AN EMPIRICAL EQUILIBRIUM MODEL OF THE MARKETS FOR RENTAL AND OWNER-OCCUPIED HOUSING*

SIDHARTH MOKTAN†

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Abstract

A large and growing share of households rent from private landlords. I empirically analyze how landlord supply constraints affect welfare and housing affordability by influencing rents, prices, and the allocation of houses between the rental and owner-occupied sectors in the UK in the presence of household borrowing constraints. I combine several novel datasets of UK property markets and document three key facts that suggest the impact of landlord supply constraints on the housing market: (i) housing quality is segmented between the rental and owner-occupied sectors, with rentals generally offering lower quality, (ii) cities with more pronounced quality segmentation tend to have higher rent-to-price ratios, and (iii) in more segmented cities landlords have fewer assets. To quantify the effect of landlord supply constraints on the housing market, I develop and estimate a two-sided assignment model which features households' optimal choice of housing quality and tenure (i.e., the choice to rent or own) in the presence of borrowing constraints, landlords' profit-maximizing choice of quality to rent out, and endogenous quality segmentation and rent-to-price ratios which are determined in equilibrium. I conduct counterfactual experiments to show that differences in landlord supply constraints explain much of the variation in quality segmentation and rent-to-price ratios observed across cities.

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†London School of Economics and Political Science, S.Moktan@lse.ac.uk

1 Introduction

Homeownership rates, prices, and rents vary dramatically both within and across local housing markets. Differences in the availability and affordability of housing have important implications for the distribution of household welfare because housing is both a major source of expenditure for households and a key determinant of access to local amenities such as schools. Understanding these differences is particularly timely given the recent rise in the private rental sector as shown in Figure 1. Despite its importance to households, we know little about why these difference arise, how they affect the distribution of household welfare across cities, and what their implications are for housing policy? I answer these questions by estimating an equilibrium model of housing markets and show that landlords' supply of rental properties plays a crucial role in generating these differences.

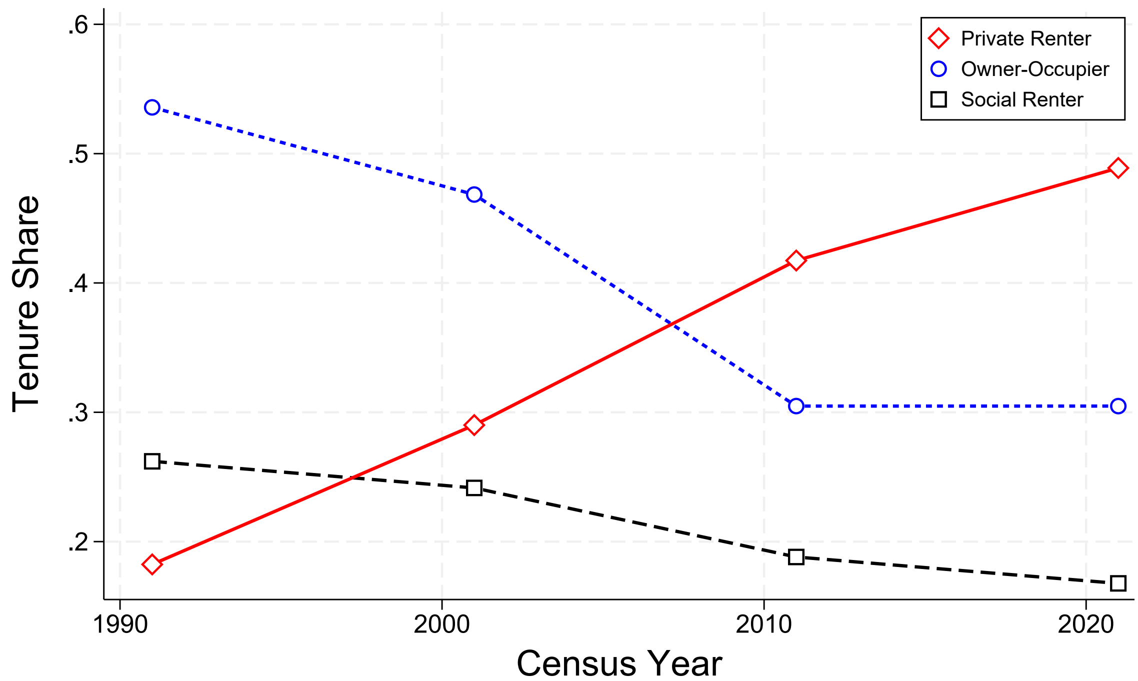
I begin by documenting several new facts which highlight differences in housing markets within and across England's largest cities and point to the potential role of landlords in generating these patterns. To illustrate these patterns, I compile data from Zoopla—the UK's second-largest property listings platform—and augment it with administrative records on property transactions from the Land Registry Price Paid dataset as well as household survey data from multiple sources.

I first develop a method for measuring differences in the distribution of house quality between the rental and owner-occupied sectors which relies on comparing the rank achieved by the same property in the local rent and price distributions.[1] To implement this approach, I exploit data on properties that were listed both for rent and for sale in Zoopla. I find that housing quality tends to be segmented between the rental and owner-occupied sectors, with the rental sector drawing more heavily from the lower quality segments of the local housing distribution. Moreover, the degree of segmentation varies widely across cities. For instance, while the quality of the housing stock is comparable between the rental and owner-occupied sectors in London, in Birmingham, the median-quality rental has lower quality than 80% of owner-occupied properties. This highlights stark disparities in the quality of rental housing relative to owner-occupied housing across cities.

Using the measure of quality segmentation, I document two correlations which point to the role of landlords in generating differences in housing markets across cities. First, I show that cities with greater levels of quality segmentation tend to have higher

[1]This approach is closely related to Diamond and Diamond (2023, 2024) who use similar rank-rank comparisons to impute rents for owner-occupied properties in the US. A key benefit of this approach is that it decouples the measurement of relative quality by tenure which depends on ranks from the measurement of the rent-to-price ratio which depends on levels. This allows me to study how differences in rents and prices impact the distribution of quality by tenure.

Figure 1: Tenure Shares Over Time in London for Households Ages 20-39



Note: This figure plots the share of households between age 20-39 who belong to one of three housing tenure categories over time. Appendix figures A1 and A2 present analogous time series of tenure shares for age groups and regions not plotted in this figure.

rent-to-price ratios. The rent-to-price would be decreasing in quality segmentation if relative rents only reflected the relative quality of rental properties (Head et al., 2023). The fact that the correlation is positive suggests that the rent-to-price may instead be driven by differences in supply constraints faced by landlords across different cities. To explore this possibility, I compare house price appreciation and landlord asset endowments across cities. This is instructive because greater capital gains directly lowers opportunity costs of capital and greater landlord asset holdings can reduce costs of capital by reducing landlords' borrowing. I find that capital gains and landlord assets both tend to be lower in cities with higher rent-to-price ratios. These correlations suggest that in cities such as London, wealthier landlords may reduce quality segmentation and increase rates of renting by increasing the relative supply of higher quality properties in the rental market for low rents relative to prices.

To understand the role of landlord supply constraints in generating the observed differences in housing markets across cities, I develop a two-sided assignment model of local housing markets which builds on Epple et al. (2020)'s equilibrium model of metropolitan housing markets with vertically differentiated housing. I enrich their model in several important directions. These extensions allow me to endogenously generate the observed patterns of quality segmentation, ownership rates, and the rent-to-price functions in the model.

On the demand side, heterogeneous households simultaneously choose a continuous level of housing quality and decide whether to rent or own. Their choices are influenced by unobserved preferences for quality and owner-occupation, observed income and asset endowments, and borrowing constraints that limit the quality range available for owner-occupied housing.

Households sort positively to house quality based on income. However, the correlation between income and quality may not be perfect for several reasons. First, the heterogeneity in unobserved preferences implies that households with the same income may choose different levels of quality. Second, the choice sets facing households may vary independently of income due to the impact of assets on borrowing constraints. Third, differences between the user costs of owner-occupation and rents may drive a wedge in the choice of quality by tenure conditional on income.

I depart from the literature by allowing unobserved user costs of owner-occupation to be different than equilibrium rents. The literature conventionally treats user costs as being equivalent to rents (Poterba, 1984, 1992). This equivalence of rents and user costs only holds when borrowing constraints and preferences for tenure do not exist because renting and owning are perfectly substitutable when these conditions are satisfied.In my model, however, rental equivalence does not hold because both of these conditions are violated. Since households are allowed to have preferences for tenure in my model, any systematic difference in preferences for renting or owning generate a wedge between rents and user costs of owning in equilibrium. Moreover, even when households do not prefer renting to owning, borrowing constraints prevent households from completely eliminating cost differences by tenure.

On the rental supply side, atomistic landlords choose the profit-maximizing house quality to own and let out in the local rental market. The choice depends on their observed asset endowments and unobserved opportunity costs of capital. The unobserved opportunity cost of capital captures costs of maintenance, depreciation, borrowing and taxes net of capital gains (Poterba, 1984, 1992). The evidence shows that the cost of borrowing is convex in the loan-to-value ratio in the UK (Benetton, 2021). I therefore define landlords' costs of capital to be a convex function of the share of property financed by borrowing. The convexity of costs with respect to borrowing implies that costs rise less steeply with quality for wealthier landlords.[2] This leads to a positive assignment of landlords to housing quality based on wealth, as wealthier landlords face lower marginal costs of operating in higher quality segments. Hence, the supply of rentals in each quality segment crucially depends on the distribution of landlord assets and the shape of their cost function.

Equilibrium in the model is characterized by two market clearing conditions for each quality segment. First, the demand for rental services must equal the supply by landlords in each quality segment. Second, the demand for ownership by landlords and households must equal the available stock of housing in each quality segment. Prices and rents are determined by the marginal household and landlord within each quality segment. The schedule of prices and rents are non-linear over quality because marginal households and landlords vary across quality segments (Landvoigt et al., 2015; Epple et al., 2020).

Quality segmentation arises in the model when the equilibrium distribution of house quality varies across the rental and owner-occupied sectors. The model provides several channels, on both the demand and supply side, through which quality segmentation can be generated. On the demand side, segmentation arises when unobserved preferences for owner-occupation are positively correlated with either preferences for housing quality or household income, both of which increase households' demand for quality. On the supply side, landlords can shape quality segmentation by influencing the wedge between rents and households' user costs of owner-occupation. The wedge in costs affects segmentation because households' optimal choice of quality depends on rents when renting and on user costs when owning.

I estimate the structural parameters of the model using a method of moments estimator exploiting data for nine markets: London, Manchester, and Birmingham for the years 2014-2016. The estimation takes as inputs the joint distribution of wealth and income for households as well as the marginal distribution of wealth for landlords, which I recover from the UK Wealth and Assets Survey. The estimation imposes market clearing for each quality segment in each market. The key moments include for each market the marginal distributions of prices and rents, ownership rates, quality segmentation, and correlations between household endowments and rents and prices, which I recover using data from Zoopla, the UK Wealth and Assets Survey and the UK Household Longitudinal Survey.

[2] I assume that landlords only borrow when they lack sufficient wealth to purchase the property outright, consistent with UK survey evidence showing landlords prefer to avoid borrowing when possible (Scanlon and Whitehead, 2016). Consequently, wealthier landlords borrow less than their less wealthy counterparts across all quality levels. Because costs are convex in borrowing, wealthier landlords tend to operate in the less convex part of the cost function, leading to a lower marginal cost of operating in higher quality segments.

The estimated model successfully replicates the empirical rent and price distributions as well as the patterns of quality segmentation observed in the data. Estimates of the household demand parameters show that heterogeneity in unobserved preferences is key to explaining differences in the choice of quality and the decision to rent or own by age, even after conditioning for differences in income and assets. However, unobserved household preferences play a smaller role in explaining differences in sorting patterns across cities. Estimates for the landlord's cost function exhibit large differences across cities. This suggests that landlords mostly account for the observed differences in quality segmentation and rent-to-price ratios across cities.

I conduct three counterfactual experiments that illustrate that landlord heterogeneity is key to explaining differences in housing markets both across cities and across quality segments within cities. In the first counterfactual, I show that within-city heterogeneity in landlord wealth almost completely explains the negative slope of the the rent-to-price function with respect to prices within cities—a robust feature of housing markets that has been documented both in the UK and the US.[3] This suggests that within-city wealth disparities among landlords are critical in shaping rental affordability over the distribution of quality. I find that reducing the variance of the landlords' wealth distribution by 90% almost completely flattens the equilibrium rent-to-price function. This occurs because a reduction in the variance of the wealth distribution increases the relative wealth of landlords operating in lower quality segments which in turn reduces their costs relative to landlords in higher quality segments.[4] Landlords respond to the relative reduction in costs by reducing rents in lower quality segments. This flattens the rent-to-price function.

In the second counterfactual, I show that the cost advantage enjoyed by landlords in London over those in Birmingham accounts for much of the cross-city difference in the housing market equilibrium. To quantify this effect, I set the opportunity cost of capital for landlords in London to Birmingham levels and solve for the resulting equilibrium in London. This reduces the gap in quality segmentation between London and Birmingham by over 60%. The change in quality segmentation is driven by a 42% reduction in the rent-to-price gap between these two cities and a 67% reduction in the user cost-to-rent gap. These results indicate that quality segmentation varies across cities largely due to differences in rental supply. In cities such as London landlords are willing to accept lower

[3]Studies which document the negative slope of the rent-to-price include Verbrugge (2008); Verbrugge and Poole (2010); Bracke (2015); Halket et al. (2020).

[4]Recall that in the model landlords sort positively to house quality based on assets. This implies that a reduction in variance increases wealth in lower segments and reduces it in higher segments. This tends to reduce relative costs in lower segments since costs are increasing in borrowing which is in turn falling in wealth.

rents from tenants and pay higher prices to acquire properties which reduces rents both in absolute terms and relative to the user costs of owner-occupation. The reduction in the costs of renting allows a greater share of renters to access housing in higher-quality segments, thereby reducing quality segmentation.

To explore the welfare implications of landlord heterogeneity, I analyze the compensating variation associated with increasing landlord costs in London to Birmingham levels. I compute the compensating variation numerically for the distribution of households in London. I find that the cost advantage enjoyed by landlords in London increases the welfare of the median renter in London by £1,448 per year and lowers the welfare of the median owner-occupier by £2,139 per year. These welfare effects reflect the trade-off arising from the the reduction of rents relative to prices as well as the increase in the relative quality of rentals. Additionally, I find that the magnitude of the welfare effects increase in household income. This occurs because in the model a reduction in landlord costs increases the profit-maximizing quality for marginal landlords in each quality segment. This causes the assignment of landlord assets to houses to cascade upwards magnifying the supply effect in higher quality segments.

In the third counterfactual, I impose a 30% tax on rental income in each city. I find that the tax elasticity of the equilibrium rent-to-price function is up to five times greater in Birmingham than in London. This shows that the same tax can lead to vastly different outcomes depending on the composition of households and landlords in the local housing market. The lower pass-through of the tax in London reflects the greater cost advantages and higher profits enjoyed by landlords London, enabling them to absorb more of the tax burden compared to their counterparts in Birmingham. These results suggest that a progressive tax structure—one that increases with landlord income—may distort the housing market less than a flat tax, as it would better account for the varying capacities of landlords across cities to absorb taxes.

1.1 Related Literature

The paper is related to a large and growing literature on housing choice. A large strand of the literature models demand for differentiated housing and/or neighbourhoods while abstracting from the choice of whether to rent or own (Bayer et al., 2007, 2016; Galiani et al., 2015; Landvoigt et al., 2015; Diamond et al., 2019; Almagro and Dominguez-Iino., 2020; Calder-Wang, 2020; Epple et al., 2020). Another strand composed mainly of macro-housing papers focuses on the choice to rent or own but assumes that housing is not differentiated (Chambers et al., 2009a,b; Sommer et al., 2013; Binner and Day, 2015; Halket and Pignatti Morano di Custoza, 2015; Sommer and Sullivan, 2018; Kaplan et al., 2020; Han et al., 2021;

Greenwald and Guren, 2024). This paper belongs to a nascent third strand which studies housing markets with differentiated housing and both a rental and an owner-occupied sector (Head et al., 2023; Higgins, 2023; Kvaerner et al., 2023).

To my knowledge, Head et al. (2023) is the only other paper that develops a model to examine cross-city differences in the composition of properties between the rental and owner-occupied sectors. While they use the price-to-rent ratio as a proxy for quality differences, I develop a measure of relative quality that is distinct from this ratio, allowing me to explore the link between relative quality and the price-to-rent ratio. In Section 3, I show that the price-to-rent ratio does not consistently capture quality segmentation, as the two measures are strongly and negatively correlated across English cities. I develop a model which explains the negative correlation observed in the data.

Several studies have found that the rent-to-price function decreases in price both in the UK and in the US (Verbrugge, 2008; Verbrugge and Poole, 2010; Bracke, 2015; Halket et al., 2020). The literature has considered a number of explanations for this phenomenon. Halket and Pignatti Morano di Custoza (2015) show that rental vacancy rates can explain some of the relationship. Halket et al. (2020) argue that this pattern can arise when high price rental properties have worse unobserved characteristics than high price owner-occupied properties. Bracke (2015) shows that the rent-to-price is falling in rent appreciation. However, using data on transaction prices I find that the rent-to-price function is increasing in capital gains within cities. This paper offers an alternative rental supply-side explanation which almost completely explains the negative slope of the rent-to-price. In particular, I show using counterfactual experiments that the slope of the rent-to-price is linked to the variance of landlord wealth distribution. I find that reducing the variance of landlord wealth by 90% almost completely flattens the rent-to-price function.

This paper also contributes to the measurement of unobserved user costs of owner-occupation. The housing choice literature often assumes that observed rents equal unobserved user costs of owner-occupation. Theoretically rental equivalence holds when households are indifferent between renting and owning and have the ability to arbitrage away cost differences between tenure types (Poterba, 1992, 1984). However, both of these conditions are violated in this paper since the model allows households to prefer owning to renting and incorporates credit constraints which restrict arbitrage. This paper provides a structural approach to estimate unobserved user costs when rental equivalence cannot be assumed. The approach is related to Díaz and Luengo-Prado (2008) which also estimates user costs as the shadow cost of owner-occupation. More broadly, the approach complements the literature measuring user costs of homeownership (Bishop et al., 2023).

2 Data

I combine information from different datasets on houses and households. To estimate distributions of prices and rents by quality of housing, I use a comprehensive dataset of rental and sale listings from Zoopla, the second-largest real estate listings platform in the UK. To measure households' characteristics and housing choices, I use data from two different household surveys.

Listings and Transactions—I obtain listing-level data from the WhenFresh-Zoopla dataset. The dataset records the universe of rental and sale listings in Zoopla, the second-largest listing platform in the UK. The dataset covers the period 2014–2020 and includes over five million listings of properties for sale and over four million properties for rent over this period. For each listing, the data records the listing date, transaction date, prices for properties on sale, weekly rents for rentals, and property characterstics including property address, number of rooms, and build type.

Household Choices, Endowments, and Characteristics—I use data from the Wealth and Assets Survey (WAS) and the UK Household Longitudinal Survey (UKHLS). The WAS is a biennial longitudinal survey of household assets and savings. During the period 2014-2016, the WAS includes approximately 18,000 households per wave or $\sim 9,000$ households per year. I use the WAS to estimate joint distributions of wealth and income by region, year, and household characteristics for households. I also use it it estimate the asset distribution for landlords by region and year.

The UKHLS is the largest longitudinal household survey in the UK. Over the period 2014-2016, the survey includes $\sim 22,000 - 25,000$ households per year. I use the UKHLS to measure the relationship between households' characteristics and housing choices by city and year. It includes ownership rates by household age, correlations between household income and rent for renters, and correlations between household income and house price for owner-occupiers.

Other Datasets—I combine data on the housing stock from the 2011 Census and Valuation Office Agency (VOA) to construct weights which are used to to estimate the distribution of prices and rents over the entire housing stock. I use the Census to measure time trends in ownership rates over a long time horizon. I also use administrative records of property transactions from the Land Registry Price Paid Data to estimate capital gains in the housing market.

3 Heterogeneity in Housing Markets Across and Within Cities

In this section, I describe how I measure the distribution of prices and rents over quality as well as differences in the distribution of quality between the rental an owner-occupied sectors. Following the literature on hedonic models and models of housing assignment (Landvoigt et al., 2015; Epple et al., 2020; Diamond and Diamond, 2023, 2024), I represent housing quality using a unidimensional index which captures all relevant features of a house and its environment. Prices and rents are increasing in this quality index. The monotonic relationship between rents and quality implies that I can use the ranking of rents in any given year to recover the quality ranking of properties in the rental sector in that year. Similarly, prices can be used to recover the quality ranking of all properties in the housing market in a given year.

However, several complications arise when trying to compare the distribution of quality, prices and rents over time and across the rental and owner-occupied sectors. First, while I need distributions over the entire stock of housing, I only observe prices or rents for listed properties. To recover distributions over the entire stock, I need to adjust for over- or under-representation of properties in the listings data. Second, the price or rent rank for a given level of quality may change over time as properties are either added or removed from the stock. To recover the path of prices or rents for a given unobserved level of quality, I need a way to trace the ranking of this quality level in the price and rent distributions over time. Third, comparing quality across the rental and owner-occupied sectors is complicated because I do not observe a common measure of quality for properties in the two sectors. Prices are not observed for rental listings and rents are not observed for sale listings. To resolve this missing data problem, I need a strategy to map the rental quality distribution to the quality distribution of the overall housing stock.

