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Objective:

Demonstrate M/M/1 Queuing Model using a suitable example.

▼ Theory:

In queueing theory, a discipline within the mathematical theory of probability, an M/M/1 queue represents the queue length in a system having a single server, where arrivals are determined by a Poisson process and job service times have an exponential distribution. An M/M/1 queue is a stochastic process whose state space is the set $\{0,1,2,3,\dots\}$ where the value corresponds to the number of customers in the system, including any currently in service.

- Arrivals occur at rate λ according to a Poisson process and move the process from state i to $i + 1$.
- Service times have an exponential distribution with rate parameter μ in the M/M/1 queue, where $1/\mu$ is the mean service time.
- A single server serves customers one at a time from the front of the queue, according to a first-come, first-served discipline.
- The buffer is of infinite size, so there is no limit on the number of customers it can contain.

Consider an Airport runway for arrivals only, arriving aircraft join a single queue for the runway,

Exponentially distributed service time with a rate, $\mu = 27$ arrivals/hour

Poisson arrivals with a rate, $\lambda = 20$ arrivals/hour

▼ Code & Output:

```
lamb = 20 # Poisson arrival rate
mu = 27 # Service rate
```

```
ls = lamb/(mu - lamb)
lq = lamb**2/(mu*(mu-lamb))
ws = ls/lamb
wq = lq/lamb
```

```
print('The Expected number of aircraft on the system:', ls)
print('The Expected number of aircraft on the runway:', lq)
print('The Average time spent on the system:', ws*60, 'minutes')
print('The Average time spent on the runway:', wq*60, 'minutes')
```

```
The Expected number of aircraft on the system: 2.857142857142857
The Expected number of aircraft on the runway: 2.1164021164021163
The Average time spent on the system: 8.571428571428571 minutes
The Average time spent on the runway: 6.349206349206349 minutes
```

Result:

Demonstrated MM1 Queuing Model using a airport runway for arrivals only.

Discussion:

The M/M/1 system is made of a Poisson arrival, one exponential (Poisson) server, queue of unlimited capacity and unlimited customer population. These assumptions are very strong, not satisfied for practical systems. Nevertheless the M/M/1 model shows clearly the basic ideas and methods of Queuing Theory.