

Today's Agenda:

1. Search in a row-wise & column-wise sorted matrix.

Starting 7:05

2. Row with maximum ones.

3. Printing boundary of a matrix.

4. Spiral Matrix

5. Sum of all submatrices sum.

1. Given a row wise & column wise sorted matrix. Check whether element K is present or not.

	0	1	2	3	K	Output
0	-5	-2	1	13		
1	-4	0	3	14	13	true
2	-3	2	6	18	0	true.
					5	false.

Brute Force

Iterate over every cell.

Q-1.

$N \times M$

Optimisation

$$0 = 0$$

Searching $\rightarrow 0$

$$-2 < 0$$

	0	1	2	3
0	-5	-2	1	13
1	-4	0	3	14
2	-3	2	6	18

$$-5 < 0$$

$$13 > 0$$

$$1 > 0$$

true.

0 7-1

-1.

-4 < -1

-3 < -1

	0	1	2	3
0	-5	-2	1	13
1	-4	0	3	14
2	-3	2	6	18

13 > -1

1 > -1

-2 < -1

Approach

- ① Start at top right cell. or bottom left
- ② If $A[i][j] < k$, move \downarrow else
move \leftarrow (reject col)
- ③ Repeat until element is found
or $(i || j)$ goes out of
bound.

Code

int i=0, j=m-1;

T.C $\rightarrow O(N+m)$ while $(i < N \text{ \& } j \geq 0)$ {

if $(arr[i][j] == k)$
return true;

else if $(arr[i][j] < k)$ {
 $i++$;

}
else {
 $j--$;

}
}

2. Given a binary sorted matrix A of size $N \times N$. Find the row with maximum number of 1.

Note:- (1) If two rows have same no, return the one having lower index.
(2) Every row is sorted.

Ex:- 1

	0	1	2
0	0	1	1
1	0	0	1
2	0	1	1

0.

Ex: 2

	0	1	2	3
0	0	0	0	0
1	0	0	0	1
2	0	0	1	1
3	0	1	1	1

3.

Quiz 3

	0	1	2	3	
0	0	1	1	1	3
1	0	0	0	1	1
2	1	1	1	1	4
3	1	1	1	1	4

(2).

Optimisation

	0	1	2	3	4	5
0	0	0	0	0	1	1
1	0	0	0	1	1	1
2	0	0	0	0	0	1
3	0	0	0	0	1	1
4	0	1	1	1	1	1
5	0	0	0	1	1	1

or.

$i = 0; j = m-1; ans = -1;$

while ($i < N$ && $j >= 0$) {

while ($j >= 0$ && $arr[i][j] == 1$)

{ $j--;$
 $ans = i;$

}

$i++;$

return ans;

3

$i = 0; j = 5; ans = 0;$
~~1 2 3 4 5 6~~ ~~7 8 9 0~~

Algorithm

① Start at $i = 0; j = m-1;$

② If 1 is present, move left (decrement j) & update ans to current row.

③ If 0 is present, move down (increment i).

Code

```
i = 0; j = m - 1; ans = -1;
```

```
while (i < N && j >= 0) {
```

```
    while (j >= 0 && arr[i][j] != 1)
```

```
        { j--;
```

```
          ans = i;
```

```
        }
```

```
    i++;
```

```
return ans;
```

T.C $\rightarrow O(N+m)$.

S.C $\rightarrow O(1)$

3

3. // Print Given a matrix of $N \times N$.
Print boundary elements in clockwise direction.

2

	0	1	2	3
0	1	2	3	4
1	5	6	7	8
2	9	10	11	12
3	13	14	15	16

Output:- 1, 2, 3, 4, 8, 12, 16, 15, 14, 13, 9, 5

Approach.

Print $N-1$ elements in each :-

- a) First Row \rightarrow
- b) Last Column \downarrow
- c) Last Row \leftarrow
- d) First Column \uparrow

Code:-

```
void print Boundary Elements (N x N) A {
```

```
    i = 0; j = 0;
```

① $N-1$ in the first row \rightarrow

```
    for (it = 1; it < N; it++) {  
        print (a[i][j]); j++;
```

```
    }
```

② $N-1$ in the last column \downarrow

```
    for (it = 1; it < N; it++) {  
        print (a[i][j]); i++;
```

```
    }
```

③ $N-1$ in the last row \leftarrow

```
    for (it = 1; it < N; it++) {  
        print (a[i][j]); j--;
```

```
    }
```

④ $N-1$ in the first column \uparrow

```
    for (it = 1; it < N; it++) {  
        print (a[i][j]); i--;
```

```
    }
```

T.C $\rightarrow O(N)$;

S.C $\rightarrow O(1)$;

4. Given an integer A , generate a square matrix of size $A \times A$ filled with elements in spiral order from 1 to A^2 .