In this section I discuss how I resolve each of these measurement issues to obtain quality distributions which are comparable over time and across the rental and owner-occupied sectors. These adjusted distributions form the basis for how I measure the evolution of rents and prices for different levels of unobserved quality. Using these adjusted distributions, I find novel evidence that the quality of the housing stock tends to be lower in the rental sector compared to the owner-occupied sector. I also show that the documented pattern of segmentation varies systematically across cities.

3.1 Measuring the distribution of quality, rents, and prices

Marginal Distributions of Rents and Prices—We are interested in the overall distribution of rents and prices at the city and year level. However, the data only lets us observe prices and rents for listed properties. Since listed properties might not be representative of the stock of houses in the rental sector and in the housing market as a whole, the observed distributions need to be adjusted for selection into listing. To obtain the adjusted distributions, I first measure the degree to which each type of property is either over- or under-represented in the listings data relative to the overall stock. I then use this information to estimate weighted distributions of rents and prices which are representative of the stock of rented properties as well as the entire housing stock.

For this exercise, I define a property type as a combination of Lower layer Super Output Areas (LSOA) and number of bedrooms.[5] I choose the LSOA as the spatial unit because the Valuation Office Agency's (VOA) data on the housing stock is provided at this level of granularity. Let P^r_{jcy} and P^s_{jcy} represent the share of type j properties among the rental stock and overall housing stock. I construct these shares by combining yearly data from the VOA with the 2011 census. Let p^r_{jcy} and p^s_{jcy} represent the share of type j properties among rental and sale listings in city c in year y. Then p^r_{jcy}/P^r_{jcy} and p^s_{jcy}/P^S_{jcy} measure the degree to which properties of type j are over or under-represented among rental and sale listings respectively. Weighting the rental and sale listings by the inverse of these ratios, I recover the distribution of rents in the rental sector $F_r(\cdot \mid city, year)$ and the distribution of prices over the housing stock $F_p(\cdot \mid city, year)$ by city and year.

Normalizing House Quality—Quality needs to be normalized because it is unobserved. In each city, I define a property's relative quality within the city by its rank in the city's overall distribution of house prices as of 2014.[6] This is equivalent to normalizing quality to equal prices as of 2014 since price ranks are monotonic in prices.[7] Under this definition of quality, we only directly observe a property's relative quality if we know its price as of 2014. We need a way to infer quality for properties for which we do not observe this information. These include properties for which we only observe prices from other years or rents. I develop a strategy which allows me to assign quality to such properties based on the rent or price information that is observed.

[5]There are 33,755 LSOAs in England each comprising 400-1,200 households with a resident population between 1,000-3,000 persons.

[6]Note that this definition of quality allows me to explore within-city differences in quality distributions by tenure. It can also be used to compare the relative quality by tenure across cities. However, I cannot directly use them to compare quality distributions across cities. In the structural estimation, I treat the distributions of quality as unobserved and estimate them.

[7]Landvoigt et al. (2015); Epple et al. (2020) normalize house quality to equal prices in a base year.

Tracing the Rank of Quality Over Time Within the Rent and Price Distributions—We need prices and rents between 2014 and 2018 for each level of unobserved quality. We have already estimated the marginal distributions of prices and rents by city and year. Therefore, to track prices and rents by quality over time, we need to trace the rank of each quality level in the price and rent distributions over time. Let $q^p_{icy} = F_p(h_{icy} \mid c, y)$ be the rank of a property of type i with unobserved quality h_{icy} in the price distribution of city c in year y. If the average quality of a type-i property remains unchanged during the period under study, i.e., $h_{icy} = h_{ic2014}$, then we can trace the price ranking of quality h_{ic2014} over time by tracing the ranking of properties of type i. Similarly, we can use q^r_{icy} , the rent rank for properties of type i, to trace the ranking of quality h_{ic2014} in the rent distribution.

I define each property type i by a combination of postcode, dwelling type, and the number of bedrooms and bathrooms. Let q^p_{icy} be the median price rank achieved by properties of type i in y. We need to estimate the mapping $q^p_{icy} = m_{pcy}(q^p_{ic2014})$ between initial price rank q^p_{ic2014} and the price rank q^p_{icy} in subsequent years $y > 2014$. I estimate m_{pcy} for each city and year using quantile regressions of q^p_{icy} on a third-degree polynomial of q^p_{ic2014}. I obtain the quantile maps in the rental sector m_{rcy} using an analogous procedure.

Appendix Figure A3 plots the functions m_{pcy} and m_{rcy} for $y = 2018$ in Birmingham, London, and Manchester. The main patterns discussed below also hold for cities and years not plotted in the figure. There are two key takeaways. First, the plots do not coincide with the diagonal which implies that the rank of a property type or quality level is not fixed over time. We therefore need a way to keep track of the rank for each quality level over time. Second, the rank in 2018 is increasing in the rank in 2014. The monotonicity implies that for any price rank q in the 2014 distribution, we can assign a unique rank $m_{pcy}(q)$ in the year-y distribution and vice-versa. Therefore, for any quality level with initial price rank q, we can trace its ranking over time by evaluating the functions $m_{pcy}(q)$ for each year between 2015 and 2018. Moreover, for properties with price rank q in 2014, we can recover its year-y price since it equals quantile $m_{pcy}(q)$ of the estimated price distribution for year y. Repeating this for all $y > 2014$ yields the path of prices for quality levels with initial price rank q. Similarly, we can obtain the path of rents given an initial rent rank.

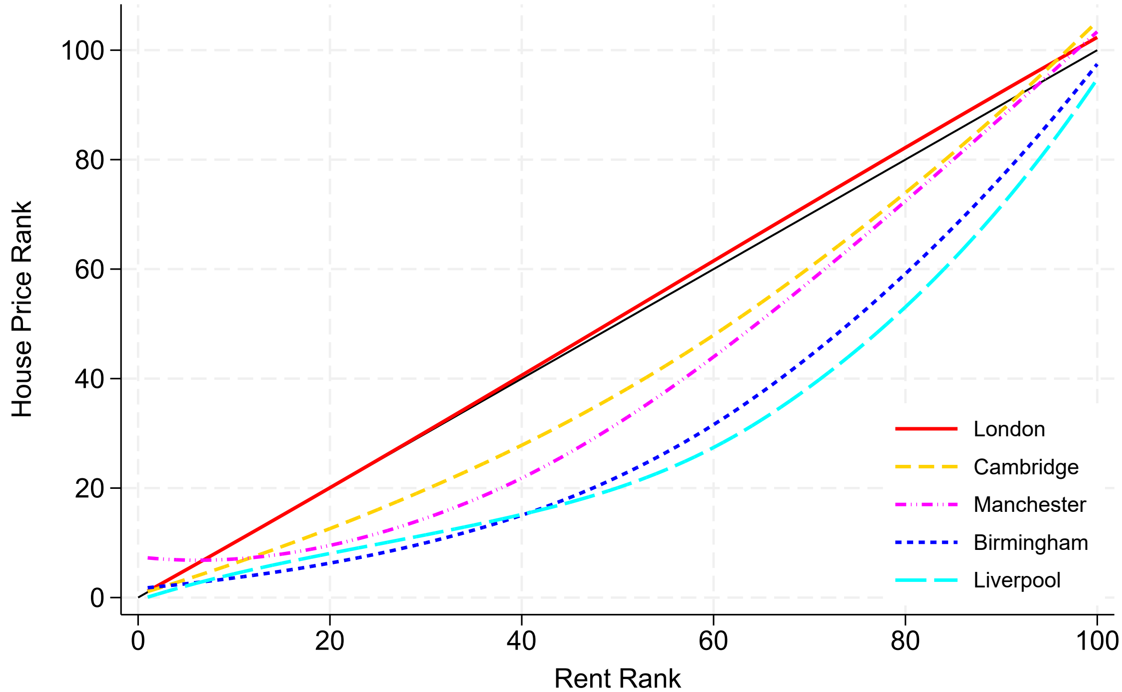
Mapping Quality Between the Rental and Sale Distributions—The preceding analysis lets us trace the path of prices given a price rank from 2014 and the path of rents given a rent rank from 2014. Since quality has been normalized to equal the price rank as of 2014, this is sufficient to track the path of prices for each level of the normalized quality. However, to track the path of rents by each level of normalized quality, we need to establish a mapping between the rent rank and the 2014 price rank. To estimate this link, I exploit information from properties which have been cross-listed for sale and rent in the dataset.

The estimation is conducted in two steps. In the first step, restricting the sample to properties that have been listed both for rent and sale at some point between 2014 and 2018, I assign each property ranks in the 2014 rent distribution and the 2014 house price distribution. For sales listings in 2014, we directly observe its sales rank as of 2014. For sales listings from future years $y > 2014$, I invert the mapping $m_{pcy}(\cdot)$ estimated above to assign a price rank in 2014 given a price rank as of y. I assign 2014 rent ranks to listings using an analogous procedure. To deal with properties that have been listed for sale more than once, I define a property's rank in the price distribution as the median 2014 price rank achieved by the property across all of its sale listings. Each property's rent rank is defined analogously. This procedure yields a pair of price and rent ranks as of 2014 (q_{p2014}, q_{r2014}) for each property in the restricted sample. In the second step, I use the assigned ranks to estimate a mapping $g_c(q)$ which predicts a property's rank in the 2014 price distribution given its rank in the 2014 rent distribution. For each city, $g_c(q)$ is estimated using a quantile regression of each property's 2014 price rank against a third-degree polynomial of its 2014 rent rank. The regression for each city is estimated over all properties in the restricted sample.

The function $g_c(q)$ links the ranks of the 2014 rent distribution to the ranks of the 2014 price distribution. Since we normalize quality to equal the 2014 price ranks, $g_c(q)$ also provides a link between the rent distribution and the distribution of quality. This mapping forms the basis for how I impute prices for rental properties and measure quality segmentation across the rental and owner-occupied markets.

Imputing Prices for Rental Properties—Consider a rental property which lists in the q^{th} quantile of the rental distribution in city c and year y. We are interested in the price of this rental unit. However prices are generally not available for rental listings. To impute the unobserved price, I apply to q the composition of mappings $m_{pcy} \circ g_c \circ m_{rcy}^{-1}$. $m_{rcy}^{-1}(q)$ converts the rent rank from year y to the rent rank as of 2014. Applying g_c to the 2014 rent rank gives the 2014 price rank which is also the measure of quality. Lastly applying m_{pcy} to the 2014 price rank gives the price rank as of y for this property. Given the price rank as of y, we can lookup the corresponding price as of y using the price CDF. Therefore, by applying the appropriate composition of mappings, we can impute prices in any year between 2014 and 2018 given a rent rank in this time period.

Figure 2: Rent Rank vs. Estimated House Price Rank



Note: This figure plots the conditional rank in the overall distribution of house prices given a property's rank in the overall rent distribution. Each plot corresponds to a different city.

3.2 Descriptive Evidence

Quality is Segmented by Tenure. Segmentation Varies by City—I use the mapping $g_c(q)$ to infer the relative quality of properties in the rental and owner-occupied sectors. To make inferences about relative quality based on $g_c(q)$, prices need to be monotonically increasing in rents. As discussed in the following paragraph, I verify that this condition is satisfied in the data. When this condition is satisfied, $g_c(q) < q$ implies that quality in quantile q of the rental distribution is lower than quality in the same location of the owner-occupied distribution. $g_c(q) > q$ implies the opposite. $g_c(q) = q$ arises as a special case when quantile q of both distributions have the same quality.

Figure 2 plots $g_c(q)$ for five cities. Three key patterns emerge. First, the price rank $g_c(q)$ is monotonically increasing in the rent rank q across all cities. This verifies that prices are monotonically increasing in rents. Second, $g_c(q)$ is almost universally below the diagonal across all cities. This implies that compared to the owner-occupied sector, the stock of properties in the rental sector draws more heavily from the lower quality segments of the housing distribution. Third, there is substantial heterogeneity across cities in the degree to which the rank-rank plots deviate from the diagonal. Whereas rental and sale listings in London have the same median quality, the median quality for rentals in Liverpool ranks in the 20^{th} percentile of the overall housing distribution in Liverpool. This implies that the relative quality of properties in the rental and owner-occupied sectors vary greatly across cities.

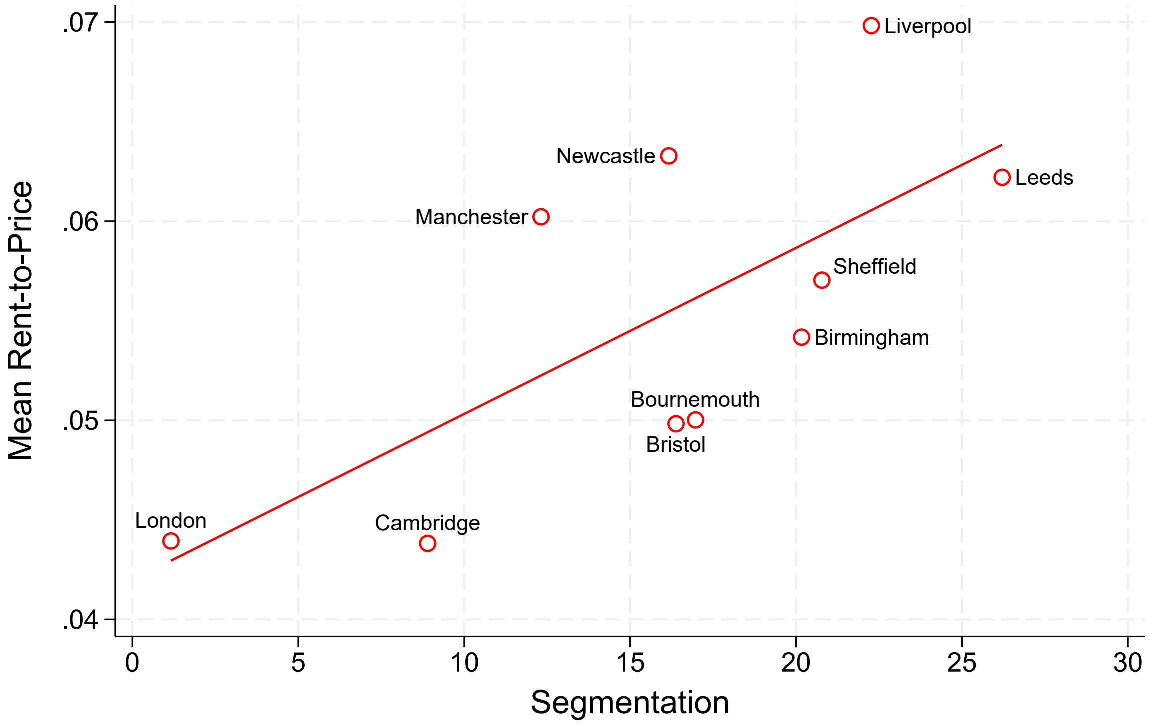
These three insights provide novel evidence that quality is segmented between the rental and owner-occupied sectors and that the degree of segmentation varies substantially across cities. Quality segmentation can arise due to several reasons. It can arise if preferences for owner-occupation are correlated households' taste for quality or their income and asset endowments, both of which influence demand for quality. Segmentation can also arise if costs of owner-occupation are different than rents for the same quality. The quantitative analysis allows for each channel.

What do these patterns of segmentation reflect? I show in Appendix Section A2 that the patterns of segmentation along the unidimensional index in Figure 2 are driven by segmentation in quality along three sub-dimensions: the location of a dwelling, its structural features, and its upkeep. The data shows that the degree of segmentation along these three dimensions co-vary across cities. The better the relative location of rentals in a city, the higher tends to be the relative quality of structural features of rented properties. Similarly, the relative upkeep of rental dwellings is increasing in both the relative quality of location and structural features of rentals. The positive correlation suggests that factors which improve the relative quality of rentals in a city do so by improving all three sub-dimensions of quality.

Cities with Higher Relative Quality of Rentals Have Lower Mean Rent-to-Price Ratios— Several studies document that the rent-to-price varies both within and across cities (Himmelberg et al., 2005; Gyourko et al., 2013; Bracke, 2015; Halket et al., 2020; Head et al., 2023). I present new evidence showing that the mean rent-to-price ratio in a city is decreasing in the relative quality of rentals in the city.

I define the degree of quality segmentation between the rental and owner-occupied sectors of a city as $\gamma_c = \sum_q (g_c(q) - q)^2$, the sum of squared differences between the rent ranks and the predicted price ranks. A higher value of γ_c implies greater segmentation. To obtain the rent-to-price function for each city, I estimate OLS regressions of the form $rent_{icy} = \alpha_c + \beta_c price_{icy} + \varepsilon_{icy}$, where i indexes quality; $rent_{icy}$ is the rent for quality i in city c as of year y; and $price_{icy}$ is the price for quality i. The literature traditionally imposes $\alpha_c = 0$ which implies a constant rent-to-price ratio (Higgins, 2023; Epple et al., 2020). I allow α_c to be non-zero because positive values of α_c help generate a negative slope between the

Figure 3: Cities with Higher Relative Quality of Rentals Have Lower Mean Rent-to-Price Ratios



Note: This figure presents a scatter plot of the mean rent-to-price ratio in a city against the degree of quality segmentation between the rental and owner-occupied sectors in the city. The figure also plots a line of best fit.

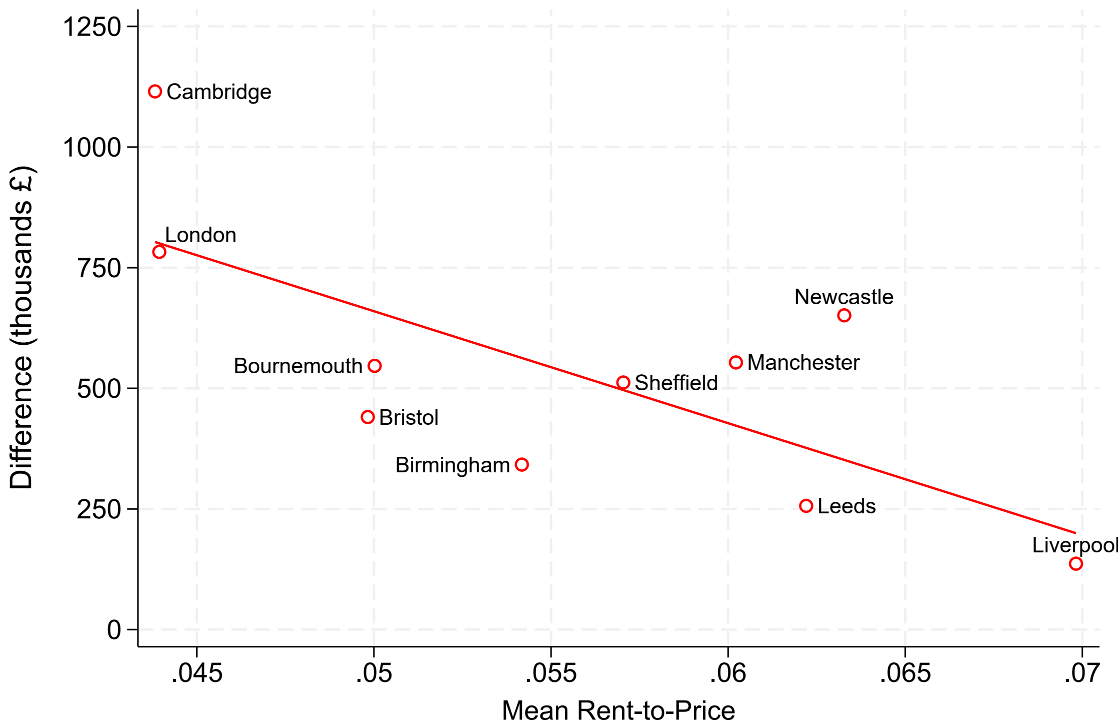
rent-to-price ratio and prices as observed in the data. Dividing predicted rents from the regression by price yields the rent-to-price function: $\widehat{rent}_{icy}/price_{icy} = \widehat{\alpha}_c/price_{icy} + \widehat{\beta}_c$.[8]

Figure 3 plots the relationship between the degree of segmentation, γ_c, and the mean rent-to-price ratio, $\widehat{\alpha}_c/\,\mathbb{E}_{iy}[price_{icy}] + \widehat{\beta}_c$, in each city over 2014-2018. The plot shows that segmentation is increasing in the average rent-to-price in a city. In other words, the cost of renting relative to prices is lower in cities with higher relative quality of rentals. Appendix Figures A4 and A5 show that this correlation is generated by more segmented cities having lower baseline rents, α_c, and greater sensitivity of rents to prices, β_c.

The positive correlation in Figure 3 implies that the difference in rent-to-price across cities cannot be explained by high rent-to-price cities having higher relative quality of rentals as assumed in Head et al. (2023). What else might explain the systematic variation in the mean rent-to-price ratios across cities? I next show evidence which suggests that

[8]We can see that $\widehat{\alpha}_c$, which represents the baseline level of rents in a city, determines the slope of the rent-to-price function. When baseline rents are higher in a city, the rent-to-price ratio drops more steeply with price. On the other hand, $\widehat{\beta}_c$, the slope on prices in the rent regression, is the asymptotic value that the rent-to-price converges to as price increases.

