# D-Q Mathematical Modelling and Simulation of Three-Phase Induction Motor for Electrical Fault Analysis

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# D-Q Mathematical Modelling and Simulation of Three-Phase Induction Motor for Electrical Fault Analysis

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**Abstract:** Building a machine that emulates the electrical fault can be expensive and implies high risk of damage. Different types of fault cannot be created in the same machine. This paper presents a mathematical model and dynamic simulation of 3-Φ squirrel cage induction motor to study the performance of the AC machine. The equations will be derived and developed that will represent the 3-Φ Induction motor. Those equations can be used to study the fault analysis of the winding as well as their performance. Mathematical equations are developed in arbitrary reference frame and simulation is done under no load condition to obtain the results. As, Direct Quadrate (d-q) model is superior to abc reference in terms of complexity and computational time, for modelling the drive system, d-q transformation method is used. Then the simulation of the system will be done and analysis of dynamic behaviour of the motor will be studied. Then the torque and speed will be calculated. The study of the transient and steady state analysis of an induction motor model will be performed through the simulation.

**Keywords:** Induction Motor, Reference Frame, Flux Linkage.

## I. INTRODUCTION

$3-\Phi$ induction motors are self-starting and can be classified in terms of difference in their design of rotor. Squirrel cage induction motor and slip ring induction motor are the two types of $3-\Phi$ induction motor. Squirrel cage induction motors are popular motors due to its performance, reliability, ease in maintenance and comparatively cheaper even though slip ring induction motor has high starting torque. At the time of starting and other operation, the driving of induction motor requires very high current i.e. 5-7 times greater than its rated current. It also produces dip in the voltage, oscillatory torque and may generate distortions. Therefore, it is important to model induction motor in order to tackle such drawbacks. A $3-\Phi$ winding can be represented mathematically by 2-Φ winding. It can be done by Park's transformation matrix [1-8]. The stator and rotor parameters like voltage, current and flux linkages of motor may remain stationary, rotate at synchronous speed and angular speed when transformed into arbitrary reference frame from natural reference frame [8-12]. From these three, any mode can be selected to develop a model depending on its need and generally development of a $3-\Phi$ induction motor model in arbitrary reference frame is considered standard [13-20]. In this paper the transient and steady state analysis of an induction motor model is simulated. Result obtained from simulation can convey the working of induction motor.

## II. MATHEMATICAL MODELLING OF 3-Φ INDUCTION MOTOR

Winding arrangement for 3-Φ induction motor is shown in Fig. 1.

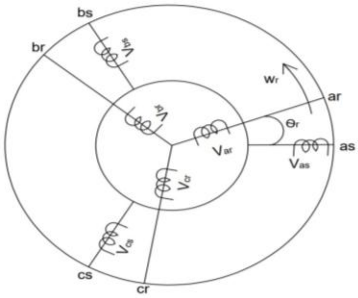


Fig. 1 Winding arrangement of 3-Φ induction motor

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Stator windings are identical and $120^o$ electrically apart from each other with $N_s$ equivalent turns and resistance $r_s$ per phase. Similarly, rotor windings are identical and $120^o$ electrically apart from each other with equivalent $N_r$ turns and resistance $r_r$ per phase [1,4,6,16,21]. The equations were derived considering windings to be identical, distributed sinusoidally around uniform air gap and magnetic saturation to be neglected.

A. Voltage Equations

Using Kirchhoff's voltage law, voltage drop across stator and rotor are given in following equation

a) voltage drop across stator windings

$$V_{Rs} = r_s i_{Rs} + \frac{d\Psi_{Rs}}{dt} \tag{1}$$

$$V_{Ys} = r_s i_{Ys} + \frac{d\Psi_{Ys}}{dt} \tag{2}$$

$$V_{Bs} = r_s i_{Bs} + \frac{d\Psi_{Bs}}{dt} \tag{3}$$

b) voltage drop across rotor windings

$$V_{Rr} = r_r i_{Rr} + \frac{d\Psi_{Rr}}{dt} \tag{4}$$

$$V_{Yr} = r_r i_{Yr} + \frac{d\Psi_{Yr}}{dt} \tag{5}$$

$$V_{Br} = r_r i_{Br} + \frac{d\Psi_{Br}}{dt} \tag{6}$$

Let k = transformation turn ratio

$$K = \frac{N_s}{N_r} = \frac{V_s}{V_r} = \frac{i_r}{i_s} \tag{7}$$

$$K^2 = \frac{Z_s}{Z_r} \tag{8}$$

It is convenient to refer all the rotor parameters to stator side and this can be done using transformation turn ratio.

$$V'_{RYBr} = KV_{RYBr} \tag{9}$$

$$i'_{RYBr} = \frac{1}{k} i_{RYBr} \tag{10}$$

$$\Psi'_{RYBr} = K\Psi_{RYBr} \tag{11}$$

$$L'_{lr} = K^2 L_{lr} \tag{12}$$

$$r'_r = K^2 r_r \tag{13}$$

The subscript $V'_{RYBr}, i'_{RYBr}, \Psi'_{RYBr}, L'_{lr}, r'_r$ shows that these rotor parameters referred to stator side.

