Siddharth Patel

Quiz 3 : Fitting to Data

1.1. Given $Ax = b$; $y_i = a x_i + b$ where $x = (a,b)^T$

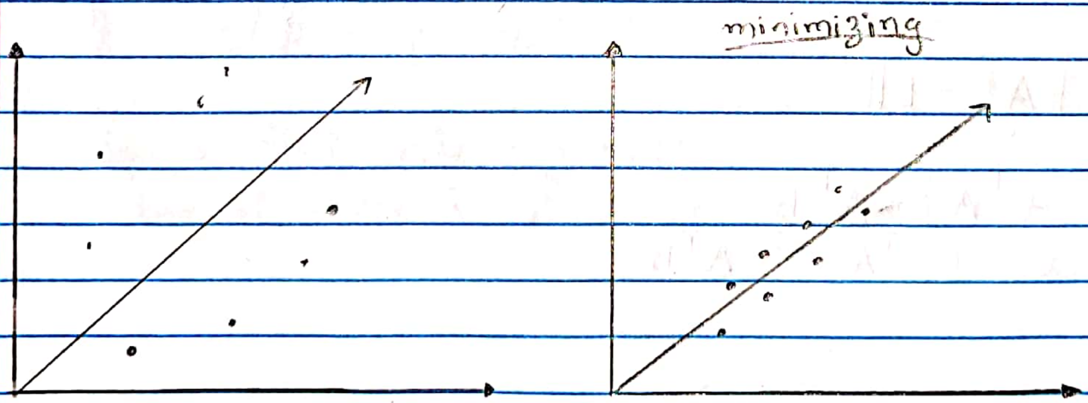
$$A = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ & \vdots \\ 1 & x_n \end{bmatrix} \quad x = \begin{bmatrix} a, b \end{bmatrix} \quad = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$Ax = b$ $\qquad x = \dfrac{b}{A}$ $\qquad x = b A^{-1}$

Because you cannot multiply b w/ A^{-1}

$$x = (A^T A)^{-1} \cdot A^T b$$

This, so to speak, algorithm



1.2 $l = (a,b,c)$ $ax_i + by_i + c = 0$ $Al = 0$

$$A = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ & \vdots & \\ 1 & x_n & y_m \end{bmatrix} l = [c, a, b] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Based on the hint provided, this approach is better because when we compute the distance of point to a line we use the general form of a straight line $Ax + By + C = 0$.
The general form has the advantage of working well with verticle lines.

1.3 As I mentioned in question 1.2, the general form that we use to solve homogenous least-squares problem has the advantage of working well with verticel lines.

Scanned with CamScanner

2.1 $ax^2 + bxy + cy^2 + dx + ey + f = 0$

Inhomogenous points $\tilde{x}_i = (x_i, y_i)$, $i = 1, \ldots, N$

$$A = \begin{bmatrix} 1 & x_1^2 & xy_1 & y_1^2 & x_1 & y_1 \\ 1 & x_2^2 & xy_2 & y_2^2 & x_2 & y_2 \\ & & \vdots & & & \\ 1 & x_n^2 & xy_n & y_n^2 & x_n & y_n \end{bmatrix} \quad x = \begin{bmatrix} f \\ a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

The way to minimize the error is:

$$\phi = \sum_{i=1}^{n} \left(Ax_i^2 + Bx_i y_i + Cy_i^2 + Dx_i + Ey_i + F \right)^2$$

2.2 To have a unique solution for c, we must have minimum of 5 values of N.