.