B. Inductance

From the above voltage equation, it is clear that flux linkages are the function of inductance thus, it is important to determine the inductances of induction motor. Inductance of induction motor consist of self-inductance, mutual inductance, magnetizing inductance and leakage inductance. Relationship between flux linkage and current is shown in Equation (14), from this equation it is clear that flux linkage is directly proportional to current.

$$\Psi = Li \tag{14}$$

$$\begin{bmatrix} \Psi_{Rs} \\ \Psi_{Ys} \\ \Psi_{Bs} \\ \Psi_{Rr} \\ \Psi_{Yr} \\ \Psi_{Br} \end{bmatrix} = \begin{bmatrix} L_{RsRs} & L_{RsYs} & L_{RsBs} & L_{RsRr} & L_{RsYr} & L_{RsBr} \\ L_{YsRs} & L_{YsYs} & L_{YsBs} & L_{YsRr} & L_{YsYr} & L_{YsBr} \\ L_{BsRs} & L_{BsYs} & L_{BsBs} & L_{BsRr} & L_{BsYr} & L_{BsBr} \\ L_{RrRs} & L_{RrYs} & L_{RrBs} & L_{RrRr} & L_{RrYr} & L_{RrBr} \\ L_{YrRs} & L_{YrYs} & L_{YrBs} & L_{YrRr} & L_{YrYr} & L_{YrBr} \\ L_{BrRs} & L_{BrYs} & L_{BrBs} & L_{BrRr} & L_{BrYr} & L_{BrBr} \end{bmatrix} \begin{bmatrix} i_{Rs} \\ i_{Ys} \\ i_{Bs} \\ i_{Rr} \\ i_{Yr} \\ i_{Br} \end{bmatrix} \tag{15}$$

C. Self-Inductance

The self-inductance of 3-Φ induction motor is an inductance generated by a single-phase winding and it consists of magnetizing and leakage inductance. As all the stator windings are identical and with same number of turns by taking this into consideration, we can say all the self-inductance of stator windings are equal. Same can be said about self-inductance of rotor windings.

$$L_{RsRs} = L_{YsYs} = L_{BsBs} = L_{ms} + L_{ls} \tag{16}$$

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Magnetizing inductance ($L_{ms}$) can be calculated by using Equation (17)

$$L_{ms} = \frac{\mu_o rlN_s^2\pi}{4g} \tag{17}$$

Same thing can be said about self-inductance of rotor windings.

$$L_{RrRr} = L_{YrYr} = L_{BrBr} = L_{mr} + L_{lr} \tag{18}$$

$$L_{mr} = \frac{\mu_o rlN_r^2\pi}{4g} \tag{19}$$

D. Mutual Inductance

Mutual inductance is the inductance existing between two windings and in induction motor there are four types of mutual inductance which are stator-stator (mutual inductance between two different stator windings), rotor-rotor (mutual inductance between two different rotor windings), stator-rotor (mutual inductance between stator and rotor windings) and rotor-stator (mutual inductance between rotor and stator windings).

stator-stator mutual inductance can be calculated by using Equation (20)

$$L_{xsys} = \frac{\mu_o rlN_r^2\pi}{4g}\cos\theta_{xsys} \tag{20}$$

Where $L_{xsys}$ is the mutual inductance between stator winding 'x' and any other stator winding 'y' and $\cos\theta_{xsys}$ is the angle between 'x' and 'y' stator winding.