Figure 4: Difference Between Landlord and Household Median Assets
Decreasing in Sensitivity of Rent to Price

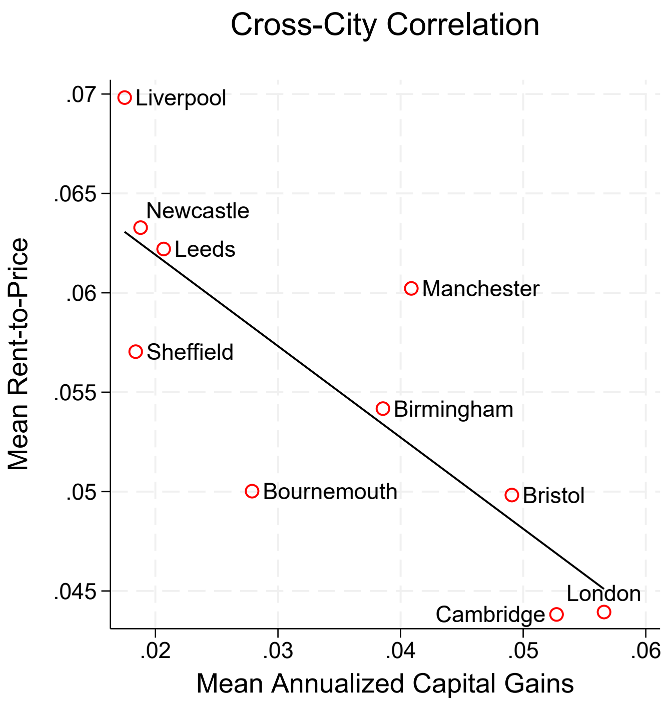


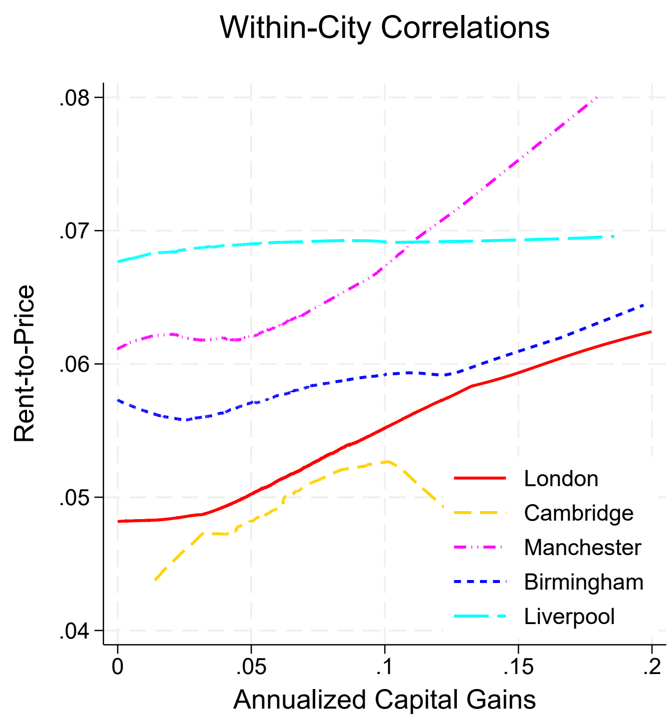
Note: This figure plots the difference in median assets between landlords and households against the mean rent-to-price ratio in the city. The figure also plots a line of best fit.

differences in the mean rent-to-price ratios across cities arise due to differences in landlords' opportunity costs of capital.

Landlords are Relatively Wealthier in Cities with Lower Mean Rent-to-Price Ratios—To explore whether opportunity costs of capital are higher in cities with high rent-to-price ratios, I start by considering landlord wealth. Comparing landlord wealth across cities is informative since higher wealth can reduce opportunity costs of capital by reducing landlords' reliance on borrowing. Borrowing can increase opportunity cost of capital through two channels. First, it increases exposure to costs of borrowing which may be greater than the cost of financing with cash. Second, increased levels of borrowing can increase interest rates charged by lenders. Consistent with the idea that landlords in low rent-to-price cities face lower opportunity costs due to reduced borrowing, Figure 4 shows that landlords are indeed wealthier relative to households in cities with lower rent-to-price ratios. Motivated by this evidence, I explicitly account for inter-city differences in the distribution of landlord endowments in the quantitative analysis and allow landlord endowments to affect rental supply.

Figure 5: Rent-to-Price Ratio and Capital Gains Covary Negatively Across Cities
But Positively Over Quality Segments Within Cities





Note: This figure the relationship between the rent-to-price ratio and capital gains both across and within cities. The capital gains are computed using data on properties that were sold both in 2010 and 2018. The figure on the left plots the mean rent-to-price in a city against the mean annualized capital gains in that city. The figure on the right presents lowess-smoothed plots for each city of the conditional mean rent-to-price ratio for properties with a given level of annualized capital gains.

Rent-to-Price Ratio and Capital Gains Covary Negatively Across Cities But Positively Over Quality Segments Within Cities–Capital gains from house price appreciation is an important determinant of the opportunity costs of capital. In Figure 5 I investigate how the rent-to-price ratio varies with annualized capital gains computed using properties that were sold both in 2010 and 2018.

The figure on the left plots the relationship between the average rent-to-price ratio and the average annualized capital gains in different cities. The relationship is negative providing further evidence that the difference in the rent-to-price ratios across cities is driven by differences in the opportunity costs of capital. The figure on the right plots the relationship between the rent-to-price ratio and annualized capital gains within cities. The relationship is reversed with rent-to-price increasing in capital gains. The positive correlation reflects the fact that both the rent-to-price and capital gains are decreasing in prices within cities. The positive correlation implies that within-city differences in the rent-to-price ratio are likely not explained by opportunity costs of capital.

The evidence suggests that opportunity costs of capital may play a role in explaining differences in the rent-to-price across cities, but not across quality segments within cities. Motivated by these patterns, in the quantitative model I allow landlords in different cities to face different unobserved opportunity costs as well as borrowing costs which increase convexly in the loan-to-value ratio. Using counterfactual simulations I show that differences in opportunity costs across cities indeed explain much of the cross-city variation in rent-to-price ratios. On the other hand, the within-city variation in the rent-to-price ratio is explained by within-city heterogeneity in landlord wealth endowments which impacts the costs facing landlords in different quality segments.[9]

4 The Model

4.1 Setup

Each market is defined by a unique combination of city and year.

Housing.—I model housing as a vertically differentiated product. Let h represent the unidimensional index of housing quality. There are $N_{ct}(h)$ houses of quality h in the market. Any house can be rented or owned at the equilibrium rents and prices. Prices $p_{ct}(h)$ evolve non-linearly over house quality. Homeownership credit constraints imply that rents may be different from the user costs of owner occupation. Consequently, I allow rents $r_{ct}(h)$ and user-costs of owner-occupation $c_{ct}(h)$ to have separate non-linear schedules over house quality.

Households.—Each market has a distribution of households that vary along four dimensions: income y, assets a, observed characteristics x, and unobserved type i. The share of households with characteristics x equals $s_{ct}(x)$. Households with observed characteristics x draw endowments from the conditional distribution $F_{ct}(a, y \mid x)$ and belong to unobserved type i with conditional probability $p_{i|x}$. The distribution of endowments for

[9]In the model landlords sort positively to quality based on wealth. An increase in the variance of the wealth distribution increases wealth for landlords operating in higher quality segments and reduces wealth for landlords operating in lower quality segments. This increases the relative borrowing of landlords in lower segments which in turn increases their relative costs since costs are increasing convexly in borrowing.

unobserved type i is given by

$$F_{ict}(a,y) = \sum_x \left(\frac{s_{ct}(x)p_{i|x}}{\sum_x s_{ct}(x)p_{i|x}} \right) F_{ct}(a,y \mid x) \tag{1}$$

The household's choice is jointly determined by its endowment and unobserved type. The endowment shapes the households' choice set through the budget constraint and homeownership credit constraint. The unobserved type determines the household's preferences for housing quality and homeownership. Conditional on the endowment and unobserved type, households' choices do not depend on observed characteristics x.

Landlords.—In each city, landlords own houses and rent out housing services to those households who choose to rent. Landlords vary in their asset endowments. They seek to maximize profits from rents and capital gains net of operating costs. The landlord's endowment shapes the costs they face to operate rentals of varying quality. This in turn impacts the quality of house they choose to own and rent out. Aggregating over the choices of all landlords gives rise to the supply of rental services over the distirbution of house quality.

4.2 Housing Demand

Each household chooses housing and non-housing consumption z to maximize utility. Housing consumption is defined by the choice of house quality h and the decision to rent or own τ, henceforth referred to as tenure choice. Households of unobserved type i with endowments (a,y) solve the following problem:

$$\begin{aligned}
&\max_{h,\tau,z} \quad u(h,z \mid \theta_i^h) + \beta_i^\tau + \varepsilon_{i\tau} && \text{subject to} \\
&\quad y = r_{ct}(h) + z && \text{if } \tau = \text{rent} \\
&\left.\begin{aligned} &y = c_{ct}(h) + z \\ &p_{ct}(h) \leq \overline{p}_{ct}(a,y) \equiv \min\left\{\phi^{ltv}a, \;\; \phi^{lti}y\right\} \end{aligned}\right\} && \text{if } \tau = \text{own}
\end{aligned} \tag{2}$$

where θ_i^h is the preference for housing quality for unobserved type i; β_i^τ is the deterministic component of households' preference for tenure τ; $\varepsilon_{i\tau}$ is an idiosyncratic preference for tenure; ϕ^{ltv} and ϕ^{lti} are the maximum loan-to-value and loan-to-income ratios which restrict the size of mortgage that a household can obtain; and $\overline{p}_{ct}(a, y)$ is the maximum house price the household can afford given its endowment.

The choice of tenure determines whether the household pays rents $r_{ct}(h)$ or user costs of owner occupation $c_{ct}(h)$ to reside in a house. The model is flexible in its treatment of rents and user costs and admits a variety of relationships between the two. For instance, the often invoked assumption of rental equivalence arises as a special case when $r_{ct}(h) = c_{ct}(h)$. I depart from the convention of assuming rental equivalence because it is only guaranteed to hold in frictionless settings in which households are indifferent between renting and owning houses of the same quality. Given the central role of credit constraints in this paper, I instead allow rents to vary from user costs of owner-occupation. Differences in rents and user costs allows the model to generate rich trade-offs between ownership and house quality. However, since user costs are unobserved, this added flexibility comes at the cost of having to estimate user costs. I exploit the equilibrium implications of the structural model to recover the unobserved costs as the shadow costs of owner-occupation.

Optimal Choice of Quality by Tenure— The household's choice of quality if it were to rent is given by the level $h^r_{ict}(y)$ which solves the first order condition

$$\frac{\frac{\partial}{\partial h} u(h, y - r_{ct}(h) \mid \theta^h_i)}{\frac{\partial}{\partial z} u(h, y - r_{ct}(h) \mid \theta^h_i)} = r'_{ct}(h) \tag{3}$$

When households do not face binding credit constraints, their choice of quality if they were to own is given by the level $h^{o^*}_{ict}(y)$ which solves the first order condition

$$\frac{\frac{\partial}{\partial h} u(h, y - c_{ct}(h) \mid \theta^h_i)}{\frac{\partial}{\partial z} u(h, y - c_{ct}(h) \mid \theta^h_i)} = c'_{ct}(h) \tag{4}$$

When rents equal user costs, the two first order conditions coincide. Therefore when costs do not vary by tenure, preferences for quality also do not vary by tenure. On the other hand, differences in rents and user costs generates differences in the optimal levels of quality by tenure. Consider the case when rents are more expensive than the user costs of owning. When this occurs, the marginal rate of substitution for house quality h in Equation 3 is lower than the marginal rate of substitution for h in Equation 4. This implies that the preferred quality will be higher when the household owns, $h^{o^*}_{ict}(y) > h^r_{ict}(y)$.

When households do not face binding homeownership credit constraints, their choice of housing exactly reflect their preferences for quality. Therefore, any difference in the preference for quality by tenure is reflected one-for-one in the choice of quality by tenure. However, this does not apply when households are credit constrained. A household is defined as being credit-constrained if the quality it prefers to owner-occupy is priced above the maximum price it can afford given its endowment, $p_{ct}(h^{o^*}_{ict}(y)) > \overline{p}_{ct}(a, y)$. Let $p^{-1}_{ct}\left(\overline{p}_{ct}(a, y)\right)$ represent the highest level of quality that the household can afford to owner-occupy. Given this definition, we can define the quality that a household would actually choose if it decided to owner-occupy as:

$$h^{o}_{ict}(a, y) = \begin{cases} h^{o^*}_{ict}(y) & \text{if } p_{ct}(h^{o^*}_{ict}(y)) \leq \overline{p}_{ct}(a, y) \\ p^{-1}_{ct}\left(\overline{p}_{ct}(a, y)\right) & \text{otherwise} \end{cases} \tag{5}$$

Tenure Choice—Households choose the tenure which maximizes their utility. Let $u^r_{ict}(y)$ and $u^o_{ict}(a, y)$ be the household's utility from renting and owning their constrained optimal quality respectively:

$$u^r_{ict}(y) \equiv u\left(h^r_{ict}(y), \quad y - r_{ct}(h^r_{ict}(y)) \quad \mid \theta^h_i \right) \tag{6}$$

$$u^o_{ict}(a, y) \equiv u\left(h^o_{ict}(a, y), \quad y - c_{ct}(h^o_{ict}(a, y)) \mid \theta^h_i \right) \tag{7}$$

A household chooses to owner-occupy $h^o_{ict}(a, y)$ if it derives greater utility from doing so:

$$u^o_{ict}(a, y) + \beta^{own}_i + \varepsilon_{iown} \geq u^r_{ict}(y) + \varepsilon_{irent} \tag{8}$$

It rents quality $h^r_{ict}(a, y)$ otherwise. The decision to rent or own depends on a quality-ownership trade-off encapsulated in Equations 5-8. The trade-off arises when households are sufficiently credit constrained. Recall that $p_{ct}(h^r_{ict}(y))$ is the price of the house that the household would rent. When $p_{ct}(h^r_{ict}(y)) \leq \overline{p}_{ct}(a, y)$ there is no trade-off because the household can afford to own a house that is atleast as good as the house it would rent. The household need not forgo quality to become an owner-occupier. However, a trade-off arises when the maximum price the household can pay falls below the price of the house it would rent, $p_{ct}(h^r_{ict}(y)) > \overline{p}_{ct}(a, y)$. When this occurs, the household cannot afford to own the house it would rent. It must forgo $h^r_{ict}(y) - h^o_{ict}(a, y)$ units of quality to become an owner-occupier. Therefore, when households are sufficiently credit constrained, the choice of tenure depends on the extent to which they value ownership over quality.

The household's preference for quality depends on the utility function $u(h, z \mid \theta^h_i)$. I defer a discussion of u to Section 5. The value from owning a property depends on cost differences by tenure and tastes for ownership. Cost differences influence the value of ownership through the budget constraint. When renting is more expensive, households that do not face binding credit constraints can increase both their housing and non-housing consumption by owning instead of renting. This increases the value of owning. The taste for tenure impacts the value of owning by shifting the household's indifference curves associated with owning relative to renting. The taste for tenure has a deterministic component β_i^τ and an idiosyncratic component $\varepsilon_{i\tau}$. These tastes capture preferences for tenure that arise due to factors not related to user cost, prices, or house quality. For instance, some households may prefer to have the option to renovate or modify their dwelling. Others may prefer the stability that comes from not having to renew a lease every year. The taste terms β_i^τ and $\varepsilon_{i\tau}$ capture preferences for tenure which arise due to such considerations.

I assume that the idiosyncratic preferences for tenure $\varepsilon_{i\tau}$ are independently and identically distributed Type I Extreme Value with variance σ_{logit}. Moreover I normalize β_i^{rent} to zero. Under this normalization β_i^{own} captures differences between households' non-idiosyncratic tastes for owning and renting, $\beta_i^{own} - \beta_i^{rent}$. Given the assumption on ε, the share of type i households with endowments (a, y) who owner-occupy houses of quality $h_{ict}^o(a, y)$ equals $o_{ict}(a, y)$:

$$o_{ict}(a,y) = \frac{\exp\{(u_{ict}^o(a,y) + \beta_i^{own})/\sigma_{logit}\}}{\exp\{u_{ict}^r(y)/\sigma_{logit}\} + \exp\{(u_{ict}^o(a,y) + \beta_i^{own})/\sigma_{logit}\}} \tag{9}$$

The remaining share $1 - o_{ict}(a, y)$ choose to rent $h_{ict}^r(y)$. The variance parameter σ_{logit} determines the sensitivity of tenure choice to differences in the non-idiosyncratic component of utility across tenure. As σ_{logit} converges to zero, the tenure choice becomes increasingly deterministic and the tenure share converges to the indicator function $\mathbb{1}\left(u_{ict}^o(a,y) + \beta_i^{own} \geq u_{ict}^r(y)\right)$. Conversely, as σ_{logit} converges to ∞, the choice of tenure becomes increasingly random and the tenure share converges to 1/2.

Demand for Quality—We can obtain the aggregate demand for quality by owner-occupiers and renters of unobserved type i by integrating over the joint distribution of income and assets for the unobserved type:

$$D_{ict}^o(h, p_{ct}, r_{ct}) = \int_{a,y} o_{ict}(a,y) \times \mathbb{1}\left(h_{ict}^o(a,y) \leq h\right) \, dF_{ict}(a,y) \tag{10}$$

$$D_{ict}^r(h, p_{ct}, r_{ct}) = \int_{a,y} (1 - o_{ict}(a,y)) \times \mathbb{1}\left(h_{ict}^r(y) \leq h\right) \, dF_{ict}(a,y) \tag{11}$$

To get the demand by tenure for households with observed characteristic x, we can weight Equations 10 and 11 by the conditional distribution of types given the observed characteristic and sum over all types:

$$D^o_{ct}(h, p_{ct}, r_{ct} \mid x) = \sum_i D^o_{ict}(h, p_{ct}, r_{ct}) \times \pi_{i|x} \tag{12}$$

$$D^r_{ct}(h, p_{ct}, r_{ct} \mid x) = \sum_i D^o_{ict}(h, p_{ct}, r_{ct}) \times \pi_{i|x} \tag{13}$$

Finally, weighting Equations 12 and 13 by the share of households with characteristics x and summing over all x gives the aggregate demand for quality h by tenure.

$$D^o_{ct}(h, p_{ct}, r_{ct}) = \sum_x D^o_{ct}(h, p_{ct}, r_{ct} \mid x) \times s_{ct}(x) \tag{14}$$

$$D^r_{ct}(h, p_{ct}, r_{ct}) = \sum_x D^r_{ct}(h, p_{ct}, r_{ct} \mid x) \times s_{ct}(x) \tag{15}$$

4.3 Landlord's Problem

Each market has a distribution of landlords with endowments drawn from $G_{ct}(a)$. Landlords are otherwise identical. Each landlord owns and rents out a single house. They choose the optimal house quality based on the conditional expected profits from buying and renting out a house in the current period and selling it the following year given their endowment. The expected profit depends on rents, costs, and expected price appreciation $\pi_{ct}(h) = E_t\left[p_{ct+1}(h)/p_{ct}(h)\right]$.

Landlords finance their house purchase using a mix of assets and loans. The financing mix determines the loan-to-value ratio $\phi^a_{ct}(h)$. The landlord's costs is variable and proportional to the current price of the property. The variable cost depends on the rate of physical depreciation of the house δ_{ct}, taxes tax_{ct}, the opportunity cost of cash i^o_{ct}, and the opportunity cost of borrowing funds $i^b_{ct}(\phi)$ which in turn depends on the loan-to-value ratio. Let $\mu_{ct}(h, a)$ be the variable cost net of expected capital gains associated with quality h for landlords with endowment a.

$$\mu_{ct}(h, a) = 1 + \delta_{ct} + tax_{ct} - \pi_{ct}(h) + \left(1 - \phi^a_{ct}(h)\right) \times i^o_{ct} + \phi^a_{ct}(h) \times i^b_{ct}(\phi^a_{ct}(h))$$

The variable cost $\mu_{ct}(h, a)$ is increasing in the cost of purchasing a property, physical depreciation, taxes, and the opportunity cost of capital. It is decreasing in expected capital gains. An increase in the loan-to-value ratio ϕ increases the landlord's exposure to the opportunity cost of borrowing $i^b_{ct}(\phi)$ and reduces exposure to the opportunity cost of cash

i^o_{ct}. I assume that $i^b_{ct}(\phi) > i^o_{ct}$ and that $i^b_{ct}(\phi)$ is increasing and convex in the loan-to-value ratio. Landlords therefore prefer financing their purchases with cash and this preference becomes more pronounced as the loan-to-value ratio increases. The landlord only borrows when the price of the house is greater than the value of their assets. This implies that the loan-to-value ratio equals $\phi^a_{ct}(h) = 1 - a/p_{ct}(h)$ for landlords who need to borrow. Since the cost of borrowing $i^b_{ct}(\phi)$ is convex in the loan-to-value ratio ϕ, the overall variable cost is also convex in ϕ. It follows that the derivative of the variable cost is decreasing in a because $\phi^a_{ct}(h)$ is decreasing in a.

