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- *2K18/ MC/ 114*

Objective:

Demonstrate the unrestricted random walk and analyse the probability of being at a given state after n steps.

Theory:

A random walk is a mathematical object, known as a stochastic or random process, that describes a path that consists of a succession of random steps on some mathematical space such as the integers. An elementary example of a random walk is the random walk on the integer number line, Z , which starts at 0 and at each step moves +1 or −1 with equal probability.

A simple random (or unrestricted random walk) walk on a line or in one dimension occurs with probability p when walker step forward (+1) and/or has probability q=1−p if walker steps back (−1). For ith step, the modified Bernoulli random variable Wi (takes the value +1 or −1 instead of {0,1}) is observed and the position of the walk at the nth step can be found by,

$$\begin{aligned} X_n &= X_0 + W_1 + W_2 + \cdots + W_n \\ &= X_0 + \sum_{i=1}^{n} W_i \\ &= X_{n-1} + W_n \end{aligned}$$

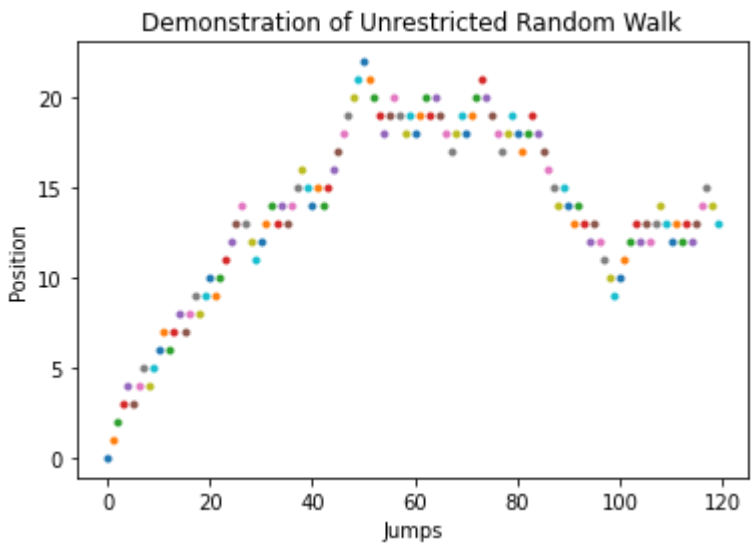
Code and Output:

```
import numpy as np
import matplotlib .pyplot as plt
```

```
p = 0.55
q = 0.45
tsteps = 120
initial = 0
step = 0


while(step<tsteps):
    plt.plot(step,initial,'.')
    if(np.random.random() < p):
        initial += 1
    else:
        initial -= 1
    step += 1

plt.ylabel('Position')
plt.xlabel('Jumps')
plt.title('Demonstration of Unrestricted Random Walk')
plt.show()
```




```
from scipy.stats import norm

p = 0.55
q = 0.45
state = 20
steps = 120
if(p+q<1):
    r = 1-p-q
    c = 0.5
else:
    c = 1
```

```
mean = p-q
sd = np.sqrt(p+q - (p-q)**2)
norm1 = norm.cdf((state+c- steps*mean)/(sd*np.sqrt(steps)))
norm2 = norm.cdf((state-c-steps*mean)/(sd*np.sqrt(steps)))
prob = norm1 - norm2

print('Probabily of being at state ',state,' after ',steps,' steps is: ',prob)
```

```
Probabily of being at state  20  after  120  steps is:  0.05588162653138029
```

Results:

We Demostrated 1D random walk assuming p = 0.55 & q = 0.45 We also calculated probabilty of being at a state after n steps using normal cdf.

Discussion:

Random Walk can give us a good understanding of the statistical processes involved in genetic drift, and they describe an ideal chain in polymer physics. They are also important in finance.