Substituting Equation (17) in Equation (20) we have

$$L_{xsys} = L_{ms}\cos\theta_{xsys} \tag{21}$$

Stator windings of induction motor are distributed $120^o$ electrical from each other thus, possible angle between any two stator windings are $120^o or\ 240^o$.

$$\cos\theta_{xsys} = \cos(\pm 120) = \cos(\pm 240) = -\frac{1}{2} \tag{22}$$

Substituting Equation (22) in Equation (21) we will get

$$L_{xsys} = -\frac{1}{2}L_{ms} \tag{23}$$

From Equation (23) we can say that mutual inductance between stator windings is simplified in Equation (24)

$$L_{RsYs} = L_{RsBs} = L_{YsRs} = L_{YsBs} = L_{BsRs} = L_{BsYs} = -\frac{1}{2}L_{ms} \tag{24}$$

rotor-rotor mutual inductance is similar to that of stator-stator mutual inductance and can be expressed as:

$$L_{RrYr} = L_{RrBr} = L_{YrRr} = L_{YrBr} = L_{BrRr} = L_{BrYr} = -\frac{1}{2}L_{mr} \tag{25}$$

stator-rotor mutual inductance depends on the rotor position as per following equation

$$L_{xsyr} = L_{sr}\cos\theta_{xsyr} \tag{26}$$

Where $L_{xsyr}$ is the mutual inductance between 'x' stator windings and 'y' rotor windings and $\theta_{xsyr}$ is the angle between them. $L_{sr}$ is the magnitude of mutual inductance between stator and rotor windings and is given by Equation (27).

$$L_{sr} = \frac{N_s^2}{2} \times \frac{N_r^2}{2} \times \frac{\mu_o rl\pi}{g} \tag{27}$$

Now using Equation (26) and Fig.1, mutual inductance between stator- rotor can be expressed as following:

$$L_{RsRr} = L_{YsYr} = L_{BsBr} = L_{sr}\cos\theta_r \tag{28}$$

$$L_{RsYr} = L_{YsBr} = L_{BsRr} = L_{sr}\cos(\theta_r + \frac{2\pi}{3}) \tag{29}$$

$$L_{RsBr} = L_{YsRr} = L_{BsYr} = L_{sr}\cos(\theta_r + \frac{4\pi}{3}) \tag{30}$$

Taking similar procedure mutual inductance between rotor-stator can be determined as follows:

$$L_{RrRs} = L_{YrYs} = L_{BrBs} = L_{sr}\cos(-\theta_r) \tag{31}$$

$$L_{RrYs} = L_{YrBs} = L_{BrRs} = L_{sr}\cos(\frac{2\pi}{3} - \theta_r) \tag{32}$$

$$L_{RrYs} = L_{YrRs} = L_{BrYs} = L_{sr}\cos(\frac{4\pi}{3} - \theta_r) \tag{33}$$

All the inductance has been calculated and now complete inductance can be developed. In order to simplify the inductance matrix, matrix is divided into sub-matrices. Inductance matrix in Equation (15) is repeated in Equation (34)

$$L = \begin{bmatrix} L_{RsRs} & L_{RsYs} & L_{RsBs} & L_{RsRr} & L_{RsYr} & L_{RsBr} \\ L_{YsRs} & L_{YsYs} & L_{YsBs} & L_{YsRr} & L_{YsYr} & L_{YsBr} \\ L_{BsRs} & L_{BsYs} & L_{BsBs} & L_{BsRr} & L_{BsYr} & L_{BsBr} \\ L_{RrRs} & L_{RrYs} & L_{RrBs} & L_{RrRr} & L_{RrYr} & L_{RrBr} \\ L_{YrRs} & L_{YrYs} & L_{YrBs} & L_{YrRr} & L_{YrYr} & L_{YrBr} \\ L_{BrRs} & L_{BrYs} & L_{BrBs} & L_{BrRr} & L_{BrYr} & L_{BrBr} \end{bmatrix} \tag{34}$$

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Sub-matrix of inductance is:

$$L = \begin{bmatrix} L_{SS} & L_{SR} \\ L_{RS} & L_{RR} \end{bmatrix} \tag{35}$$

Where $L_{SS}, L_{SR}, L_{RS}, L_{RR}$ is stator-stator, stator-rotor, rotor-stator and rotor-rotor mutual inductance respectively.

$$L_{SS} = \begin{bmatrix} L_{ms} + L_{ls} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{ms} + L_{ls} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{ms} + L_{ls} \end{bmatrix} \tag{36}$$

$$L_{RR} = \begin{bmatrix} L_{mr} + L_{lr} & -\frac{1}{2}L_{mr} & -\frac{1}{2}L_{mr} \\ -\frac{1}{2}L_{mr} & L_{mr} + L_{lr} & -\frac{1}{2}L_{mr} \\ -\frac{1}{2}L_{mr} & -\frac{1}{2}L_{mr} & L_{mr} + L_{lr} \end{bmatrix} \tag{37}$$

$$L_{SR} = L_{sr} \begin{bmatrix} \cos\theta_r & \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r + \frac{4\pi}{3}) \\ \cos(\theta_r + \frac{4\pi}{3}) & \cos\theta_r & \cos(\theta_r + \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r + \frac{4\pi}{3}) & \cos\theta_r \end{bmatrix} \tag{38}$$

$$L_{RS} = (L_{SR})^T \tag{39}$$

From Equations (37) & (38) it is clear that stator-rotor and rotor-stator mutual inductance depends on rotor position $\theta_r$ which continuously changes with time when rotor is rotating. In order to develop dynamic model de-coupling is done by converting in arbitrary reference frame. Before transforming it the motor parameters to arbitrary reference frame it is necessary to refer all the rotor parameters to stator side.