Landlord's Choice—The expected profit from houses of quality h for landlords with endowments a is given by

$$\Pi_{ct}(h \mid a) = r_{ct}(h) - \mu_{ct}(h,a)p_{ct}(h) \tag{16}$$

The first order condition for the landlord's optimal choice of quality is

$$\frac{\partial r_{ct}(h)}{\partial h} = \frac{\partial p_{ct}(h)}{\partial h}\mu_{ct}(h,a) + \frac{\partial \mu_{ct}(h,a)}{\partial h}p_{ct}(h) \tag{17}$$

Let $s_{ct}(a, r_{ct}, p_{ct})$ be the house quality which satisfies the first order condition for landlords with endowment a. Then the aggregate supply of rentals of quality h is given by

$$S_{ct}(h \mid r_{ct}, p_{ct}) = \int_a \mathbb{1}\{s_{ct}(a, r_{ct}, p_{ct}) \leq h\}\, dG_{ct}(a) \tag{18}$$

The derivative of the variable cost is decreasing in a as discussed above. This implies that marginal profits are increasing in a. This is easily verified by the second derivative of the profit function which is positive:

$$\frac{\partial^2 \Pi_{ct}(h \mid a)}{\partial h \partial a} = -\left(\frac{\partial p_{ct}(h)}{\partial h}\frac{\partial \mu_{ct}(h,a)}{\partial a} + \frac{\partial^2 \mu_{ct}(h,a)}{\partial h \partial a}p_{ct}(h)\right) > 0 \tag{19}$$

When marginal profits are increasing in a, the indirect profit function satisfies a single-crossing property in the quality-rent space. Therefore there is positive sorting between landlords and house quality based on endowments: i.e., the landlord's choice of housing quality $s_{ct}(a, r_{ct}, p_{ct})$ is increasing in their endowment a. This allows us to define the inverse function $a_{ct}(h) = s^{-1}_{ct}(a, r_{ct}, p_{ct})$ which gives the endowment of the landlord who rents out a house of quality h. Evaluating the landlord's first order condition at the assigned quality-endowment pair $(h, a_{ct}(h))$ gives the gradient of the rent function with respect to quality at h:

$$\frac{\partial r_{ct}(h)}{\partial h} = \frac{\partial p_{ct}(h)}{\partial h}\mu_{ct}(h, a_{ct}(h)) + \frac{\partial \mu_{ct}(h, a_{ct}(h))}{\partial h} p_{ct}(h) \tag{20}$$

With the rent gradient in hand, we can compute the level of rents for a given quality level by integrating over the distribution of house quality

$$\begin{aligned} r_{ct}(h_j) &= r_{ct}(0) + \int_{h=0}^{h_j} \frac{\partial r_{ct}(h)}{\partial h} dh \\ &= r_{ct}(0) + \int_{h=0}^{h_j} \frac{\partial p_{ct}(h)}{\partial h}\mu_{ct}(h, a_{ct}(h)) + \frac{\partial \mu_{ct}(h, a_{ct}(h))}{\partial h} p_{ct}(h) \;\; dh \end{aligned} \tag{21}$$

Equation 21 shows that the level of rents depends on the rent derivatives and a constant $r_{ct}(0)$ which represents the baseline level of rents in the market.

Rent-to-price Ratio and Exposure to the Cost of Borrowing—The landlord's model establishes a link between the shape of the rent-to-price function in a city and the distribution of asset endowments for the landlord. The model predicts that the rent-quality gradient will be shallower in cities where landlords' have relatively high asset endowments compared to house prices in the market. To see this, differentiate the rent derivative in Equation 20 with respect to the landlord's endowment:

$$\frac{\partial^2 r_{ct}(h)}{\partial h \partial a_{ct}(h)} = \frac{\partial p_{ct}(h)}{\partial h}\frac{\partial \mu_{ct}(h, a_{ct}(h))}{\partial a_{ct}(h)} + \frac{\partial^2 \mu_{ct}(h, a_{ct}(h))}{\partial h \partial a_{ct}(h)} p_{ct}(h) < 0 \tag{22}$$

The cross derivative in Equation 22 is negative because landlord costs are increasing and convex in the loan-to-value ratio, and the loan-to-value ratio is decreasing in the landlord's endowment. A negative value for the cross-derivative implies that an increase in the landlord's endowment makes the rent-quality gradient shallower holding fixed the price-quality gradient. This relative decrease in the rent-quality gradient compared to the price-quality gradient generates a decrease in the slope of the rent-to-price function over quality. The model thus generates a correlation between the gradient of the rent-to-price function and the relative distribution of landlord asset endowments in a city.

4.4 Housing Supply

We have previously derived household demand for housing services by tenure and quality, as well as landlord supply of rental services by quality. To complete the model, it is essential to specify the supply of housing quality across different markets. Let m and m'

represent two distinct markets. I express the relative housing supply by quality between these two markets using the following recursive relationship:

$$N_{m'}(h) = f^{n}\left(N_m(h), p_m, p_{m'}, \kappa_{m'}, \kappa_m \mid \zeta\right) \tag{23}$$

Here, the relative stock of housing of quality h in markets m and m' is influenced by the relative price levels, p_m and $p_{m'}$, and the relative population sizes, $\kappa_{m'}$ and κ_m, in the two markets. The parameter ζ captures adjustment costs that prevent the housing stock from perfectly responding to price and population differences across markets.

Suppose $N_0(h)$ represents the housing stock in a baseline market. The recursive specification allows us to compute the housing stock in all other markets, provided $N_0(h)$ is known. This property proves useful in the estimation process, as $N_0(h)$ can be directly measured in the data. This normalization enables me to recover the quality distribution in all other markets. I elaborate on this approach in Section 5.

4.5 Equilibrium

Equilibrium in market ct is characterized by the vector of rents and prices (r_{ct}, p_{ct}) which satisfy the following market clearing conditions for each quality level h:

$$N_{ct}(h \mid N_{ct-1}, p_{ct}, p_{ct-1}) = D^{o}_{ct}(h, p_{ct}, r_{ct}) + D^{r}_{ct}(h, p_{ct}, r_{ct}) \tag{24}$$

$$D^{r}_{ct}(h, p_{ct}, r_{ct}) = S_{ct}(h, p_{ct}, r_{ct}) \tag{25}$$

Equation 24 states that the overall demand for housing of quality h by renters and owner-occupiers must equal the stock of such housing. Equation 25 states that the demand for quality h by renters must equal the supply of rental services in the quality segment by landlords.

5 Estimation and Identification

5.1 Discretizing the Quality Space

The estimation involves solving the equilibrium conditions in Equation 24 and 25 numerically. Since prices and rents are non-linear over quality, I need to numerically solve for the value of prices and rents at each quality level h. I discretize the quality space into a grid of J points $h_1, ..., h_J$ to make the estimation feasible. The discretization requires that we work with the discrete analogs of the choices and quantities defined in Section 4. I discuss the discrete analogs below.

I start with the landlord's case because it is more straightforward. Optimal behaviour of the landlord implies that there exist J asset thresholds $a_{1ct} < ... < a_{Jct}$ such that all rental properties of quality h_j are operated by landlords with assets in the range $a_{jct} \leq a < a_{j+1ct}$. The asset threshold a_{jct} is implicitly defined by the asset value which makes the landlord indifferent between qualities h_j and h_{j+1}: $\Pi_{ct}(h_j \mid a) = \Pi_{ct}(h_{j+1} \mid a)$. Given the thresholds, the supply of rental properties in quality segment h_j is given by $G_{ct}(a_{j+1ct}) - G_{ct}(a_{jct})$, the share of landlords with assets between the cutoff points a_{jct} and a_{j+1ct}. A similar approach is used by Epple et al. (2020) to define household demand for house quality.

In our case, the household's choice is less straightforward as households choose both tenure and quality subject to a homeownership credit constraint which depends on both income and assets. These additional features make it difficult to identify endowment cutoffs which neatly segment households into different quality bins by tenure. To overcome this issue, I simulate households and aggregate their choices to obtain simulated demand functions by tenure and quality bin. For each unobserved household type i in each city and year, I draw n^{sim} asset and income values $\left(a^l_{ict}, y^l_{ict}\right)$ from the joint distribution $F_{ict}(a, y)$. For each simulated household $l = 1, ..., n^{sim}$, I solve the discrete analog of the first order conditions in Equations 3 and 4 to identify each simulated household's optimal quality by tenure:

$$h^r_{ict}(y) = \underset{h_j}{\operatorname{argmax}} \quad u(h_j, y - r_{ct}(h_j) \mid \theta^h_i) \tag{26}$$

$$h^o_{ict}(a, y) = \underset{h_j : p_{ct}(h_j) \leq \overline{p}_{ct}(a,y)}{\operatorname{argmax}} \quad u(h_j, y - c_{ct}(h_j) \mid \theta^h_i) \tag{27}$$

I then compute the tenure shares in Equation 9 based on the optimal quality levels $h^o_{ict}(a^l, y^l)$ and $h^r_{ict}(y^l)$. Finally, I evaluate the integrals in the demand functions in Equations 10–15 by numerically integrating over the simulated households given their housing choices. This gives the simulated demand for each quality bin by tenure.

5.2 Parametrizing the Model

We have yet to specify the household's indirect utility $u(h, z \mid \theta^h_i)$ and unobserved user costs of owner-occupation $c_{ct}(h)$. We also need to specify a function to approximate the landlord's variable cost curves over different loan-to-value ratios. I now specify each in turn.

Household Utility from Consumption—Following Epple et al. (2020), I define the indirect utility from living in a house of quality h at cost $r(h)$ given income y as

$$u(h, z \mid \theta_i^h) = \ln\left(1 - \phi_i(h + \eta_i)^{\gamma_i}\right) + \frac{1}{\alpha_i}\ln(y - r(h)) \tag{28}$$

where $\alpha_i, \eta_i, \phi_i > 0$ and $\gamma_i < 0$. The utility function is well-defined when the following constraints are satisfied $1 - \phi_i(h + \eta_i)^{\gamma_i} > 0$ and $y - r_{ct}(h) > 0$. The estimation imposes both constraints. The parameter vector $\theta_i^h = (\alpha_i, \phi_i, \eta_i, \gamma_i)$ summarizes the taste for housing quality for a household of unobserved type i. The term α_i captures the household's relative preference for housing services compared to non-housing consumption. The term $1 - \phi_i(h + \eta_i)^{\gamma_i}$ may be viewed as the level of housing services the household generates from housing quality h.

Household Unobserved Usercosts of Owner-Occupation—I specify the household's unobserved user cost of owner-occupation as a linear function in price:

$$c_{ct}(h) = \omega_0 + \omega_1 p_{ct}(h) \tag{29}$$

where ω_0 represents the baseline user cost the household needs to pay for being a homeowner. ω_1 is the amount by which user costs increase for a unit change in prices. While this specification is linear in prices, it admits non-linearities in user costs over quality.

Landlord's Variable Costs—We need to estimate the landlord's variable costs as a function of the loan-to-value ratio. As discussed in Section 4.3, the variable cost plays an important role in the model. When variable costs are convex in the loan-to-value ratio, there is positive sorting between landlords and house quality based on landlord assets. However, it is challenging to directly measure the variable cost since we do not observe many of its components including physical depreciation and the opportunity cost of capital. I instead treat the landlord's cost function as a latent object to be estimated. To approximate the landlord's variable costs net of capital gains, I use the following exponential function with parameters λ_1, λ_2 and λ_3:

$$vc_{ct}(h \mid a) = \exp\left\{\lambda_1 + \lambda_2\left(\lambda_3 + 1 - \frac{a}{p_{ct}(h)}\right)\right\} p_{ct}(h) \tag{30}$$

where λ_1 influences the baseline variable cost which depends on the opportunity cost of funds, physical depreciation, taxes, and expected capital gains. The term $1 - a/p_{ct}(h)$

represents the loan-to-value ratio on a house of quality h for landlords with endowment a. The coefficient λ_2 captures the impact of an increase in the loan-to-value ratio on the variable cost. The marginal cost $\partial vc/\partial h$ is increasing in λ_2 when $a/p_{ct}(h) < 1+\lambda_3$. I impose $a/p_{ct}(h) < 1+\lambda_3$ in the estimation. When $\lambda_3 = 0$, the variable cost is decreasing in a until $a = p_{ct}(h)$ and is constant for higher levels of assets. Positive values for λ_3 allows the variable cost to decrease in assets past this point.

The landlord's choice is jointly determined by their participation constraint and the discrete analog of their first order condition. The participation constraint is given by:

$$a \geq \left(1+\lambda_3+\frac{\lambda_1}{\lambda_2}-\frac{1}{\lambda_2}\ln\left(\frac{r_{ct}(h_j)}{p_{ct}(h_j)}\right)\right)p_{ct}(h_j) \tag{31}$$

where the right hand side of Equation 31 defines the minimum level of assets at which it is profitable for a landlord to operate houses of quality h_j. The participation threshold is increasing in λ_1 and λ_3. The participation threshold is increasing in λ_2 whenever the rent is greater than $\exp\{\lambda_1\}p_{ct}(h_j)$ which represents the lowest possible cost a landlord can face. This condition is always satisfied as landlords do not operate below the break even point.

As discussed in Section 5.2, the asset value which satisfies the indifference condition $\Pi_{ct}(h_j \mid a) = \Pi_{ct}(h_{j+1} \mid a)$ represents the cut-off a_{jct} at which the landlord is indifferent between renting out houses of quality h_j and h_{j+1}. Given the definition of variable costs in Equation 30, the indifference condition becomes:

$$\begin{aligned} & r_{ct}(h_{j+1}) - r_{ct}(h_j) = \\ & \exp\left\{\lambda_1+\lambda_2\left(\lambda_3+1-\frac{a}{p_{ct}(h_{j+1})}\right)\right\}p_{ct}(h_{j+1}) - \exp\left\{\lambda_1+\lambda_2\left(\lambda_3+1-\frac{a}{p_{ct}(h_j)}\right)\right\}p_{ct}(h_j) \end{aligned} \tag{32}$$

The solution to this indifference condition represents the asset cut-off a_{jct}. It is not possible to explicitly solve for the asset value which satisfies Equation 32. However, we can solve for it numerically provided a unique solution exists. To verify that a unique solution does exist, consider the the right hand side of Equation 32. We know that it intersects with the positive constant $r_{ct}(h_{j+1}) - r_{ct}(h_j)$ at most once because it is strictly decreasing in a.[10]

[10]This difference on the right hand side is the discrete analog of the derivative of variable costs with respect to quality. We know this difference is increasing in $\lambda_3 + 1 - a/p_{ct}(h_j)$ due to the convexity of the variable cost function. Since $\lambda_3 + 1 - a/p_{ct}(h_j)$ is decreasing in a, it follows that the difference in the right hand side is also strictly decreasing in a.

We also know that it intersects with $r_{ct}(h_{j+1}) - r_{ct}(h_j)$ at least once because it spans the entire non-negative range from zero to ∞. This verifies that a solution to Equation 32 exists and is unique.

Evolution of the Housing Stock—Let m and m' index two different markets. Similar to Epple et al. (2020), I specify the evolution of the housing stock by quality segment by the following constant elasticity function:

$$\frac{N_{m'}(h_j)}{N_m(h_j)} = \nu_{m'} \left(\frac{\kappa_{m'}}{\kappa_m}\right)^{h_j \times \zeta_{pop}} \tag{33}$$

where $\nu_{m'}$ is a constant to ensure that the housing stock in m' sums to one. Equation 33 implies that a one percent increase in the relative population density $\kappa_{m'}/\kappa_m$ generates a $h_j \times \zeta_{pop}$ percent change in $N_{m'}(h_j)/N_m(h_j)$, the housing stock in m' relative to m. When $\zeta_{pop} > 0$, the relative supply of high quality houses is greater in denser cities. In contrast, when $\zeta_{pop} < 0$, the relative supply of high quality houses is greater in less dense cities.

5.3 Approximating the Empirical Distribution of Endowments, Prices and Rents

Endowments, prices, and rents are important inputs in the structural estimation. I model the marginal distributions of prices, rents, household income and household and landlord assets using a generalized beta distribution of the second kind, henceforth referred to as GB2. The GB2, also known as the generalized beta prime distribution, is a flexible four parameter distribution which nests many of the distributions commonly used to model income, assets and prices including the log normal, Singh-Maddala, and generalized gamma distributions (McDonald, 1984; McDonald and Xu, 1995). I use a Student-t copula to model the dependence between household income and assets. The joint distribution of household endowments is allowed to vary by market and demographic group, while the distribution of landlord assets, as well as prices and rents, varies by market. The estimation is conducted using Maximum Likelihood.

5.4 Solving for the Equilibrium

Since house quality is latent, I start by normalizing the J levels of house quality based on the house price distribution in a base market. I define the base market to be London in 2014. I divide the house price distribution in the base market into J bins. I then define quality h_j to equal the price at the j^{th} cut-off. Given the normalization the equilibrium price simply equals the quality, $p_{00}(h_j) = h_j$ in the base market. While prices are linear in quality in the base market due to the normalization, it need not be so in other markets. We can also directly measure the equilibrium rent $r_{00}(h_j)$, overall housing stock $N_{00}(h_j)$, stock of rentals $N^r_{00}(h_j)$, and stock of owner-occupied houses $N^o_{00}(h_j)$ in each quality segment of the base market. We therefore do not need to solve for the equilibrium in the base market.

We need to solve for the equilibrium in all other markets. Given the normalization of the base market, we can use the recursion in Equation 33 to compute the housing stock in London in 2015 and in all other cities in 2014. Suppose we have computed the housing stock based on this recursion for some market (c, t). We can then solve for the vector of rents and prices which satisfy the equilibrium conditions in Equations 24 and 25 for market (c, t). The equilibrium in market (c, t) in turn provides the initial conditions required to recursively compute the housing stock for market $(c, t + 1)$. Repeating this process yields the housing stock and the equilibrium in all markets.

5.5 Method of Moments Estimator

I estimate the model using a method of moments estimator.

Parameters—We need to estimate three sets of parameters which shape the choices of households, landlords, and the builder. First, households belong to type i with probability $p_{i|x}$ and their preferences over housing quality and tenure depend on the parameters $\theta^h_i = (\eta_i, \phi_i, \gamma_i, \alpha_i)$ and $\theta^\tau_i = (\beta^{own}_i, \sigma_{logit})$ respectively. Moreover households' user costs of owner-occupation depend on $\omega = (\omega_0, \omega_1)$ and they face credit constraints which depend on the mortgage thresholds $\phi = (\phi^{lti}, \phi^{ltv})$. In the estimation, I fix $\phi^{ltv} = 0.9$ and only estimate ϕ^{lti}. Second, the landlord's profit function depends on variable cost parameters $\lambda = (\lambda_1, \lambda_2, \lambda_3)$. Lastly, the evolution of the housing stock depends on the elasticity ζ_{pop}. Let $\Omega = (p_{i|x}, \theta^h_i, \theta^\tau_i, \omega, \phi, \lambda, \zeta_{pop})$ be the collection of all the parameters that need to be estimated.

Moments—The estimation uses the following set of moments for each city and year: (i) marginal distributions of prices and rents; (ii) distribution of rent-to-price ratio over price; (iii) correlation between income and prices and rents by age group; (iv) assets-to–house price ratio for owner-occupiers by age group; (v) ownership rates by age group; and (vi) rank-rank mapping of house quality between rented and owned properties. These moments characterize the sorting of households and landlords into tenure and quality segments in equilibrium across cities and over time.

Estimation Algorithm—The estimation imposes market clearing for each quality segment and each type of tenure in each market. The method of moments estimator selects parameters which minimize the distance between equilibrium prices, rents and allocations in the model and in the data across all cities and years. The algorithm is implemented as follows:

1. Initialize the parameter vector to Ω_0
2. Draw $N^{sim} = 5000$ households per unobserved type per market. The households are drawn from $F_{ict}(a, y)$, the joint distribution of assets and income for unobserved type i. Recall from Equation 1 that $F_{ict}(a, y)$ depends on the type probability $p_{i|x}$.
3. Normalize quality using prices in London in 2014. Directly measure the housing stock and the schedule of prices and rents by quality in the base market.
4. Calculate excess demand in the base market given the normalization:
   (a) Given the normalized quality, prices and rents, compute the simulated demand by tenure and quality segment, $D^r_{ct}(h, p_{ct}, r_{ct})$, $D^o_{ct}(h, p_{ct}, r_{ct})$, and the supply of landlord services by quality segment, $S_{ct}(h, p_{ct}, r_{ct})$.
   (b) Check if the equilibrium conditions in Equations 24 and 25 are satisfied. The base market will not clear for an arbitrary guess of Ω. Define $\text{ED}_{00}(\Omega)$ as the excess demand in the base market which needs to be minimized.
5. Solve for the equilibrium in all markets other than the base market (London in 2014):
   (a) Initialize the vector of prices p_{ct} and rents r_{ct}.
   (b) Compute the housing stock by quality segment recursively given p_{ct}
   (c) Compute the simulated household demand by tenure and quality segment
   (d) Compute the supply of rental services by landlords by quality segment
   (e) Update the price and rent vectors and repeat steps 5b–5d until the equilibrium conditions are satisfied for the market in question
6. Form moment conditions based on differences between the model-based moments and their empirical counterparts. Construct an objective function $M(\Omega)$ based on the moment conditions and excess demand in the base market $\text{ED}_{00}(\Omega)$.
7. Update Ω and repeat steps 2–6 until $M(\Omega)$ is minimized.

5.6 Identification

Separately Identifying the Parameters for Households, Landlords, and Builders—We can directly measure the housing stock in the base market given our normalization which equates house quality to prices in the base market. Since the baseline stock is normalized outside the structural model, the structural estimates of the housing supply parameter ζ do not impact the equilibrium in the base market. This exclusion restriction helps identify the parameters for the household and landlord separately from that of the builder. Variation in household and landlord endowment distributions across markets helps separately identify the parameters for households and landlords.

Identifying the Household's Parameters—The parameter governing the household's taste for tenure β_i^{own} and the idiosyncratic shock $\varepsilon_{i\tau}$ drop out of the first order conditions in Equations 3 and 4 as they do not affect the choice of quality conditional on tenure. This exclusion restriction helps identify the taste for tenure parameters separately from the user costs of owner-occupation which impact both tenure choice as well as differences in the choice of quality conditional on tenure. Additive separability of the household's taste for tenure is not necessary to generate this exclusion restriction. The exclusion restriction arises even when the taste for tenure is introduced multiplicatively in Equation 2.

The relative distribution of household endowments and house prices varies by household characteristic x, city, and time. The credit constraint parameters, ϕ^{ltv} and ϕ^{lti}, influence how ownership rates respond to variation in the distribution of household endowments relative to local house prices. This correlation helps identify the credit constraints.

