

Yes, because after applying the low pass filter we are interpolating the image. Therefore, we are trying to restore the pixels of downsampled image.

When you have a high res. image and you apply a low-pass filter your image will get blurred. I believe after downsampling a blurred image, it will look similar to the original one, but in just smaller size.



# Quiz 1: Filters, Fourier Transforms, & Resizing

$$1. f(g(x)) = \int_{-\infty}^{\infty} f(y) g(x-y) dy$$

$$= \int_{-\infty}^{\infty} e^{\frac{-y^2}{2\sigma^2}} e^{-\frac{(x-y)^2}{2\sigma^2}} dy \quad u = \frac{2y-x}{2\sigma}$$

$$dy = \sigma du$$

$$= \int \sigma e^{-\frac{u^2}{4\sigma^2}} \frac{-x^2}{4\sigma^2} du$$

$$= \int \frac{2e^{-u^2}}{\sqrt{\pi}} du = \operatorname{erf}(u)$$

$$= \frac{-x^2}{4\sigma^2} \int \frac{2e^{-u^2}}{\sqrt{\pi}} du = \frac{-x^2}{4\sigma^2} \operatorname{erf}(u)$$

$$= \frac{\sqrt{\pi} \sigma e^{-\frac{x^2}{2\sigma^2}} \operatorname{erf}\left(\frac{2y-x}{2\sigma}\right)}{2} + C$$

$$= \frac{-x^2}{4\sigma^2}$$

$$= \frac{\sqrt{\pi} \sigma e}{2}$$

The relationship between  $\sigma_1$  and  $\sigma_2$  is that their computation equals  $\sigma_3$



2 let  $K = [x_0, x_1, x_2, x_3, x_4, x_5, x_6]$

$$K = \begin{bmatrix} c & b & a & b & c \end{bmatrix}$$

$$\therefore c + b + a + b + c = a + 2b + 2c = 1$$

$$\text{odd: } a + b + 2c \quad \text{Even: } a + b + c$$

odd = even

$$a + b + 2c = \frac{1}{2} \quad a + b + c = \frac{1}{2}$$

$$2c = \frac{1}{2} - b - a \quad c = \frac{1}{2} - b - a$$

$$c = \frac{1/2 - b - a}{2} \quad a = \frac{1}{2} - b - c$$

$$b = \frac{1}{2} - a - c$$

$$a = \frac{1}{2} - b - 2c$$

$$K = \begin{bmatrix} c \\ b \\ a \\ c \\ b \\ a \\ c \end{bmatrix} = \begin{bmatrix} \frac{1}{4} - \frac{b}{2} - \frac{a}{2} \\ \frac{1}{2} - a - c \\ \frac{1}{2} - b - 2c \\ \frac{1}{2} - b - a \\ \frac{\frac{1}{2} - b - a}{2} \end{bmatrix}$$