Relationship between $L_{ms}$ & $L_{sr}$by evaluating Equation (17) & (27) and is given in Equation (40)

$$L_{ms} = \frac{N_s}{N_r} L_{sr} = L'_{sr} \tag{40}$$

stotor- rotor mutual inductance referred to stator side is expressed as:

$$L'_{SR} = L_{ms} \begin{bmatrix} \cos\theta_r & \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r + \frac{4\pi}{3}) \\ \cos(\theta_r + \frac{4\pi}{3}) & \cos\theta_r & \cos(\theta_r + \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r + \frac{4\pi}{3}) & \cos\theta_r \end{bmatrix} \tag{41}$$

$$L_{ms} = \frac{N_s^2}{N_r^2} L_{mr} = L'_{mr} \tag{42}$$

$$L'_{RR} = \begin{bmatrix} L_{ms} + L'_{lr} & -0.5L_{ms} & -0.5L_{ms} \\ -0.5L_{ms} & L_{ms} + L'_{lr} & -0.5L_{ms} \\ -0.5L_{ms} & -0.5L_{ms} & L_{ms} + L'_{lr} \end{bmatrix} \tag{43}$$

$$L'_{RS} = (L'_{SR})^T \tag{44}$$

Flux linkage is now expressed as:

$$\begin{bmatrix} \Psi_{RYBs} \\ \Psi'_{RYBr} \end{bmatrix} = \begin{bmatrix} L_{SS} & L'_{SR} \\ L'_{RS} & L'_{RR} \end{bmatrix} \begin{bmatrix} i_{RYBs} \\ i'_{RYBr} \end{bmatrix} \tag{45}$$

Now flux linkages can be transformed to arbitrary reference frame by using Park's transformation matrix.

$$K_s = \frac{2}{3} \begin{bmatrix} \sin\theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{4\pi}{3}) \\ \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta - \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{46}$$

$$K_s^{-1} = \begin{bmatrix} \sin\theta & \cos\theta & 1 \\ \sin(\theta - \frac{2\pi}{3}) & \cos(\theta - \frac{2\pi}{3}) & 1 \\ \sin(\theta - \frac{4\pi}{3}) & \cos(\theta - \frac{2\pi}{3}) & 1 \end{bmatrix} \tag{47}$$

$$K_r = \frac{2}{3} \begin{bmatrix} \sin\beta & \sin(\beta - \frac{2\pi}{3}) & \sin(\beta - \frac{4\pi}{3}) \\ \cos\beta & \cos(\beta - \frac{2\pi}{3}) & \cos(\beta - \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{48}$$

$$\beta = \theta - \theta_r \tag{49}$$

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$K_s, K_s^{-1}$ is Park's transformation and inverse transformation matrix for stator respectively.
$K_r, K_r^{-1}$ is Park's transformation and inverse transformation matrix for rotor respectively.

$$\begin{bmatrix} \Psi_{dqos} \\ \Psi'_{dqor} \end{bmatrix} = \begin{bmatrix} K_s L_{SS} K_s^{-1} & K_s L'_{SR} K_s^{-1} \\ K_r L'_{RS} K_s^{-1} & K_r L'_{RR} K_r^{-1} \end{bmatrix} \begin{bmatrix} i_{dqos} \\ i'_{dqor} \end{bmatrix} \tag{50}$$

Evaluating Equation (49) we get the flux linkages in arbitrary reference frame and is shown in Equation (51).

$$\begin{bmatrix} \Psi_{qs} \\ \Psi_{ds} \\ \Psi_{0s} \\ \Psi'_{qr} \\ \Psi'_{dr} \\ \Psi'_{0r} \end{bmatrix} = \begin{bmatrix} L_{ms}+L_{ls} & 0 & 0 & 0.5L_{ms} & 0 & 0 \\ 0 & L_{ms}+L_{ls} & 0 & 0 & 0.5L_{ms} & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 \\ 0.5L_{ms} & 0 & 0 & L_{ms}+L'_{lr} & 0 & 0 \\ 0 & 0.5L_{ms} & 0 & 0 & L_{ms}+L'_{lr} & 0 \\ 0 & 0 & 0 & 0 & 0 & L'_{lr} \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{qs} \\ i_{0s} \\ i'_{qr} \\ i'_{dr} \\ i'_{0r} \end{bmatrix} \tag{51}$$

$$L_m = \frac{2}{3} L_{ms} \tag{52}$$

From Equation (50) it is clears that by converting flux linkage into arbitrary reference frame rotor position $(\theta_r)$is no longer the function of flux linkages which reduces the model complexity.