Ex:- $A = 4$. $A \neq 4$.
 1 to 16

Output :-

	0	1	2	3
0	1	2	3	4
1	12	13	14	5
2	11	16	15	6
3	10	9	8	7

Ex:- $A = 6$

	0	1	2	3	4	5
0	1	2	3	4	5	6
1	20	21	22	23	24	7
2	19	32	33	34	25	8
3	18	31	36	35	26	9
4	17	30	29	28	27	10
5	16	15	14	13	12	11

Q: 35.

i

j

A

0

0

6

↙ -2

1

1

4

↙

2

2

2

↙

3

3

0

Iterate 5 times in 4 directions

Iterate 3 times in 4 directions

Iterate 1 times in 4 directions

Code -

$i=0; j=0; A$; $count=0;$
 $nt(A)(A);$

while ($A > 1$) {

① $A-1$ in the first row \rightarrow

for ($it = 1; it < A; it++$) {
 $A[it](j) = ++count; j++;$

}

② $A-1$ in the last column \downarrow

for ($it = 1; it < A; it++$) {
 $A[it](j) = ++count; i++;$

}

③ $A-1$ in the last row \leftarrow

for ($it = 1; it < A; it++$) {
 $A[it](j) = ++count; j--;$

}

④ $A-1$ in the first column \uparrow

for ($it = 1; it < A; it++$) {
 $A[it](j) = ++count; i--;$

}

$i++; j++; A -= 2;$

}

if ($A == 1$) {

1

$A[i](j) = ++count;$

}

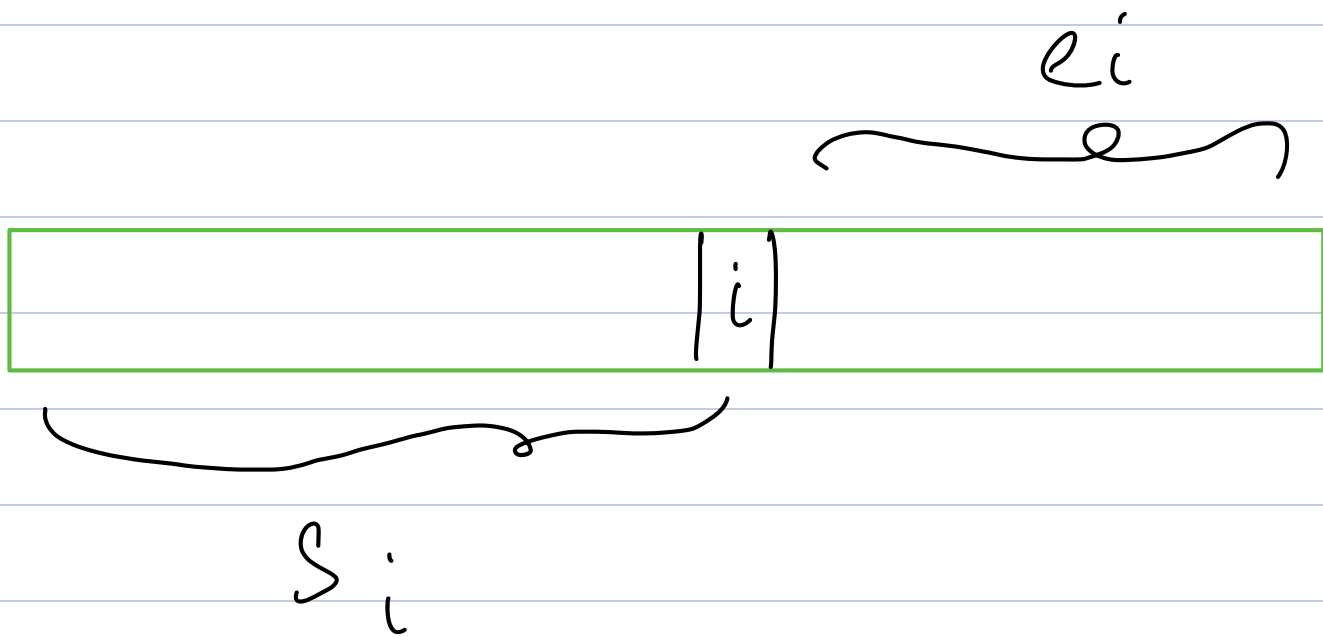
}

$$A = S \dots$$

	1	2	3	4	
0	1	2	3	4	5
1	16	17	18	19	6
2	15	24	25	20	7
3	14	23	22	21	8
4	13	12	11	10	9