The type probabilities $p_{i|x}$ are identified by two forces. First, differences in the type probabilities by observed characteristic x generate differences in aggregate demand by x. Second, as households with characteristic x become more homogenous (i.e., as $p_{i|x}$ converges to one for any given i), the correlation between income and rents converges to one for households with x. In the data the correlations are substantially lower than one implying the existence of household heterogeneity. These two forces discipline the estimates for $p_{i|x}$.

6 Empirical Results

6.1 Parameter Estimates

This section presents the parameter estimates for the household, landlord, and housing supply.

Household Preference Parameters—

Table 1: Household Preference Parameters

	Type 1	Type 2	Type 3	Type 4	Type 5	Type 6
α	18.538	5.11	4.596	1.94	0.908	3.687
η	7.447	3.013	1.75	0.652	0.618	2.456
γ	-1.349	-3.037	-2.548	-3.644	-1.377	-3.621
ϕ	5.385	5.047	0.593	6.182	2.899	19.779
$\ln \beta_{logit}$	1.488	0.495	-10.858	1.425	2.867	11.795
σ_{logit}	0.303	0.303	0.303	0.303	0.303	0.303

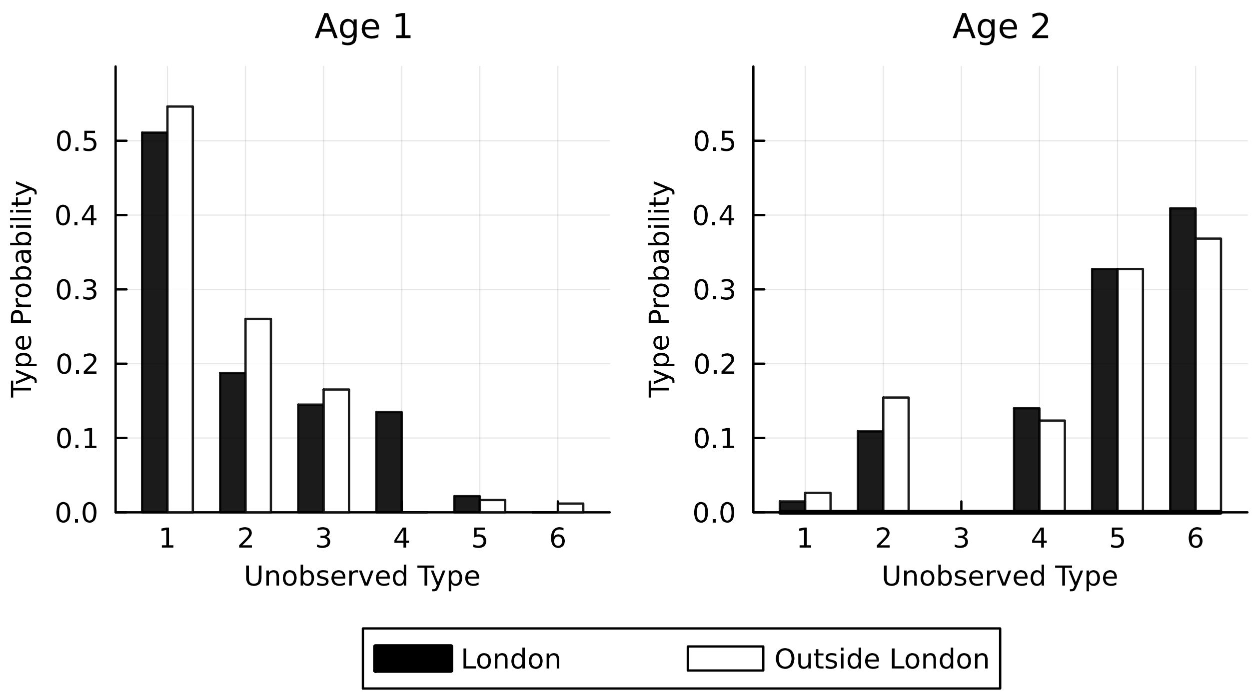
Note: This table presents estimates for the household preference parameters

The model is estimated with six unobserved household types, with type probabilities varying by age and by whether the household resides in London. Table 1 presents the utility parameters for each unobserved type, while Figure 6 plots the distribution over types by household age, both within and outside London. The types are ordered by the share of young households belonging to the type. Figure 6 reveals a strong negative correlation in type probabilities across age groups. Whereas young households are most likely to be type 1 and least likely to be type 6, the opposite is true for older households. Moreover, the type probabilities exhibit substantial heterogeneity within each age group, with over 70% of households in either age group belonging to one of two types. These patterns indicate that the demand system effectively captures rich heterogeneity in preferences among households conditional on income and assets.

The estimates for α generally decrease with the type, suggesting that younger households have a higher relative preference for housing consumption compared to non-housing consumption. Similarly, the estimates for η tend to decrease with the type, indicating that younger households derive greater utility from lower levels of housing compared to their older counterparts. The estimates for γ and ϕ also exhibit heterogeneity, however, they do not vary as systematically with the age-specific type probabilities.

Turning to households' deterministic preferences for tenure, $\ln \beta_{logit}$, Table 1 indicates that all types except type 3 prefer owner-occupation to renting. Type 3 households have a large and negative estimate for $\ln \beta_{logit}$ implying a strong preference for renting. Approximately 15% of younger households fall into this category, while none of the older households do, suggesting that all older households prefer owning over renting. Among households that prefer owning, the ownership premia tends to be significantly larger for older households. The estimate for the scale parameter σ_{logit} is smaller in magnitude than the deterministic ownership preference across all types.

Figure 6: Conditional Distribution of Households Over Unobserved Types



Note: This figure plots estimates for the share of households who belong to each unobserved type by age and location

Mortgage Thresholds— The loan-to-income threshold is estimated to be 5.469, implying that households can borrow up to approximately 5.5 times their annual income. The estimation fixes the loan-to-value ratio to 0.9 which implies that households need to make a minimum deposit of 10% on the value of the house.

Unobserved Usercosts of Owner-Occupation—The estimated parameters for the user cost functions, ω_0 and ω_1, are 0.446 and 0.029 respectively. The intercept ω_0 is ignorable considering that prices are in the hundred thousands. This implies that the user-cost-to-price ratio is effectively constant and equals 0.029.

Landlord's Parameters— Table 2 presents estimates for the parameters of the landlord's cost function by city. The estimate for λ_1 shows that baseline costs are lowest for landlords in London and highest for landlords in Birmingham. Conversely, the estimate for λ_2 suggests that landlord costs are most sensitive to borrowing in London and least sensitive in Birmingham. The estimate for λ_3 is greatest in Birmingham. Higher values of λ_3 tend to make the participation constraint more binding.

Housing Supply Parameters—The estimate for the housing supply parameter ζ_{pop} is -1.259. indicating that the quality distribution of the housing stock is strongly and

Table 2: Landlord Cost Parameters

	London	Manchester	Birmingham
λ_1	-5.361	-4.667	-4.435
λ_2	2.112	0.821	0.348
λ_3	0.901	3.481	4.321

Note: This table presents estimates of the landlord's cost parameters by city

negatively related to the density or congestion in that market. Higher quality housing is relatively less abundant in denser cities such as London.

6.2 Model Fit

This section presents results which illustrate the model's fit. In the interest of space, I only present comparisons for 2015. The fit is similar in other years.

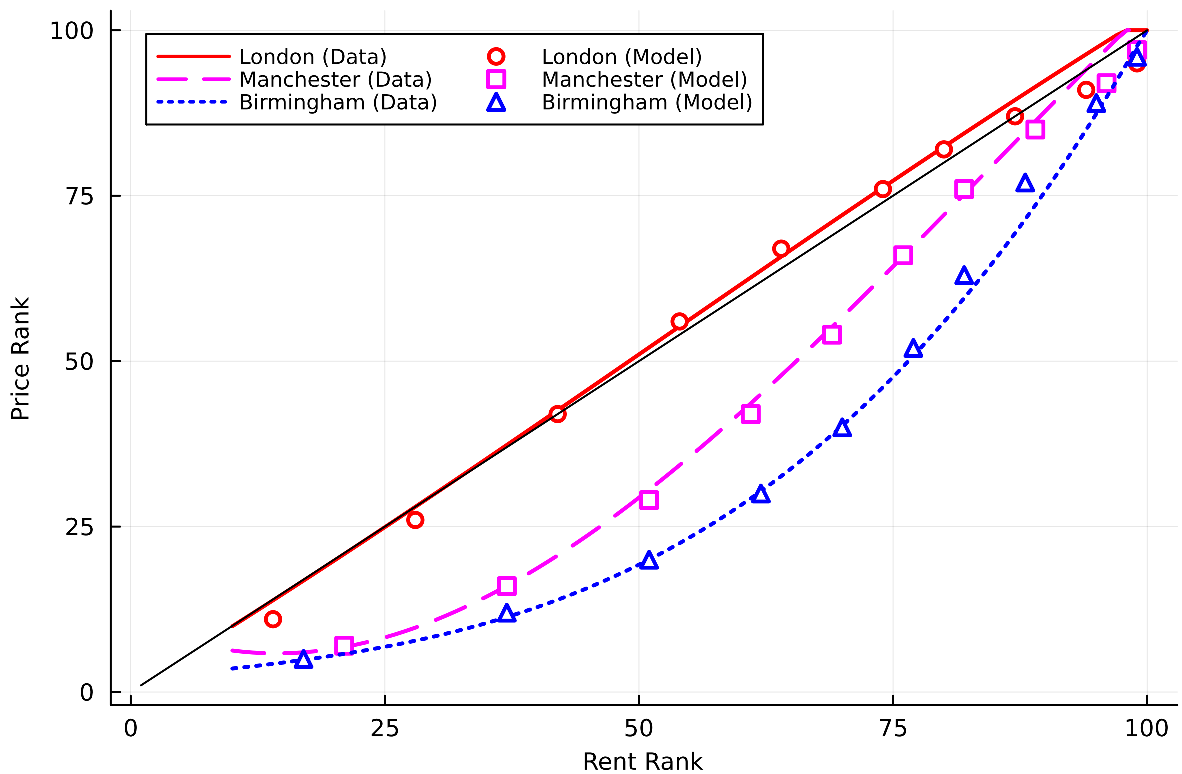
Market Clearing—The model successfully achieves market clearing. Appendix Figure A9 plots the cumulative demand and supply functions for the overall housing market and for the rental market in 2015. The figure illustrates that demand very closely corresponds to supply over the range of quality in all cities.

Selected Targeted Moments—Figure 7 compares the pattern of quality segmentation by tenure generated by the model to the pattern of segmentation estimated directly from the data. The fit is very good. The model is able to replicate differences in the patterns of segmentation across cities with a high degree of accuracy.

Appendix Figure A10 plots the cumulative distribution of prices and rents for each city. The model successfully generates the empirical price and rent distributions. Appendix Figure A11 compares ownership rates predicted by the model to rates obtained from the data. The model performs fairly well for both age groups in Manchester and Birmingham. In London, it correctly generates a higher ownership rate for older households as observed in the data, however, it understates the magnitude of this difference.

Landlord Assignment to Quality—The model predicts that landlords sort positively to quality based on their asset endowments. In the discretized version of the model with ten quality levels, the assignment is determined by nine asset cutoffs which satisfy the indifference condition in Equation 32 for each quality level. Appendix Figure A12 plots the asset cutoffs for each quality level by city. As expected, the asset cutoffs are monotonically increasing in house quality.

Figure 7: Quality Segmentation in Rental vs. Owner-Occupied Markets Across Cities

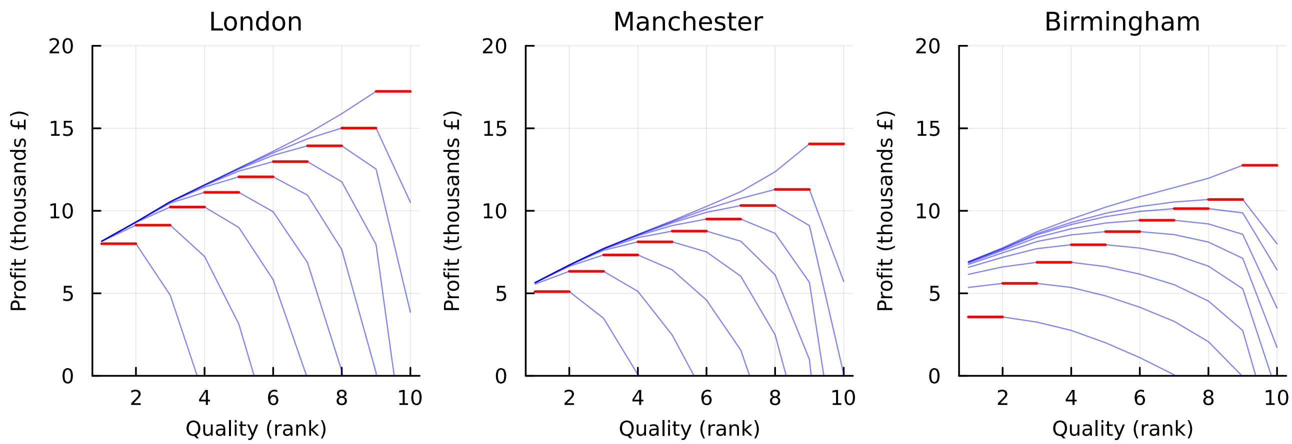


Note: This figure compares the relative distribution of quality between the rental and owner-occupied segments of the housing market across cities. The lines represent the relationship observed in the data for each city, while the scatter points show the relationship as predicted by the estimated equilibrium model.

Figure 8 presents the profit curves for landlords at each of the nine asset cutoffs. Consistent with the model's prediction, the profit curves are flat at the indifference point corresponding to each asset cutoff. This pattern illustrates how the assignment model partitions the landlord asset space into different quality levels based on profit maximization. The minimum profit for landlords operating within each quality level is represented by the red step function defined by the flat sections of each of the nine profit curves in the figure. This step function indicates that realized profits increase with the quality of the property and landlord assets.

The results presented in this section indicate that the estimated model adequately captures the key forces which determine the joint equilibrium in the rental and owner-occupied sectors of the housing market. In the following sections, I use the estimated model to analyze the role of landlord heterogeneity in generating patterns observed both within and across housing markets.

Figure 8: Estimated Landlord Profit Curves by Landlord Asset Endowment (2015)



Note: This figure shows estimated profit curves for landlords at nine different asset cutoffs. The topmost curve represents profits for landlords with assets which make them indifferent between the highest and second-highest quality levels. The bottommost curve represents profits for landlords with assets which make them indifferent between the lowest and second-lowest quality levels. The indifference point of each curve is highlighted red.

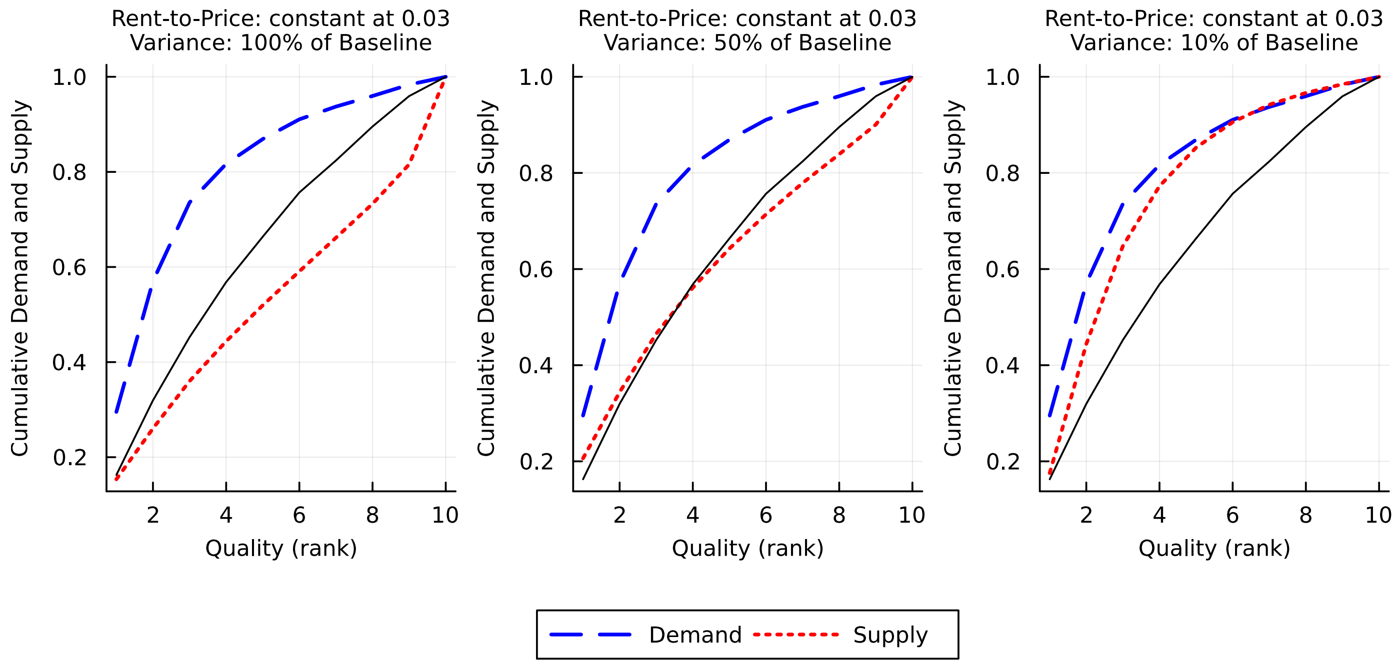
7 Counterfactuals

The model captures landlord heterogeneity in two ways—through variations in observed wealth within cities and through differences in unobserved opportunity costs of capital both within and across cities. In this section I conduct three counterfactual simulations to quantify the impact of this heterogeneity on housing markets across cities. I find three key insights from these simulations. First, in Section 7.1, I show that within-city variation in landlord wealth almost completely explains the downward slope of the rent-to-price function with respect to property prices within cities. Second, in Section 7.2, I show that cross-city differences in landlords' opportunity costs significantly drive the observed patterns of quality segmentation and rent-to-price ratios across cities. Moreover I show that these differences have important implications for the distribution of household welfare. Finally, in Section 7.3, I illustrate how the impact of landlord taxes on rents and prices varies across cities and quality segments due to differences in landlord composition.

7.1 Impact of Within-City Heterogeneity in Landlord Endowments

Several studies have found that the rent-to-price function decreases in price (Verbrugge, 2008; Verbrugge and Poole, 2010; Bracke, 2015; Halket et al., 2020) both in the UK and US. The literature has proposed several explanations including differences in rental vacancy

Figure 9: Demand and Supply of Rental Properties When the Rent-to-Price Ratio is Fixed (London 2015)

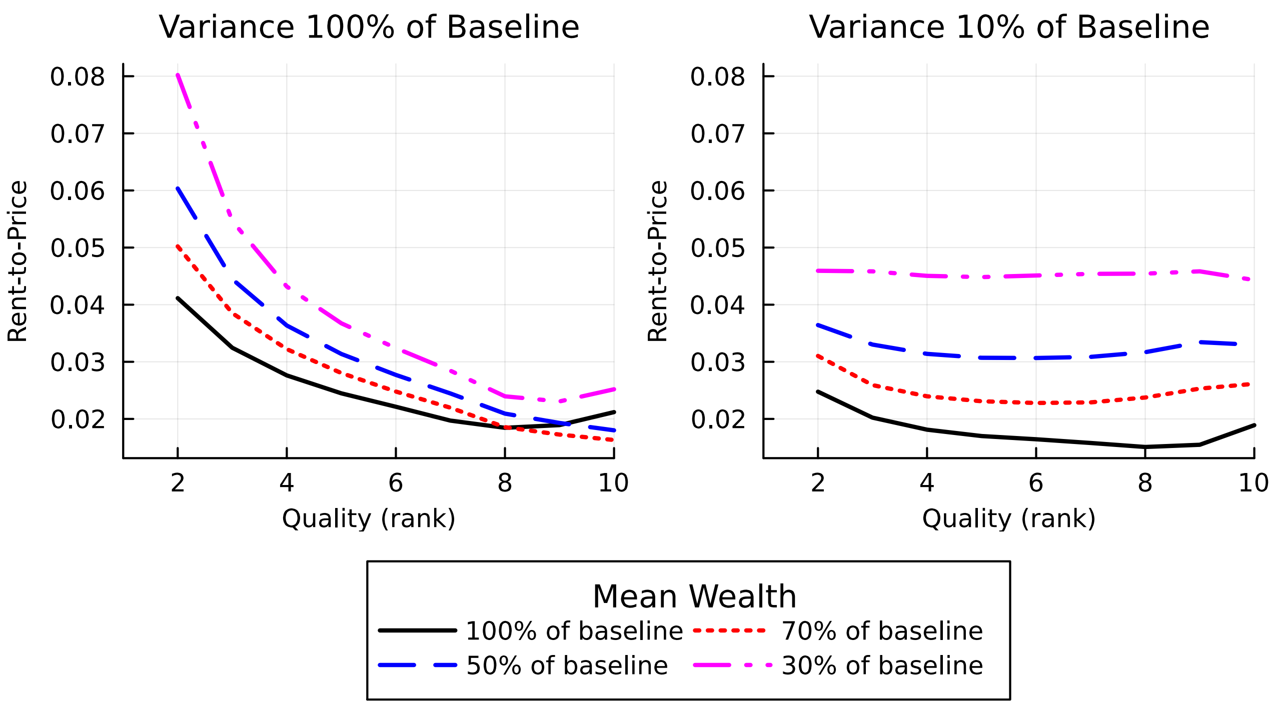


Note: This figure presents cumulative demand (blue dashed curve) and cumulative supply (red dotted curve) when the rent-to-price ratio is fixed to equal 0.03. For reference, the figure also plots the baseline equilibrium cumulative quantities when the rent-to-price ratio is allowed to vary (solid black curve). The center and right-most plots exogenously reduce the variance of the landlord's asset distribution by 50% and 90% respectively. The left-most plot leaves the landlord's asset distribution unchanged. The gap between the demand and supply curves represents the extent by which the rental market fails to clear. To fix the rent-to-price at 0.03, I set annual rents to equal three percent of the equilibrium price at baseline.

rates and unobserved building characteristics. In this section I show that heterogeneity in landlord wealth endowments naturally generate a downward sloping rent-to-price function in equilibrium, even when one controls for unobserved quality. Moreover, using counterfactual simulations I show that the downward slope of the rent-to-price function can be eliminated by reducing the heterogeneity in landlord wealth endowments.