E. Voltage Equation in Arbitrary Reference Frame

Voltage equation in natural reference frame is given in following equation:

$$V_{RYBs} = r_s i_{RYBs} + \frac{d\Psi_{RYBs}}{dt} \tag{53}$$

$$V'_{RYBr} = r'_r i'_{RYBr} + \frac{d\Psi'_{RYBr}}{dt} \tag{54}$$

Taking only the stator voltage and transferring it to arbitrary reference frame

$$V_{dq0s} = K_s r_s K_s^{-1} i_{dqos} + K_s \frac{d(K_s^{-1}\Psi_{dq0s})}{dt} \tag{55}$$

$$K_s r_s K_s^{-1} = r_s = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix} \tag{56}$$

$$K_s \frac{d(K_s^{-1}\Psi_{abcs})}{dt} = K_s \frac{dK_s^{-1}}{dt} \Psi_{dq0s} + K_s K_s^{-1} \frac{d\Psi_{dq0s}}{dt} \tag{57}$$

After solving Equation (57) we get stator voltage in arbitrary reference frame as following:

$$\begin{bmatrix} V_{ds} \\ V_{qs} \\ V_{0s} \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{0s} \end{bmatrix} + \omega_e \begin{bmatrix} -\Psi_{qs} \\ \Psi_{ds} \\ 0 \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \Psi_{ds} \\ \Psi_{qs} \\ \Psi_{0s} \end{bmatrix} \tag{58}$$

Rotor voltage in arbitrary reference frame can be determined using same approach.

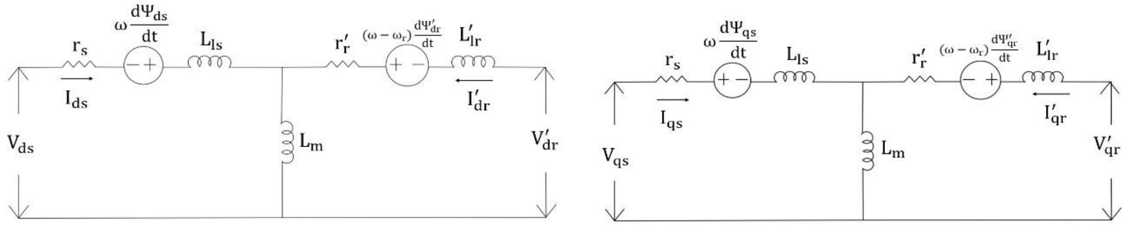
$$\begin{bmatrix} V'_{dr} \\ V'_{dr} \\ V'_{0r} \end{bmatrix} = \begin{bmatrix} r'_r & 0 & 0 \\ 0 & r'_r & 0 \\ 0 & 0 & r'_r \end{bmatrix} \begin{bmatrix} i'_{dr} \\ i'_{qr} \\ i'_{0r} \end{bmatrix} + (\omega_e - \omega_r) \begin{bmatrix} -\Psi'_{qr} \\ \Psi'_{dr} \\ 0 \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \Psi'_{dr} \\ \Psi'_{qr} \\ \Psi'_{0r} \end{bmatrix} \tag{59}$$

Where $\omega_e$ is the rotational speed of the reference frame and $\omega_r$ is the rotational speed of the rotor. The model developed in Equation (59) is a general model in arbitrary reference frame which can take the form of any reference frame depending on the value substituted for $\omega_e$, therefor it is called arbitrary reference frame when $\omega_e = 0$ stationary reference frame as dq axes do not rotate.

$\omega_e = \omega_s$ synchronous reference frame when $\omega_e$ is set to angular frequency of supply frequency.
$\omega_e = \omega_r$ rotor reference frame when $\omega_e$ is set to angular frequency of rotor frequency.

## III. EQUIVALENT CIRCUIT

Complete mathematical model of 3-Φ induction motor is given by Equation (51), (58) & (59). These equations are used to draw an equivalent circuit of 3-Φ induction motor and it is shown in Fig. 2.