i, j	A
0, 0	5
1, 1	3
2, 2	1
3, 3	-1

$$TC \rightarrow O(A^2)$$



$i+1$

↓

$\times (N-i)$

↓

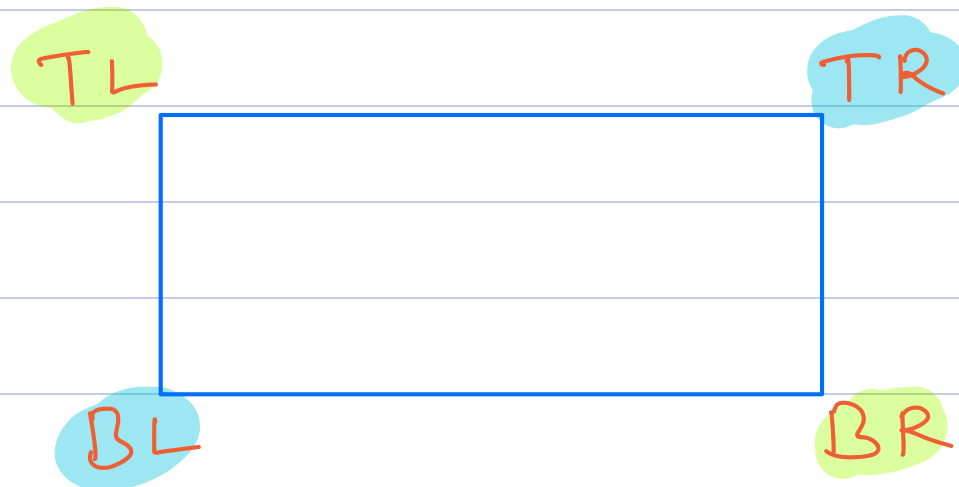
Submatrices

→ Same concept as a subarray.

→ A continuous part of a matrix.

	0	1	2	3
0	1	2	3	4
1	5	6	7	8
2	9	10	11	12
3	13	14	15	16

Uniquely identifying a rectangle



Either

TL & BR

OR BL & TR.

5. Given a matrix. Find the sum of all submatrix sum.

$1 \times 3 \times 2 \times 1$

	0	1	2
0	4	9	6
1	5	-1	2

All submatrices

1×1

- 4 - 4
- 9 - 9
- 6 - 6
- 5 - 5
- 1 - -1
- 2 - 2

1×2

- 4, 9 - 13
- 9, 6 - 15
- 5, -1 - -4
- 1, 2 - -1

1×3

- 4, 9, 6 - 19
- 9, -1, 2 - -6

2×1

- 4 - 9
- 6
- 9 - 8
- 1
- 6 - 8
- 2

2×3

✓ 25

2×2

4 9
5 -1

17

9 6
-1 2

16

166

Total Submatrices $\rightarrow O(N^4)$

Iterate on each submatrix

$O(N^2)$

$O(N^6)$

Apps each

	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

(3, 2)

12: $\times 12 = 144$

Quiz

	0	1	2	3	4
0					
1					
2					
3					

6 \times 9 = 54

	0	...	j	...	m-1	
0	✓	✓	✓	✓		
⋮	✓	✓	✓	✓		
⋮	✓	✓	✓	✓		
i	✓	✓	✓	✓	✓	✓
⋮				✓	✓	✓
N-1				✓	✓	✓

$(0, j)$

$$\bar{j} = 0 + 1$$

$(i, N-1)$

$(j, m-1)$

$$T.C \rightarrow (i+1) * (j+1).$$

$$B.R \rightarrow (N-i) * (m-j).$$

Contribution of element at index i, j

$$= (i+1) * (j+1) * (N-i) * (m-j) * AC[i][j];$$

Code:-

ans = 0;

for (i=0; i < N; i++) {

for (j=0; j < m; j++) {

ans += (i+1) * (j+1) * (N-i) * (m-j) * A[i][j],

}

}

return ans;

T.C $\rightarrow \underline{\underline{O(N * m)}}$.