Figure 9 plots the cumulative demand and supply of rental services over quality in London when the rent-to-price function is exogenously fixed to equal 0.03 throughout the quality distribution. To fix the rent-to-price at 0.03, I set annual rents to equal three percent of the equilibrium price at baseline. When there is excess supply in a quality segment, the cumulative supply function is steeper than the cumulative demand function. The opposite is true when there is excess demand. The leftmost plot shows that fixing the rent-to-price to 0.03 generates excess demand at lower levels of quality and excess supply at higher levels. The market does not clear because when the rent-to-price is constant, landlords

Figure 10: Equilibrium Rent-to-Price Functions Under
Counterfactual Landlord Asset Distributions (London 2015)



Note: This figure presents equilibrium rent-to-price functions for different landlord asset endowments. The figure on the left plots the rent-to-price function holding the variance fixed and varying the mean of the landlord wealth distribution. The figure on the right presents analogous rent-to-price functions with the variance reduced by 90%.

are more attracted to higher levels of quality than households. For the market to clear the attractiveness of higher quality must fall for landlords. This is achieved when the slope of the rent-to-price function with respect to price becomes more negative. This illustrates how the negative slope of the rent-to-price function arises due to market clearing forces.

To explore why the market does not clear when the rent-to-price is constant, the center and rightmost subfigures of Figure 9 plot demand and supply for rental services with the variance of the landlord asset distribution reduced by 50% and 90% respectively. The market clearing condition comes close to being satisfied when the variance of the landlord endowment distribution is reduced by 90%. These plots suggest that the rent-to-price function is steeper when landlord assets are more spread out. This can occur in the model because landlords sort positively to quality based on wealth which implies that the increased variance increases landlord wealth in the higher quality segments and reduces it in the lower segments. Since landlord costs are increasing in borrowing, this increases relative costs in the lower quality segments giving rise to a steeper rent-to-price function.

To directly test whether reducing the variance of landlord endowments indeed flattens the rent-to-price function, Figure 10 plots counterfactual rent-to-price functions obtained by resolving the housing market equilibrium under different landlord asset distributions. The figure on the left plots the rent-to-price function in London with the mean landlord wealth set to different levels but with the variance fixed to the baseline level. The figure on the right plots analogous rent-to-price functions with the variance reduced to 10% of the baseline variance. The differences are stark. The reduction in variance almost completely flattens the rent-to-price function regardless of the mean level of wealth.[11] These results illustrate the significant influence of landlord heterogeneity on the shape of the rent-to-price function over quality segments within cities.

7.2 Impact of Cross-City Heterogeneity in Landlord Costs

The estimates of the landlord's cost function in Table 2 show that at low levels of borrowing, the opportunity cost for landlords is significantly lower in London than in Birmingham.[12] I show in this section that these cross-city differences in unobserved landlord costs help explain much of the observed differences in housing markets across cities. Moreover, these cost differences have important consequences for the distribution of household welfare.

To quantify the impact of cross-city differences in landlord costs, I simulate a counterfactual in which I fix the cost function in London to equal the cost function in Birmingham and solve for the resulting equilibrium in London. Figure 11 compares equilibrium outcomes in the counterfactual scenario in London against the baseline equilibrium in London and Birmingham. Figures 11a and 11b show that the change in landlord costs causes rents to increase relative to both prices and user costs of owner-occupation in London.[13] This is consistent with landlord costs being higher in Birmingham in baseline.

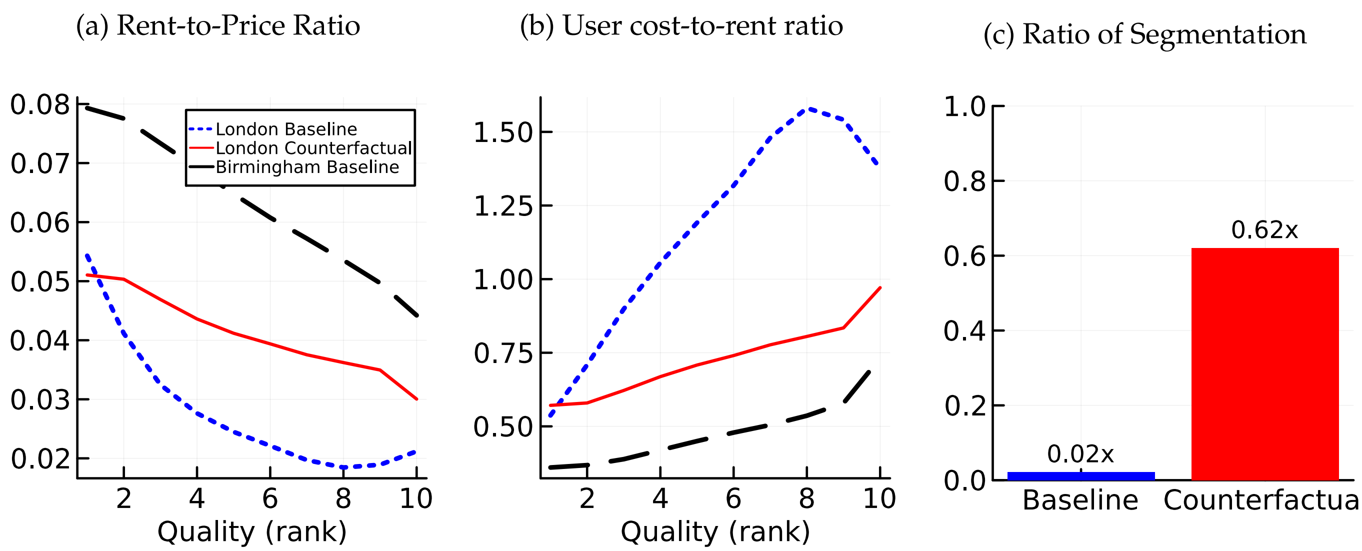
Comparing the gap between London and Birmingham in the counterfactual and baseline scenarios shows that the cross-city difference in landlord costs accounts for much of the differences in housing markets across cities. In particular, the cross-city difference in landlord costs explains up to 40% of the gap in the rent-to-price ratio between the two cities and up to 67% of the gap in the ratio of user costs of owner-occupation and rents. These

[11]The mean of the wealth distribution tends to influence the level of the rent-to-price function.

[12]For instance when landlord wealth exactly equals the house price, the opportunity cost is 40% lower in London than in Birmingham. The difference in costs is increasing in the asset to price ratio.

[13]The sensitivity of the rent-to-price and user cost-to-rent ratios to landlord costs is increasing in the quality segment. This occurs because in the model an increase in costs reduces the optimal quality for the marginal landlord in each segment. The supply effect of a cost increase is less pronounced in lower quality segments because marginal landlords in these segments can more easily be replaced by landlords who were previously operating in higher segments. This substitution effect is less pronounced in higher quality segments.

Figure 11: Impact on Rents, Prices, and Allocations
Counterfactual: Increasing Landlord Costs in London to Birmingham Levels

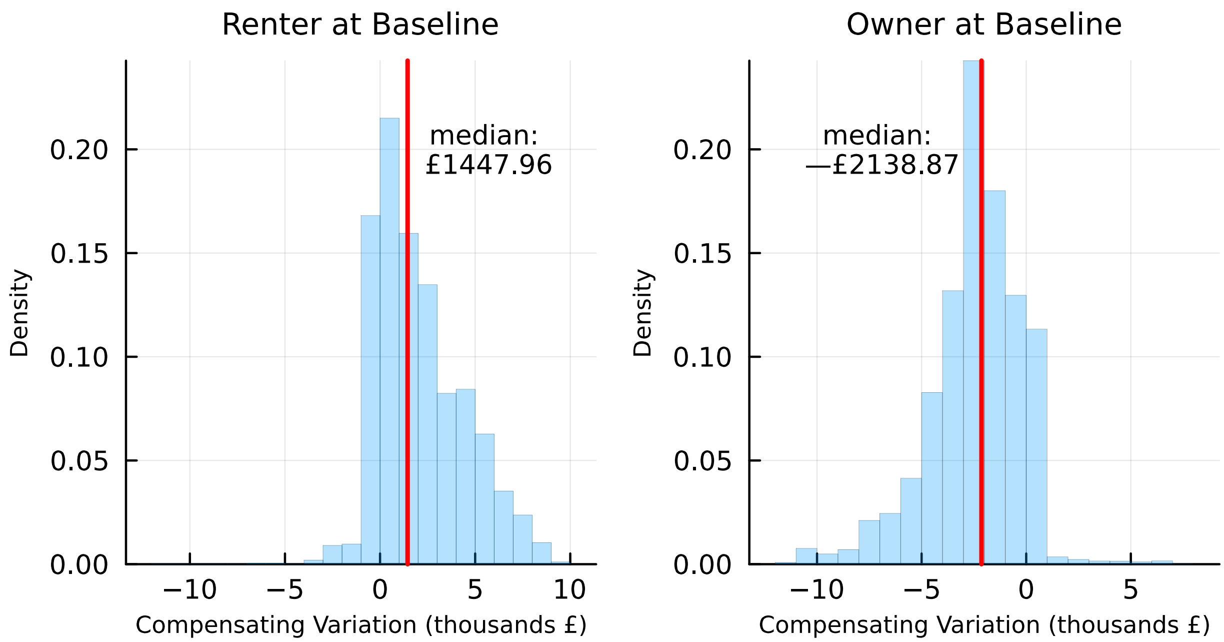


Note: This figure compares equilibrium outcomes between the counterfactual and baseline scenarios. Figure 11a compares the counterfactual rent-to-price function in London (in red) against the baseline rent-to-price functions in London (in blue) and Birmingham (in black). Similarly, Figure 11b compares the counterfactual ratio of user costs-to-rents in London (in red) against the baseline in London (in blue) and Birmingham (in black). Figure 11c compares the ratio of segmentation between London and Birmingham in the counterfactual (in red) and baseline (in blue) scenarios.

results show that the cost advantage faced by landlords in London generates important differences in rental supply between the two cities which in turn impacts the overall housing market in each city. Due to the cost advantage, landlords in London are willing to accept significantly lower rents while paying much higher prices to acquire properties. These differences in rental supply have large effects on the allocation of housing quality across the rental and owner-occupied sectors. Figure 11c shows that equalizing landlord costs between London and Birmingham reduces the cross-city gap in quality segmentation between the rental and owner-occupied sectors by 60%.

Welfare Effects of Heterogeneity in Landlords Costs—The results so far show that cross-city heterogeneity in unobserved landlord costs generate large cross-city differences in the allocation and costs of housing across the rental and owner-occupied sectors. I now explore the welfare consequences of these differences. Figure 12 plots the distribution of compensating variation for households in London when landlord costs in London are increased to Birmingham levels. The distributions are presented separately by the tenure of the household at baseline. The plots show that the welfare effects vary greatly by tenure.

Figure 12: Distribution of Welfare Effects by Tenure
Counterfactual: Increasing Landlord Costs in London to Birmingham Levels

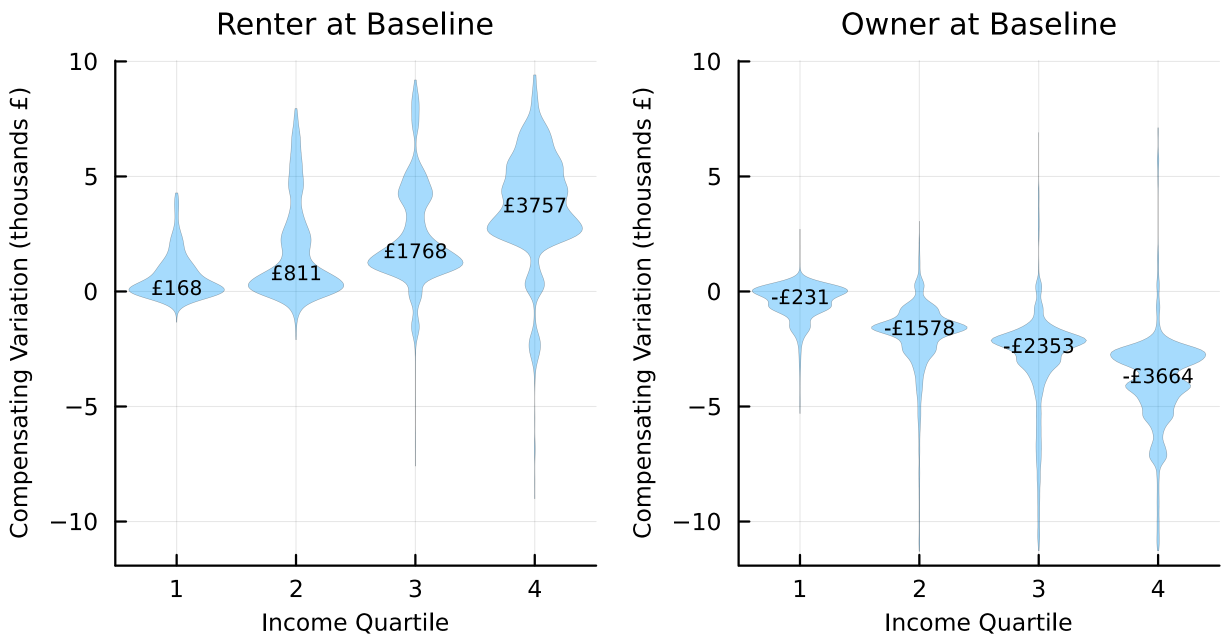


Note: This figure plots the distribution of compensating variation for households associated with increasing landlord costs in London to Birmingham levels. The plot on the left presents the compensating variation for households who rented at baseline, whereas corresponds to households who owner-occupied at baseline.

The cost advantage faced by landlords in London increase the welfare of the median renter household in London by £1,448 per year (3.6% of the median household income in London). This reflects the presence of lower costs and higher quality in the rental sector of London as shown in Figure 11. On the other hand, the median owner-occupier is worse-off by £2,139 per year (5.3% of the median household income in London), reflecting the higher costs and worse relative quality in the owner-occupied sector of London.

Figure 12 shows that there is large heterogeneity in the welfare effects even conditional on tenure. To explore the source of this heterogeneity, Figure 13 plots the distribution of compensating variations by household income quartile. The figure shows that the magnitude of the welfare effects are increasing in income for both renters and owner-occupiers. This occurs because in the model a reduction in landlord costs increases the profit-maximizing quality for marginal landlords in each quality segment. This tends to increase the relative supply of rental services in higher quality segments in turn magnifying the effect on rents and prices in those segments.

Figure 13: Heterogeneous Welfare Effects by Household Income and Tenure
Counterfactual: Increasing Landlord Costs in London to Birmingham Levels



Note: This figure plots the distribution of compensating variation for households associated with increasing landlord costs in London to Birmingham levels by household income. The plot on the left presents the compensating variation for households who rented at baseline, whereas corresponds to households who owner-occupied at baseline. The median compensating variation is noted for each distribution.

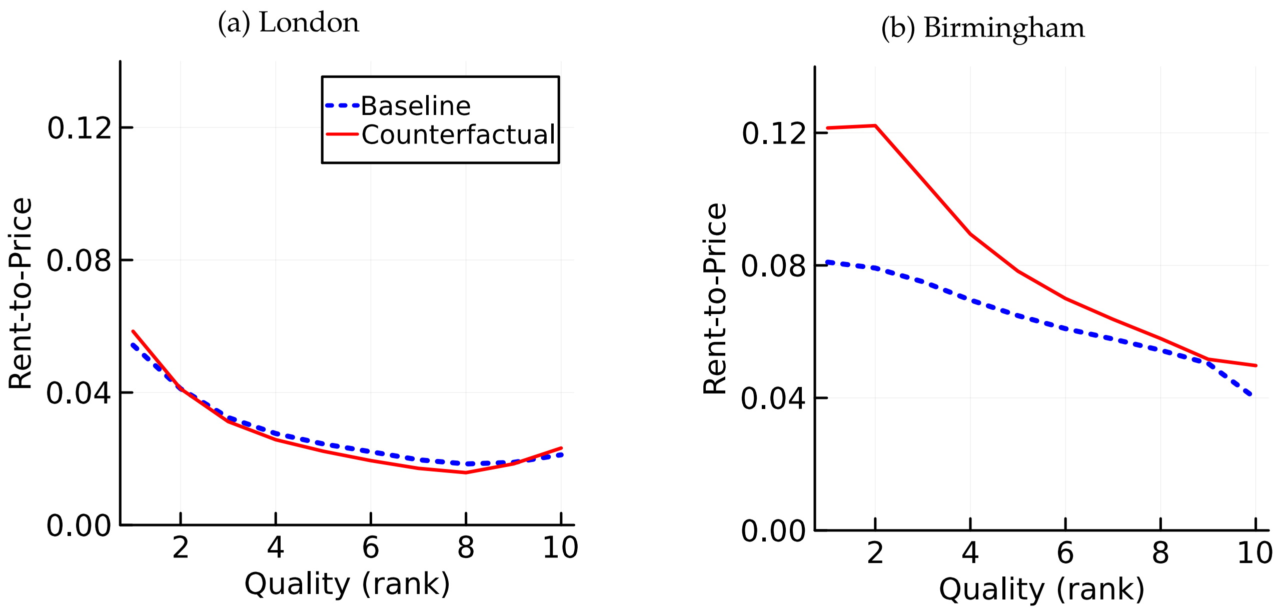
7.3 Influence of Landlord Heterogeneity on the Impact of Housing Policy

The previous sections show that landlord heterogeneity in wealth and costs play a crucial role in shaping housing markets both within and across cities. In this section I examine how the equilibrium effects of housing policy are influenced by the composition of landlords across local housing markets. Specifically, I explore the effect on the equilibrium rent-to-price function of introducing a 30% tax on rental income in both London and Birmingham.

Figure 14 presents the equilibrium rent-to-price functions in both cities for the baseline and counterfactual scenarios. Two key findings emerge. First, the rent-to-price function is substantially more sensitive to the rent tax in Birmingham than in London. Second, the sensitivity of the rent-to-price function is more pronounced in the lower quality segments in Birmingham.

The variation in tax sensitivity across cities and quality segments is driven by differences in landlord costs and profitability. Lower-quality segments in Birmingham exhibit greater sensitivity because landlords in these segments typically earn lower baseline

Figure 14: Sensitivity of Rent-to-Price to Rent Tax by City



Note: This figure compares the baseline rent-to-price function with the rent-to-price function in the counterfactual in which landlords are levied an additional 30% tax on rental income.

profits.[14] Landlords with lower profits are more likely to pass increased tax costs onto tenants through higher rents, as they have limited capacity to absorb these additional expenses. This amplifies the tax sensitivity of the rent-to-price ratio within low-profit market segments.

The findings show that tax pass-through decreases as landlord profits and income increase. This relationship has important implications for the design of taxes for landlords. It suggests that a progressive tax, which increases with landlord income, would generate fewer distortions in the housing market than a flat tax.

8 Conclusion

In this paper, I analyze how landlord heterogeneity shapes housing market outcomes across cities in England. I develop and estimate a two-sided assignment model in which heterogeneous households and landlords match to differentiated housing. I find that heterogeneity in observed landlord wealth within cities and unobserved opportunity costs across cities play a key role in explaining variations in rental affordability, segmentation between rental and owner-occupied housing, and the relative supply of rental properties.

[14] Figure 8 illustrates that landlord profits are generally lower in Birmingham than in London, with the lowest profits concentrated in Birmingham's lower-quality housing segments.

Using counterfactual experiments, I quantify the impact of landlord heterogeneity on each of these outcomes. First, I show that within-city heterogeneity in landlord wealth almost completely explains differences in the rent-to-price ratio across quality segments within a city. Second, I demonstrate that heterogeneity in landlords' unobserved opportunity costs explains 60% of the gap in quality segmentation between London and Birmingham and up to 67% of the gap in the user cost-to-rent ratio. Finally, I find that landlord heterogeneity moderates the equilibrium impact of housing policy. I show that a tax on rental income has lower pass-through in cities with wealthier landlords. These findings suggest that landlord composition is crucial in determining housing affordability and accessibility, with significant implications for housing policy.