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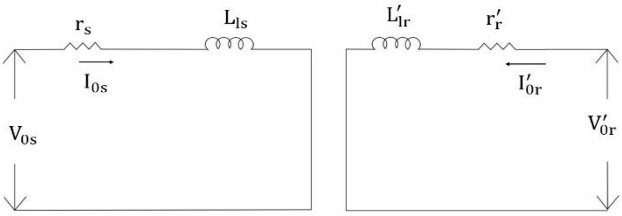


Fig. 2 Circuit diagram of 3-Φ induction motor in dq0

Repeating Equation (51) & (59) in the following equations. Zero sequence parameters are neglected.

$$V_{ds} = r_s i_{ds} + \frac{d\Psi_{ds}}{dt} - \omega_e \Psi_{qs} \tag{60}$$

$$V_{qs} = r_s i_{qs} + \frac{d\Psi_{qs}}{dt} + \omega_e \Psi_{ds} \tag{61}$$

$$V'_{dr} = r'_r i'_{dr} + \frac{d\Psi'_{dr}}{dt} - (\omega_e - \omega_r)\Psi'_{qr} \tag{62}$$

$$V'_{qr} = r'_r i'_{qr} + \frac{d\Psi'_{qr}}{dt} + (\omega_e - \omega_r)\Psi'_{dr} \tag{63}$$

$$\Psi_{ds} = L_{ls} i_{ds} + L_m(i_{ds} + i'_{dr}) \tag{64}$$

$$\Psi_{qs} = L_{ls} i_{qs} + L_m(i_{qs} + i'_{qr}) \tag{65}$$

$$\Psi'_{dr} = L'_{lr} i'_{dr} + L_m(i_{ds} + i'_{dr}) \tag{66}$$

$$\Psi'_{qr} = L'_{lr} i'_{qr} + L_m(i_{qs} + i'_{qr}) \tag{67}$$

$$\Psi_{dm} = L_m(i_{ds} + i'_{dr}) \tag{68}$$

$$\Psi_{qm} = L_m(i_{qs} + i'_{qr}) \tag{69}$$

Values for currents are obtained by evaluating Equation (64), (65), (66), (67), (68) & (69):

$$i_{ds} = \frac{\Psi_{ds}(L'_{lr} + L_m) - L_m \Psi'_{dr}}{L'_{lr} L_m + L_{ls} L_m + L_{ls} L'_{lr}} \tag{70}$$

$$i_{qs} = \frac{\Psi_{qs}(L'_{lr} + L_m) - L_m \Psi'_{qr}}{L'_{lr} L_m + L_{ls} L_m + L_{ls} L'_{lr}} \tag{71}$$

$$i'_{dr} = \frac{\Psi'_{dr}(L_{ls} + L_m) - L_m \Psi_{ds}}{L'_{lr} L_m + L_{ls} L_m + L_{ls} L'_{lr}} \tag{72}$$

$$i'_{qr} = \frac{\Psi'_{qr}(L_{ls} + L_m) - L_m \Psi_{qs}}{L'_{lr} L_m + L_{ls} L_m + L_{ls} L'_{lr}} \tag{73}$$

$$X_{ml} = \frac{1}{\frac{1}{x_m} + \frac{1}{x_{ls}} + \frac{1}{x'_{lr}}} \tag{74}$$

$$\Psi_{qm} = X_{ml}\left[\frac{\Psi_{qs}}{X_{ls}} + \frac{\Psi'_{qr}}{x'_{lr}}\right] \tag{75}$$

$$\Psi_{dm} = X_{ml}\left[\frac{\Psi_{ds}}{X_{ls}} + \frac{\Psi'_{dr}}{x'_{lr}}\right] \tag{76}$$

$$i_{qs} = \frac{1}{x_{ls}}(\Psi_{qs} - \Psi_{qm}) \tag{77}$$

$$i_{ds} = \frac{1}{x_{ls}}(\Psi_{ds} - \Psi_{dm}) \tag{78}$$

$$i'_{qr} = \frac{1}{x'_{lr}}(\Psi'_{qr} - \Psi_{qm}) \tag{79}$$

$$i'_{dr} = \frac{1}{x'_{lr}}(\Psi'_{dr} - \Psi_{dm}) \tag{80}$$

Substituting Equation (78), (79), (80) & (81) in Equation (60), (61), (62) & (63) we get the following equations:

$$\frac{d\Psi_{ds}}{dt} = \omega_b\left[V_{ds} + \frac{\omega_e}{\omega_b}\Psi_{qs} + \frac{r_s}{x_{ls}}(\Psi_{dm} - \Psi_{ds})\right] \tag{81}$$

$$\frac{d\Psi_{qs}}{dt} = \omega_b\left[V_{qs} - \frac{\omega_e}{\omega_b}\Psi_{ds} + \frac{r_s}{x_{ls}}(\Psi_{qm} - \Psi_{qs})\right] \tag{82}$$

