Task 1 (20 points, written)

You are a meteorologist that places temperature sensors all of the world, and you set them up so that they automatically e-mail you, each day, the high temperature for that day. Unfortunately, you have forgotten whether you placed a certain sensor S in Maine or in the Sahara desert (but you are sure you placed it in one of those two places) . The probability that you placed sensor S in Maine is 5%. The probability of getting a daily high temperature of 80 degrees or more is 20% in Maine and 90% in Sahara. Assume that probability of a daily high for any day is conditionally independent of the daily high for the previous day, given the location of the sensor.

Part a: If the first e-mail you got from sensor S indicates a daily high under 80 degrees, what is the probability that the sensor is placed in Maine?

Part b: If the first e-mail you got from sensor S indicates a daily high under 80 degrees, what is the probability that the second e-mail also indicates a daily high under 80 degrees?

Part c: What is the probability that the first three e-mails all indicate daily highs under 80 degrees?

Task 2 (5 points, written)

Function P is a function defined on a set of samples S = {A, B, C, D}. We do not know the value of P for all samples, but we know that P(A) = 0.3 and P(B) = 0.6. What can you say about whether P is a valid probability function? Is P definitely a probability function, possibly a probability function, or definitely not a probability function? Justify your answer.

Task 3 (5 points, written)

Function P is a function defined on the set of real numbers. We do not know the value of P for all cases, but we know that P(x) = 0.3 when 0 <= x <= 10. What can you say about whether P is a valid probability density function? Is P definitely a probability density function, possibly a probability density function, or definitely not a probability density function? Justify your answer.

Task 4 (70 points, programming)

In this task you will implement naive Bayes classifiers based on Gaussians.

Arguments

You must implement a Matlab function or a Python executable file called `naive_bayes` that learns a naive Bayes classifier for a classification problem, given some training data and some additional options. In particular, your function can be invoked as follows:

```
naive_bayes(<training_file>, <test_file>)
```

If you use Python, just convert the Matlab function arguments to command-line arguments. The arguments provide to the function the following information:

- The first argument is the path name of the training file, where the training data is stored. The path name can specify any file stored on the local computer.
- The second argument is the path name of the test file, where the test data is stored. The path name can specify any file stored on the local computer.

Both the training file and the test file are text files, containing data in tabular format. Each value is a number, and values are separated by white space. The i-th row and j-th column contain the value for the j-th feature of the i-th object. The only exception is the LAST column, that stores the class label for each object. **Make sure you do not use data from the last column (i.e., the class labels) as attributes (features).**

The training and test files will follow the same format as the text files in the UCI datasets directory. A description of the datasets and the file format can be found on this link. For each dataset, a training file and a test file are provided. The name of each file indicates what dataset the file belongs to, and whether the file contains training or test data. Your code should also work with ANY OTHER training and test files using the same format as the files in the UCI datasets directory.

As the description states, **do NOT use data from the last column (i.e., the class labels) as features**. In these files, all columns except for the last one contain example inputs. The last column contains the class label.

Training:

You should model P(x | class) as a Gaussian SEPARATELY for each dimension of the data and (obviously) for each class. The output of the training phase should be a sequence of lines like this:

```
Class %d, attribute %d, mean = %.2f, std = %.2f
```

The output lines should be sorted in increasing order of class number. Within the same class, lines should be sorted in increasing order of attribute number. Attributes should be numbered starting from 1, not from 0.

In certain cases, it is possible that value computed for the standard deviation is equal to zero. Your code should make sure that the variance of the Gaussian is NEVER smaller than 0.0001. Since the variance is the square of the standard deviation, this means that the standard deviation should never be smaller than sqrt(0.0001) = 0.01. Any time the value for the standard deviation is computed to be smaller than 0.01, your code should replace that value with 0.01.

Matlab and numpy have predefined constants for π, you should use these constants in your formulas, instead of hardcoding a different value for π.

In your answers.pdf document, provide the output produced by the training stage of your program when given yeast_training.txt as the input file.

Classification

For each test object x and each class C, your program should compute P(C | x), so as to identify the class C that maximizes P(C | x). To compute P(C | x), you can use Bayes rule, which means that you need to first compute P(x | C), p(C), and P(X). Note that P denotes probability densities, and p denotes probabilities.

P(x | C) should be computed using the Naive Bayes assumption, that each dimension is independent of all other dimensions. Thus, p(x | C) is a product of as many terms as the number of dimensions of x. Each term is the output of a Gaussian, that is computed based on the parameters you learned during the training stage. For p(C), you should use the fraction of training objects that belong to class C. P(x) can be computed using the sum rule.

For each test object you should print a line containing the following info:

- object ID. This is the line number where that object occurs in the test file. Start with 1 in numbering the objects, not with 0.
- predicted class (the result of the classification). If your classification result is a tie among two or more classes, choose one of them randomly.
- probability of the predicted class given the data.
- true class (from the last column of the test file).
- accuracy. This is defined as follows:
    - If there were no ties in your classification result, and the predicted class is correct, the accuracy is 1.
    - If there were no ties in your classification result, and the predicted class is incorrect, the accuracy is 0.

    - If there were ties in your classification result, and the correct class was one of the classes that tied for best, the accuracy is 1 divided by the number of classes that tied for best.
    - If there were ties in your classification result, and the correct class was NOT one of the classes that tied for best, the accuracy is 0.

The output of the classification phase should be a sequence of lines like this:

```
ID=%5d, predicted=%3d, probability = %.4f, true=%3d, accuracy=%4.2f\n
```

Object IDs should be numbered starting from 1, not 0. Lines should appear sorted in increasing order of object ID.

After you have printed the results for all test objects, you should print the overall classification accuracy, which is defined as the average of the classification accuracies you printed out for each test object. The classification accuracy should appear in a line following this format:

```
classification accuracy=%6.4f
```

In your answers.pdf document, provide ONLY THE LAST LINE (the line printing the classification accuracy) of the output by the test stage of your program, when given yeast_training.txt as the input file.

Grading

- 30 points: Correct estimation of Gaussians, correct mean and standard deviation estimated for each Gaussian.
- 20 points: Correct application of a naive Bayes classifier based on Gaussians for classification of test data.
- 20 points: Following the specifications in producing the required output.