9 Appendix

Figure A1: Tenure Shares Over Time in London

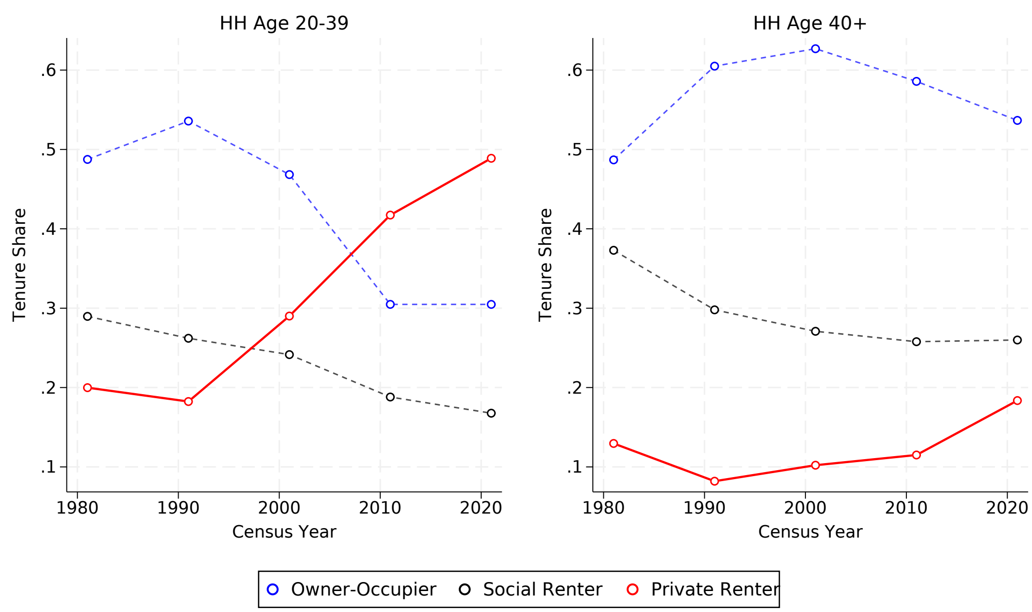
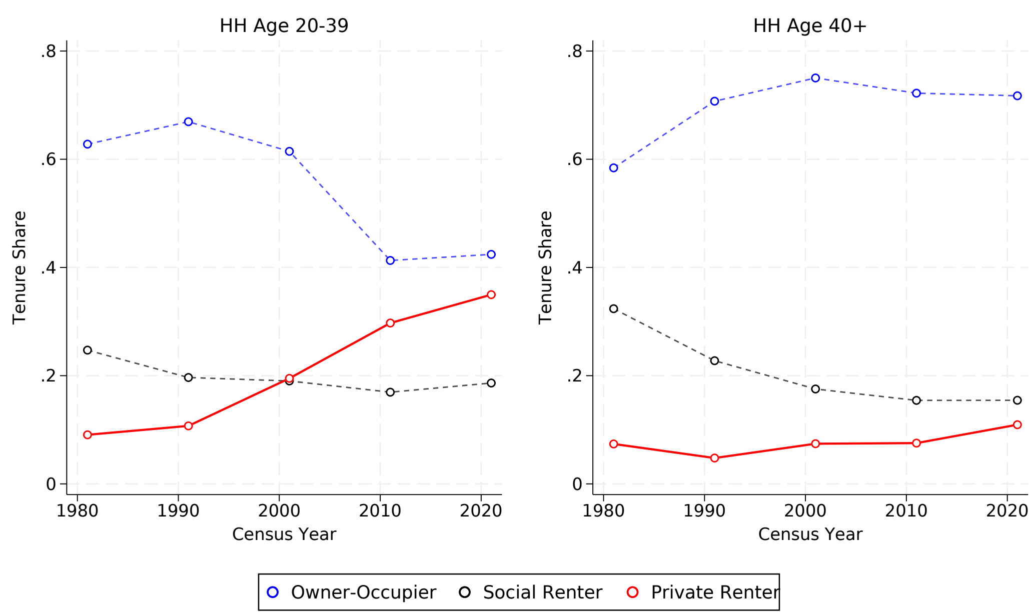


Figure A2: Tenure Shares Over Time Outside London



A1 Measuring the distributions of quality, rents, and prices by tenure

Figure A3: Evolution of Property Type Rankings in the Rent and Price Distributions

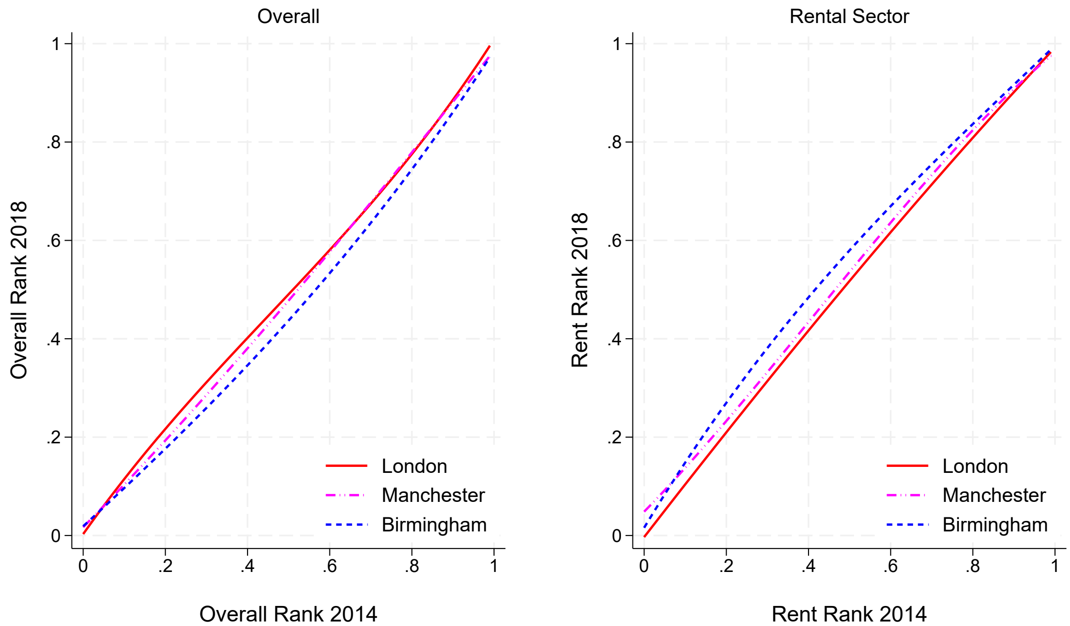


Figure A4: Correlation: Sensitivity of Rent to Price Decreasing in Relative Quality of Rentals

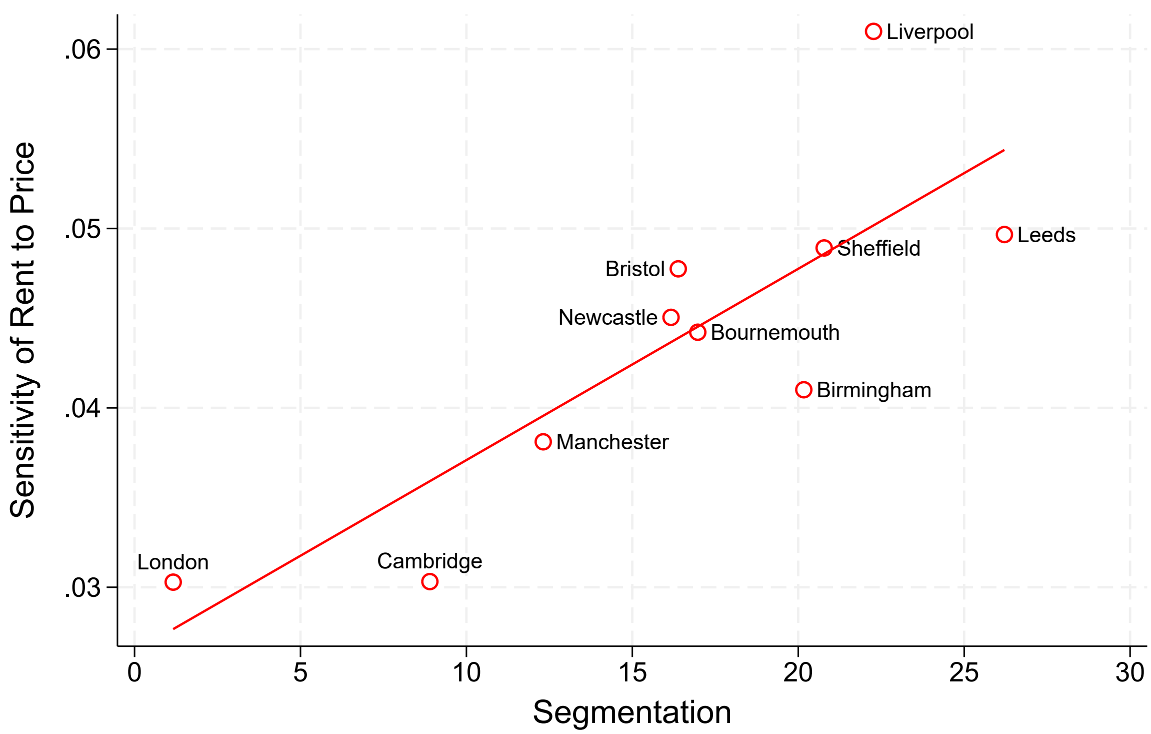
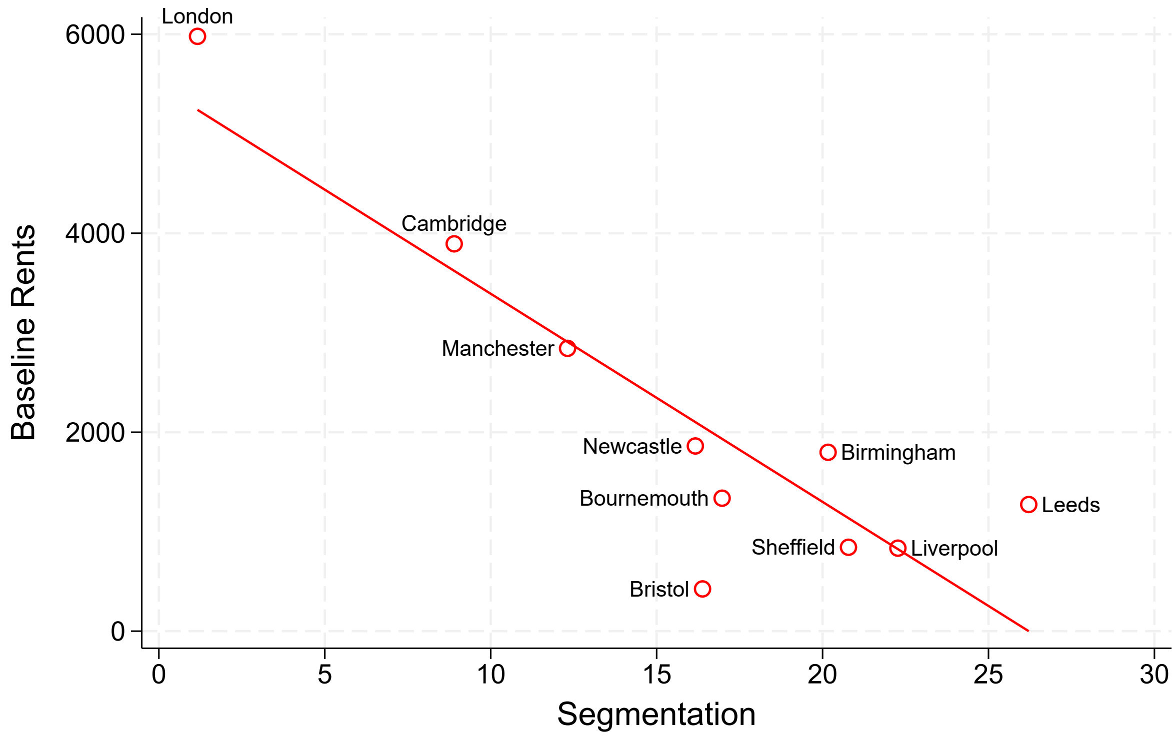


Figure A5: Correlation: Baseline Rents Increasing in Relative Quality of Rentals

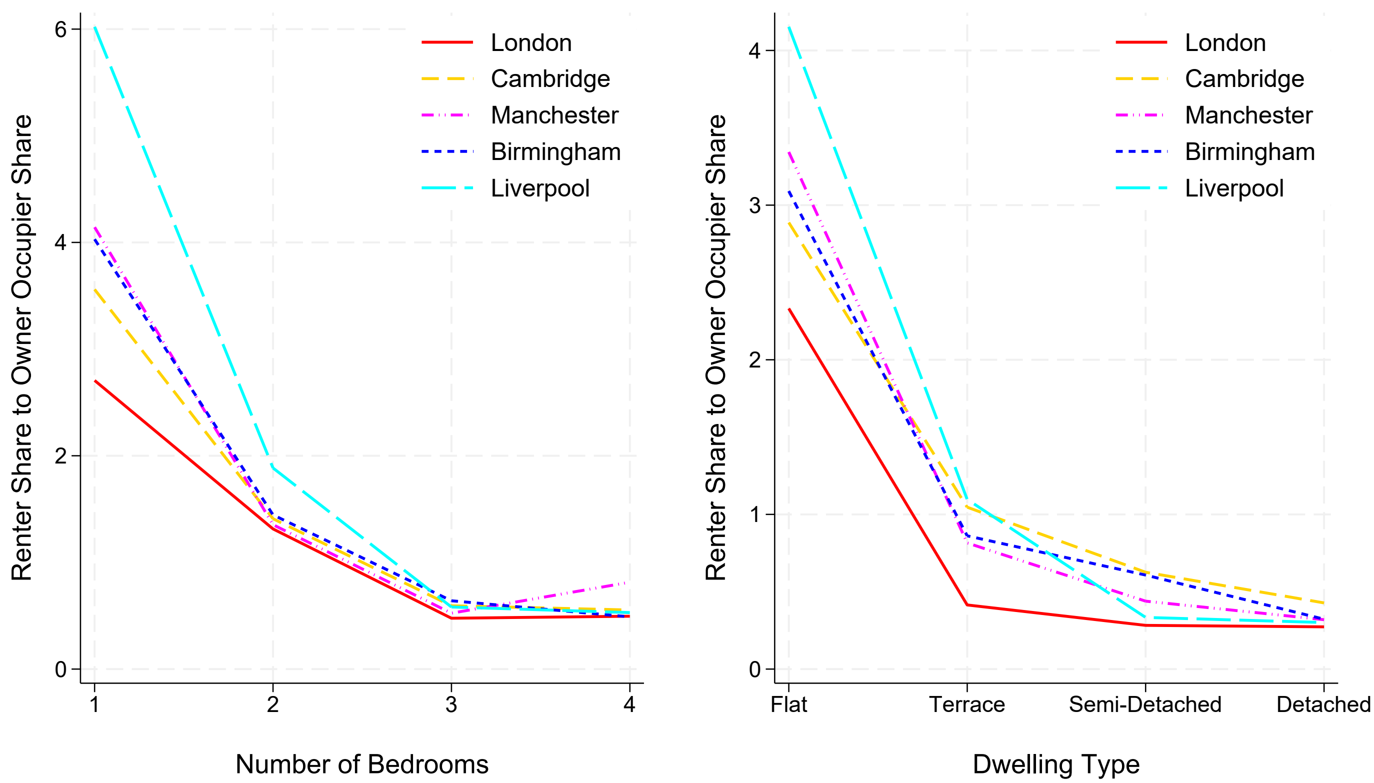


A2 What do the Patterns of Segmentation Reflect?

The patterns of segmentation along the unidimensional index in Figure 2 are driven by segmentation in quality along three sub-dimensions: the location of a dwelling, its structural features, and its upkeep. The data shows that the degree of segmentation along these three dimensions vary across cities. Moreover, segmentation along these dimensions dimensions are correlated across cities. The higher the relative quality of the location of rented properties compared to owner-occupied properties in a city, the higher tends to be the relative quality of structural features of rented properties. Similarly, the upkeep of rental dwellings is correlated with the other two sub-dimensions. The positive correlation suggests that factors which improve the relative quality of rented dwellings in a city do so by improving all three sub-dimensions of quality.

Figure A6 shows differences in structural features of rented and owner-occupied dwellings across cities. The figure on the left plots the share of rented dwellings of a given size divided by the share of owner-occupied dwellings of the same size. The figure on the right plots the relative shares of properties with different builds. There are two key patterns to note. First, rented dwellings are smaller and more likely to be flats in every city. Rented dwellings thus tend to have structural features that command lower value in

Figure A6: Ratio of Share of Dwelling Type in Rented and Owner-Occupied Dwellings



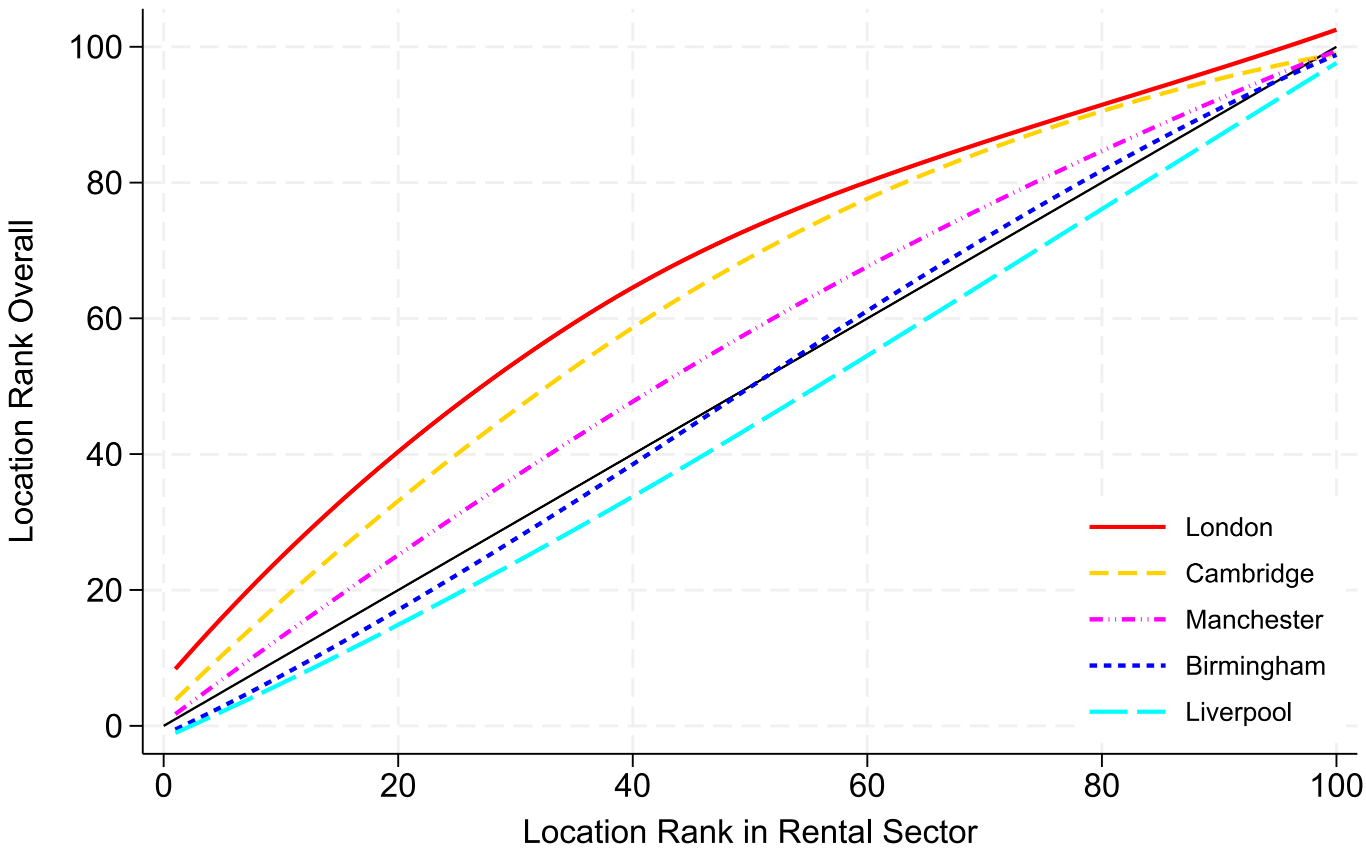
the market. Second, the relative quality of structural features of rented dwellings tends to be lower in cities which are more segmented along the unidimensional index of quality. These patterns reveal that the segmentation along the unidimensional index is at least partly driven by segmentation with respect to the quality of dwellings' structural features.