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$$\frac{d\Psi'_{dr}}{dt} = \omega_b\left[V'_{dr} + \frac{(\omega_e-\omega_r)}{\omega_b}\,\Psi'_{qr} + \frac{r'_r}{x'_{lr}}(\Psi_{dm} - \Psi'_{dr})\right] \tag{83}$$

$$\frac{d\Psi'_{qr}}{dt} = \omega_b\left[V'_{qr} - \frac{(\omega_e-\omega_r)}{\omega_b}\,\Psi'_{dr} + \frac{r'_r}{x'_{lr}}(\Psi_{qm} - \Psi'_{qr})\right] \tag{84}$$

Based on above equations torque and speed can be calculated as follows:

$$T_e = \frac{3}{2}\times\frac{p}{2}(\Psi_{ds}i_{qs} - \Psi_{qs}i_{ds}) \tag{85}$$

$$\omega_r = \frac{p}{2J}\int(T_e - T_L)dt \tag{86}$$

Now all the required mathematical equations for 3-Φ induction motor are developed and ready to implement in MATLAB simulation.

## IV. RESULT AND ANALYSIS

In this section 3-Φ induction motor is simulated in MATLAB/SIMULINK using the equations in above section. Developed Simulink model is shown in Fig. 3.

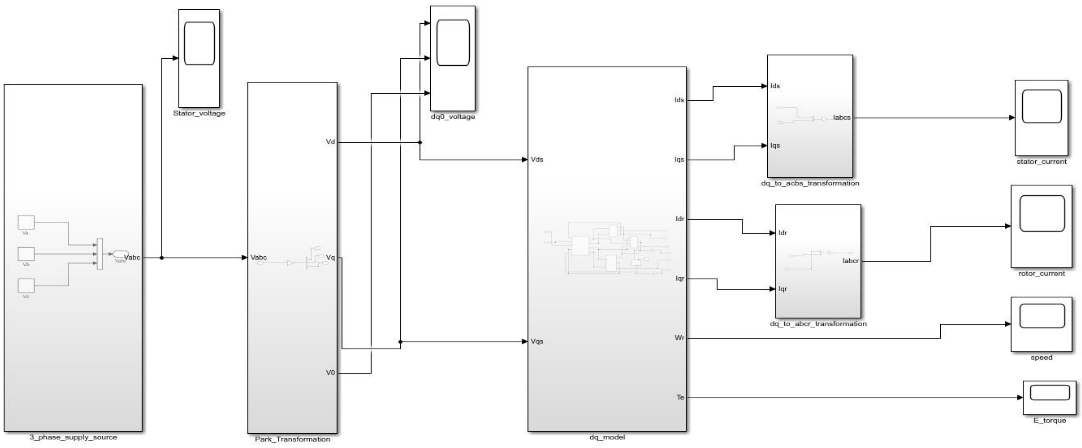


Fig. 3 Simulink model of 3−Φ induction motor

The 3−Φ squirrel cage induction motor of 3 Hp, 230 V, 50 Hz is tested in simulation model. The parameters of the model are given in Table 1.

Table 1 Parameters of 3−Φ squirrel cage induction motor

| Sl No. | Parameters | Values |
|---|---|---|
| 1 | Power | 3 Hp |
| 2 | Phase voltage | 230 V |
| 3 | Frequency | 50 Hz |
| 4 | Pole | 4 |
| 5 | $r_s$ | 0.435Ω |
| 6 | $r_r$ | 0.816 Ω |
| 7 | $x_m$ | 26.13 Ω |
| 8 | $x_{ls}$ | 0.754 Ω |
| 9 | $x_{lr}$ | 0.754 Ω |
| 10 | J | 0.089 |
| 11 | $\omega_b$ | $100\pi$ |

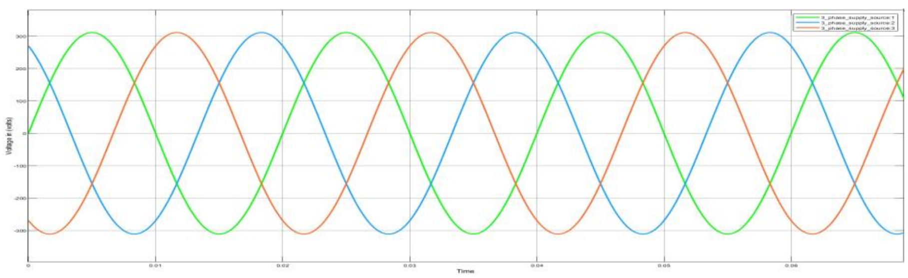

Fig. 4: 3 −Φ stator voltage in natural reference frame

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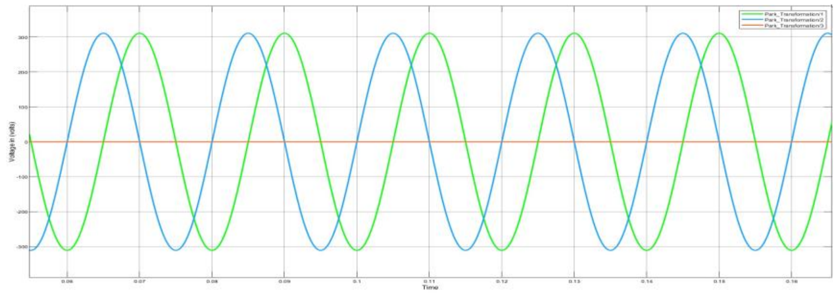

Fig. 5: Stator voltage in arbitrary reference frame

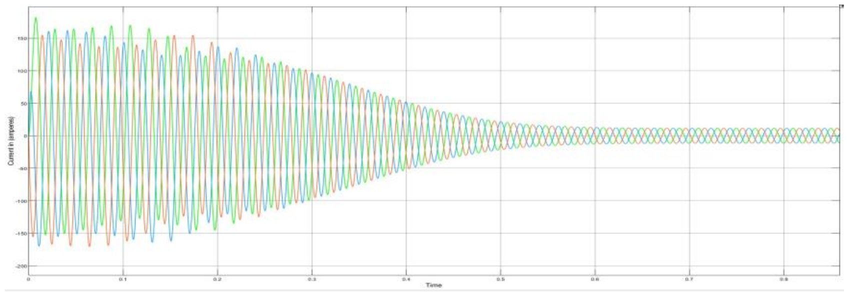

Fig. 6: Stator current

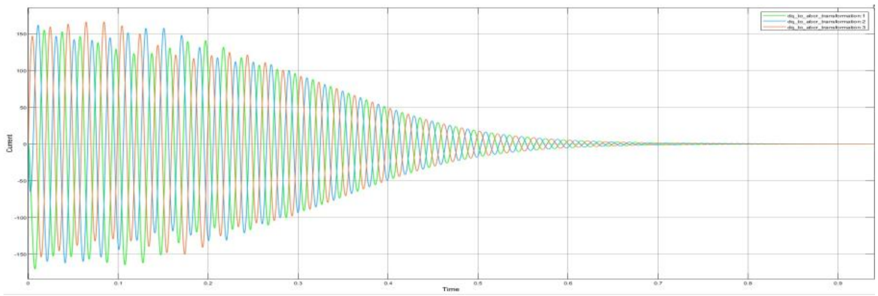

Fig. 7: Rotor current

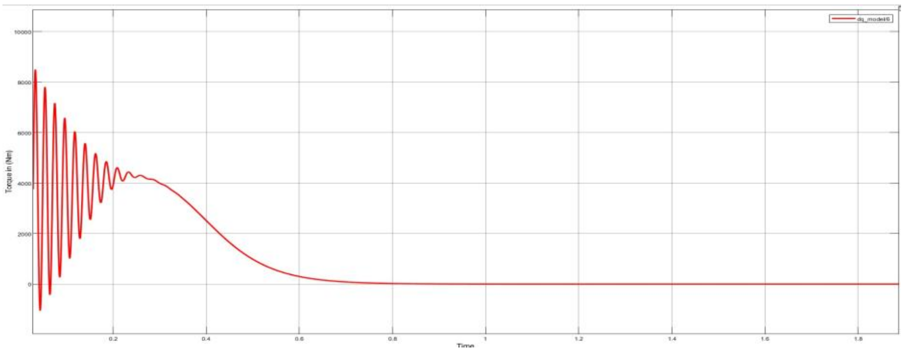

Fig. 8: Electromagnetic torque characteristics

Fig. 9: Angular velocity of rotor in r.p.m

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## V. CONCLUSION

Mathematical model for 3-Φ squirrel cage induction motor has been developed in arbitrary reference frame. Developed model was simulated in MATLAB and its results were compared with conventional model. Comparison was done on the basis of developed electromagnetic torque and rotor speed characteristics. Obtained result showed similar characteristics. Thus, we can conclude that our developed model in MATLAB/SIMULINK software is reliable, comparatively cheap, user-friendly and safer to study the performance and to predict the behaviour of 3-Φ squirrel cage induction motor. Our developed model can be used to study the performance of both the squirrel cage and slip ring induction motor.

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