To measure segmentation with respect to the quality of dwellings' location, I decompose the unidimensional index of quality into a location component and a component capturing the quality of dwellings' structural features. I estimate the following fixed effects regression using data on sale listings:

$$price_{ilcy} = \alpha_y^c + \alpha_l^c + \alpha_{build(i)}^c + \varepsilon_{ilcy}$$

where i indexes property type; l indexes the LSOA; α_y^c is a fixed effect for year y; α_l^c is a fixed effect for LSOA l; and $\alpha_{build(i)}^c$ is a vector of dwelling structure fixed effects obtained by interacting the type of each dwelling with the number of bedrooms and bathrooms. The regression allows me to isolate the location component of quality in α_l^c and the structural features component in $\alpha_{build(i)}^c$. A higher value of α_l^c implies a higher quality for location

Figure A7: Segmentation of Location Quality Between the Rental and Owner-Occupied Sectors

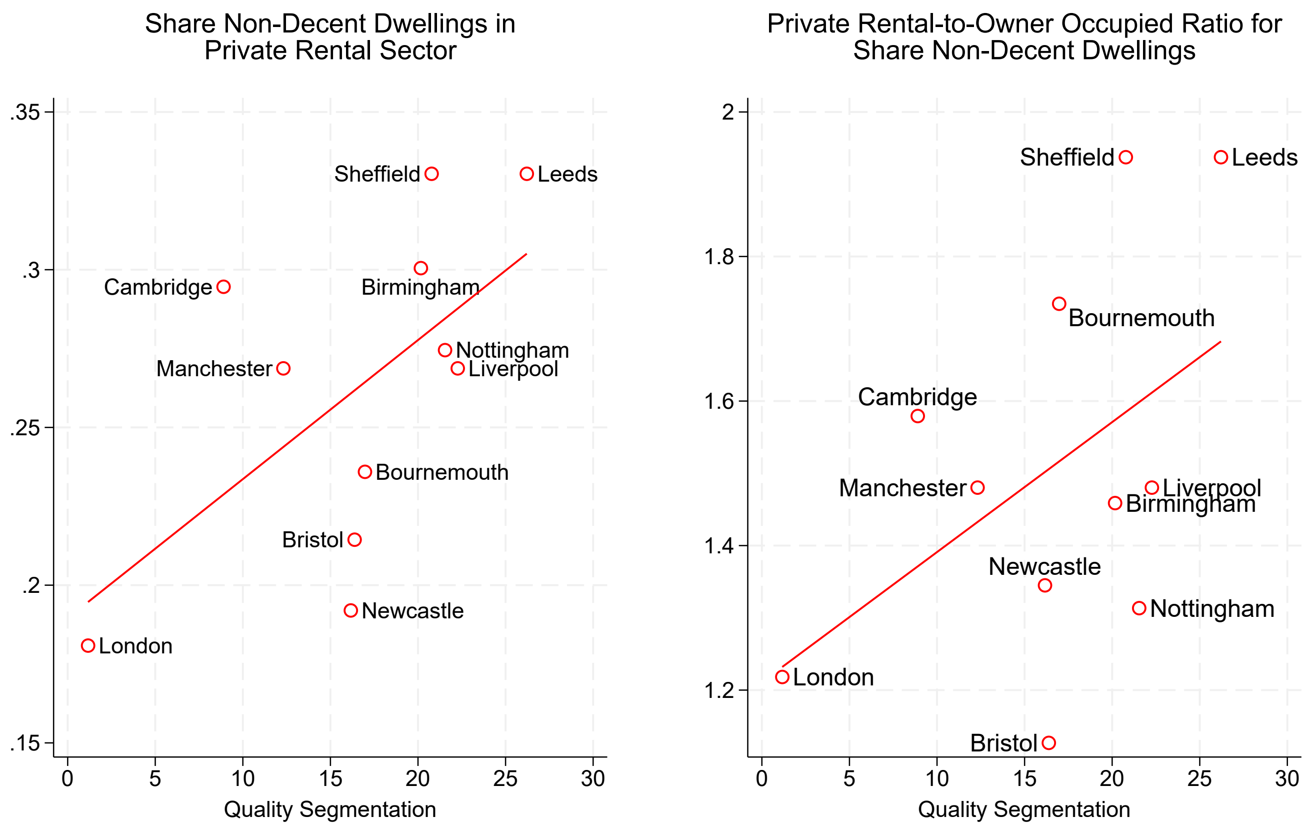


l. Similarly, a higher value for $\alpha^{c}_{build(i)}$ implies a higher quality for dwelling feature of type $build(i)$. Using this decomposition, I construct year- and city-specific distributions of location quality in the rental sector and in the housing stock as a whole. Based on these distributions, Figure A7 plots the relative ranking of location quality in the rental and overall housing distributions. The figure shows that in most cities rental properties tend to be located in higher quality LSOAs compared to owner-occupied properties. Moreover, the relative quality of LSOAs of rented properties is higher in cities in which rental properties have higher relative values in the unidimensional index of quality.

Lastly, Figure A8 plots the degree of segmentation in a city against: (i) the share of non-decent dwellings in the private rental sector, and (ii) the ratio of the share of non-decent dwellings in the private rental and owner-occupied sectors. I construct the shares of non-decent dwellings at the city level using the English Housing Survey. The survey categorizes dwellings as decent or non-decent based on the Decent Homes Standard which assesses hazards, state of repair, modern facilities, and thermal comfort.[15] Figure A8 shows that in more segmented cities the condition of rental dwellings tends to be lower both in absolute terms and relative to the condition of owner-occupied dwellings.

Taken together, Figures A6, A7 and A8 show that the three dimensions of quality tend to move in tandem across cities. This suggests that trade-offs along these margins are not key to generating the patterns of segmentation observed across cities. Instead factors which improve relative quality in the rental sector of a city do so by simultaneously improving the structural features, location, and upkeep of rented dwellings.

Figure A8: Prevalence of Non-Decent Rental Dwellings Correlated with Segmentation



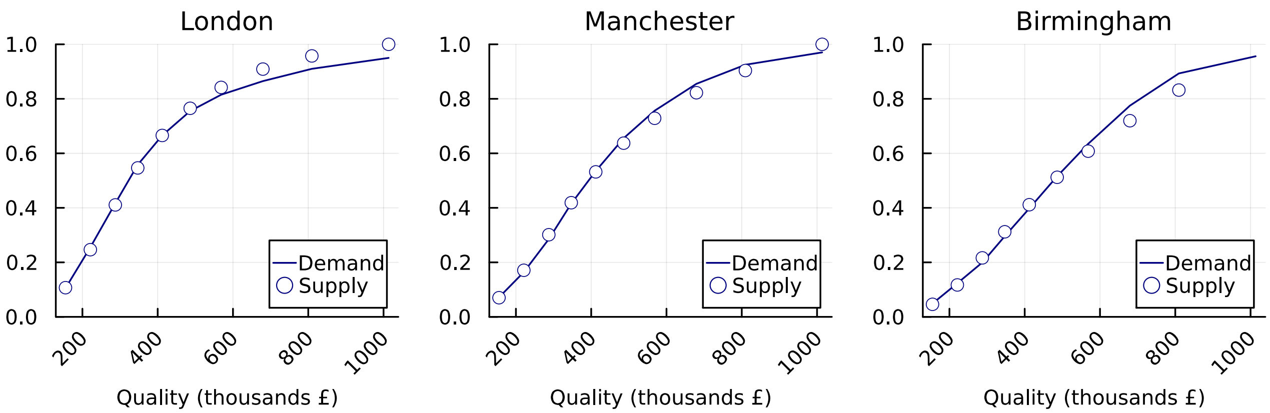
A3 Model Fit

Figure A9 plots the cumulative demand and supply functions in the estimated model by city in 2015.

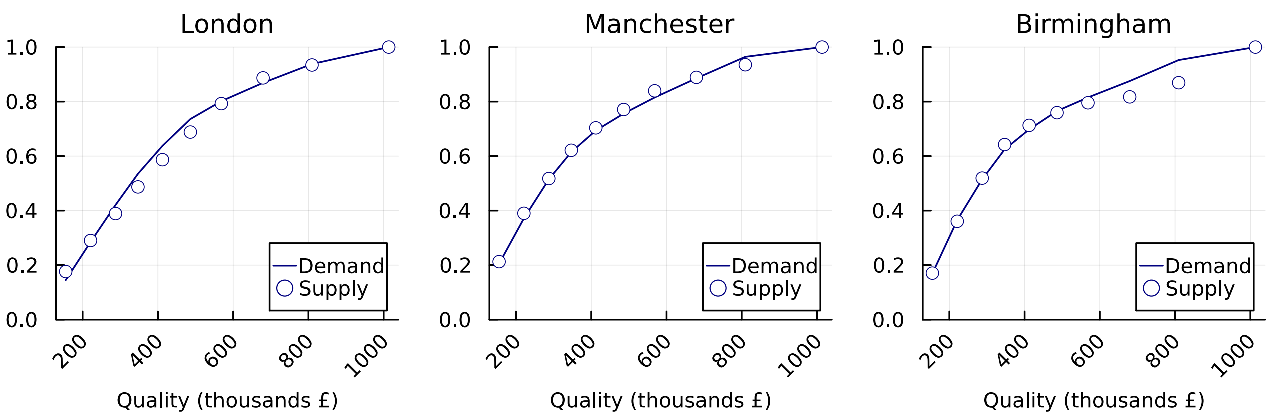
[15]The Decent Homes Standard was introduced by the UK government in 2000 and is overseen by the Department for Levelling Up, Housing and Communities (DLUHC). It sets the minimum criteria for housing quality in terms of safety, condition, amenities, and energy efficiency, particularly in social housing.

Figure A9: Aggregate Cumulative Demand and Supply (2015)

(a) Overall Housing Market



(b) Market for Rental Housing

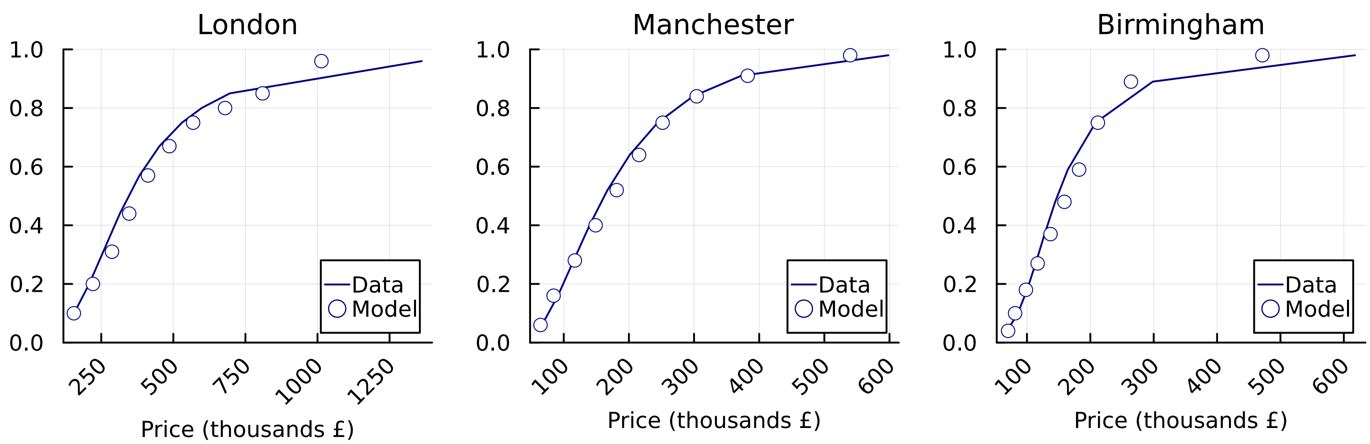


Note: This figure presents the cumulative demand and supply functions by quality at equilibrium based on the estimated model. The vertical axis measures the cumulative share for each quality.

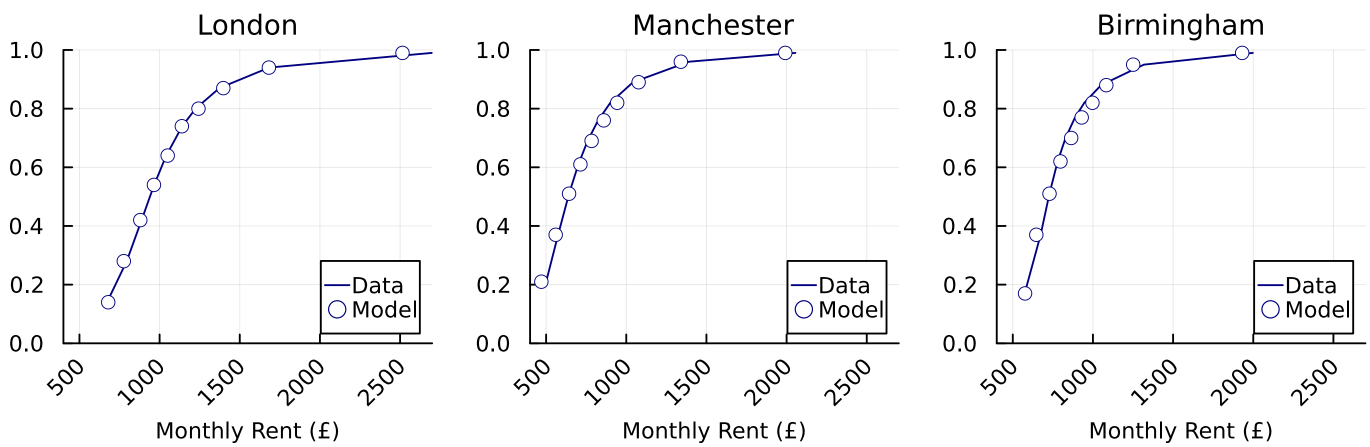
Figure A12 plots the asset cutoffs at which landlords are indifferent between different quality levels.

Figure A10: Distribution of Equilibrium Prices and Rents (2015)

(a) House Price

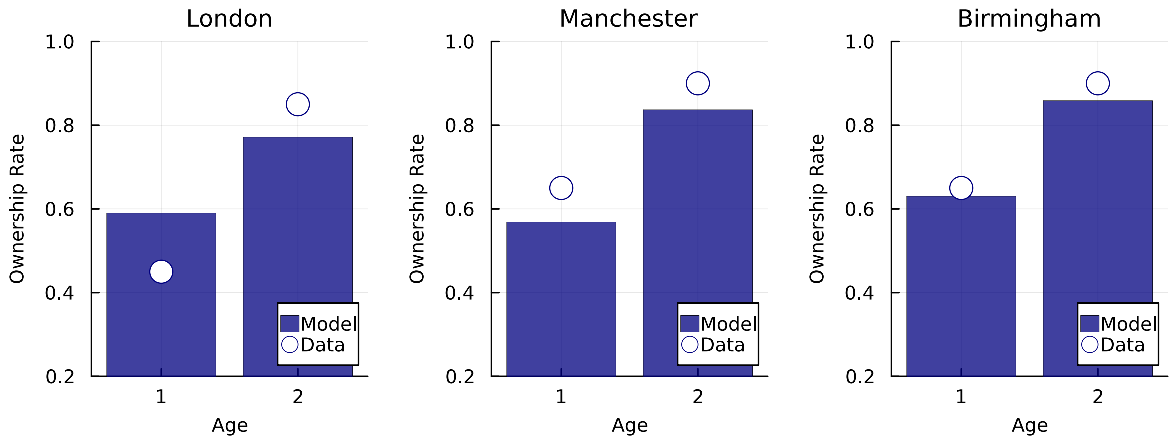


(b) Monthly Rents



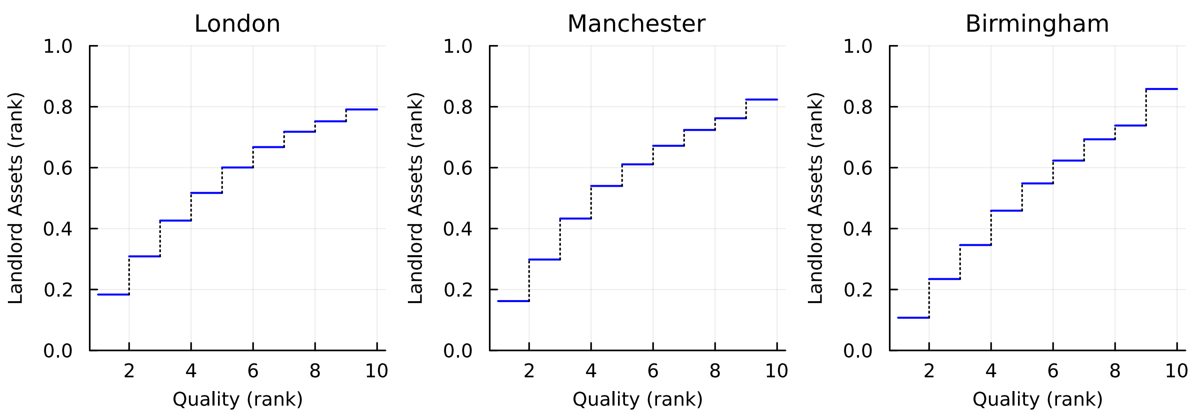
Note: This figure compares the cumulative distribution of equilibrium prices and rents implied by the estimated model to the distribution in the data. The solid curves labelled Data plot the empirical CDFs obtained by fitting the GB2 distribution. Each scatter point corresponds to one of ten quality levels in the model. The height of the price and rent scatters depends on the quality distribution of properties in the overall and rental markets respectively.

Figure A11: Ownership Rates (2015)



Note: This figure compares the ownership rates implied by the estimated model against ownership rates in the data by city and age group in 2015

Figure A12: Landlord Assignment Asset Cutoffs by House Quality (2015)

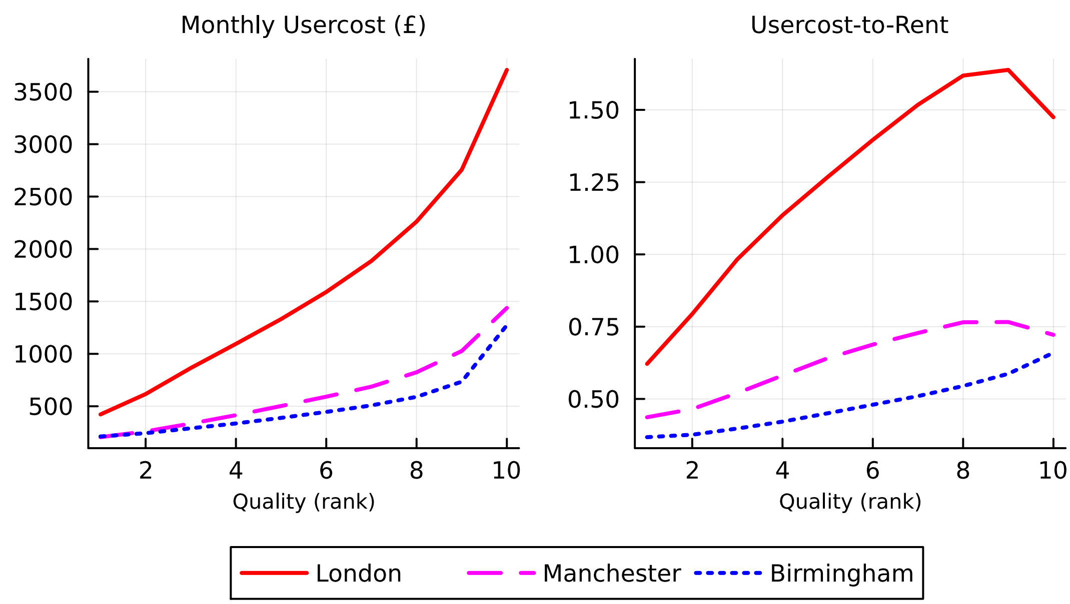


Note: This figure presents the nine asset cutoffs which define the assignment of landlords to house quality. The leftmost step represents the asset threshold which makes landlords indifferent between operating in the first and second quality levels. The rightmost step represents the asset indifference threshold between the ninth and tenth quality levels.

A4 Quality Segmentation and the Usercost-to-Rent Ratio

The pattern of quality segmentation observed in the data implies a specific sorting of households into tenure and quality across cities. Why do households sort in this manner? In this section, I show that this pattern of sorting is explained by differences in the relative costs of owning and renting across cities.

Figure A13: Estimated Usercosts of Owner-Occupation in Levels and Relative to Rents 2015

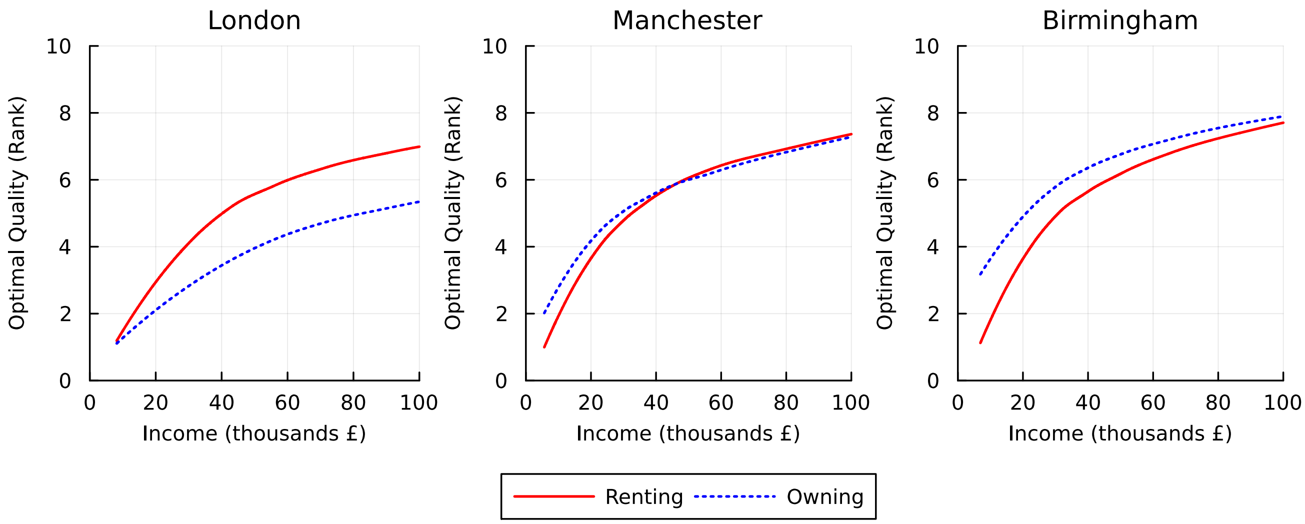


Note: This figure plots estimated unobserved usercosts of owner-occupation and the ratio of usercosts-to-observed rents by quality across cities in 2015

Figure A13 plots the monthly unobserved user cost of owner-occupation in levels and in comparison to rents. While user costs increase convexly with housing quality across all cities, the increase is dramatically more pronounced in London, reflecting disparities in the price distribution. Since differences in rents are more muted, this generates a corresponding divergence in the user costs-to-rent ratio between London and the comparison cities.

The demand model implies that these differences in relative costs should generate a wedge between households' optimal choice of quality by tenure. In particular, households in London should prefer more quality when they rent compared to when they own given the steep growth in the relative cost of owning. Figure A14 shows that this is indeed the case. The figure plots a weighted average of the optimal quality preferred by households when they do not face credit constraints by tenure at different levels of income. The weighted average is taken over unobserved types and household assets. The figure shows that households in London prefer more quality when renting across almost all levels of income, and that the difference becomes more pronounced as income increases. The pattern is reversed in Birmingham where households prefer more quality when owning across the entire income range. Manchester is an intermediate case where the preference for quality when renting is greater only at higher incomes.

Figure A14: Average Unconstrained Optimal Quality by Tenure and Income



Note: This figure plots estimated income-specific means of the optimal quality a household would choose to rent or own in the absence of credit constraints. The estimates are based on a local regression of optimal quality against household income.

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