Session 13 Additional Exercise

Problem Statement 1:

A company manufactures LED bulbs with a faulty rate of 30%. If I randomly select 6 chosen LEDs, what is the probability of having 2 faulty LEDs in my sample? Calculate the average value of this process. Also evaluate the standard deviation associated with it.

Solution:

p = Success = 0.7;

q = Failure = 0.3

1. Calculate Mean

μ = E(x) = n. p

Where:

n = No. of Trials = 6

p = Success ratio = 0.7

$\therefore$ Mean = 6*0.7

= 4.2

2. Standard Deviation

$$\boldsymbol{\sigma = \sqrt{npq}}$$

Where:

n = No. of selected Leds = 6

p = Success ratio = 0.7

q = Failure ratio= 0.3

$\therefore$ Standard Deviation = √6*0.7*0.3

= 1.12

Problem Statement 2:

Gaurav and Barakha are both preparing for entrance exams. Gaurav attempts to solve 8 questions per day with a correction rate of 75%, while Barakha averages around 12 questions per day with a correction rate of 45%. What is the probability that each of them will solve 5 questions correctly? What happens in cases of 4 and 6 correct solutions? What do you infer from it? What are the two main governing factors affecting their ability to solve questions correctly? Give a pictorial representation of the same to validate your answer.

Solution:

Gaurav:

n = 8

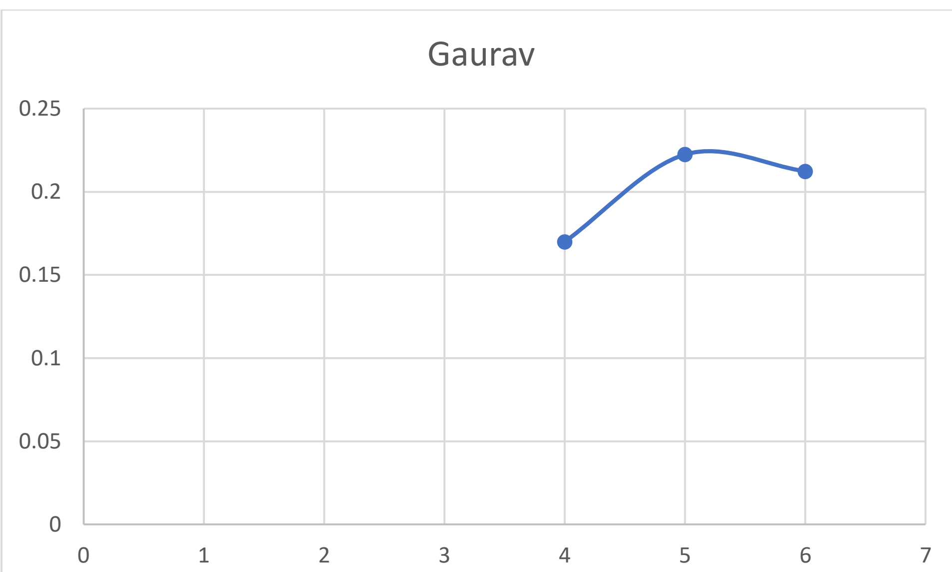
p = 0.75

q = 0.25

p(x =5) = $8c_5 * (0.75)^5 * (0.25)^3$ = 0.155728125

p(x =4) = $8c_4 * (0.75)^4 * (0.25)^4$ = 0.086515625

p(x =6) = $8c_6 * (0.75)^6 * (0.25)^2$ = 0.31146



Barakha:

n = 12

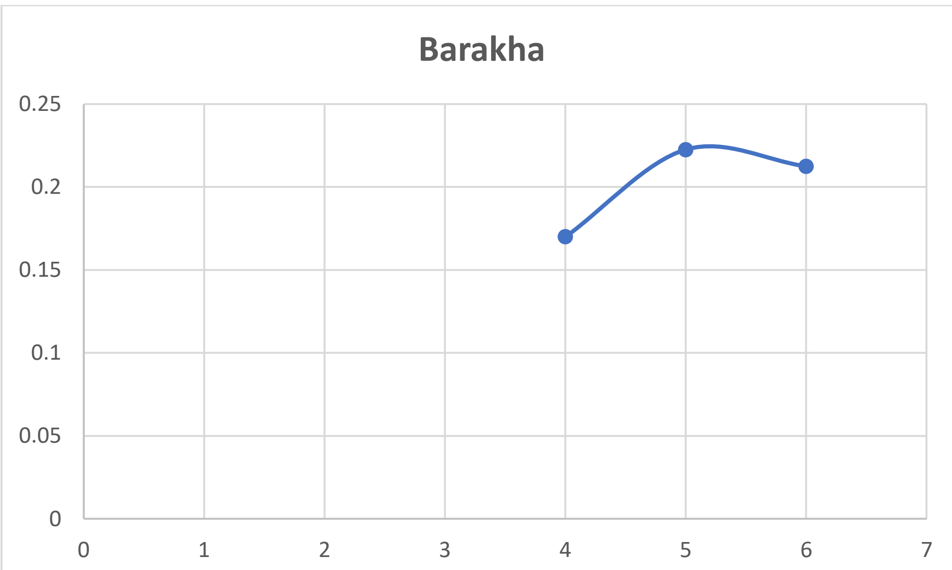
p = 0.45

q = 0.55

p(x =5) = $12c_5 * (0.45)^5 * (0.55)^7$ = 0.22249

p(x =4) = $12c_4 * (0.45)^4 * (0.55)^8$ = 0.169962

p(x =6) = $12c_6 * (0.45)^6 * (0.55)^6$ = 0.212378



Problem Statement 3:

Customers arrive at a rate of 72 per hour to my shop. What is the probability of k customers arriving in 4 minutes? a) 5 customers, b) not more than 3 customers, c) more than 3 customers. Give a pictorial representation of the same to validate your answer.

Solution:

60 minutes - 72 customers are arriving

4 minutes - ?

$\frac{4}{60} * 72 = 4.8$

$*2548.09$

a) **5 Customer**

$$\frac{e^{(-4.8)} * (4.8)^5}{5!} = \frac{0.008229 * 2548.09}{120} = 0.1747$$

b) **Not more than 3**

X=0

$$\frac{e^{(-4.8)} * (4.8)^0}{0} = 0.008229$$

X=1

$$\frac{e^{(-4.8)} * (4.8)^1}{1!} = 0.008229 * 4.8 = 0.0394992$$

X = 2

$$\frac{e^{(-4.8)} * (4.8)^2}{2!} = \frac{0.008229 * 23.04}{2} = 0.09479$$

X = 3

$$\frac{e^{(-4.8)} * (4.8)^3}{3!} = \frac{0.008229 * 110.592}{6} = 0.151676$$

P(xi<=3) = $0.008229 + 0.0394992 + 0.09479 + 0.151676$

= 0.2941942

c) **More than 3 customers**

X = 4

$$\frac{e^{(-4.8)} * (4.8)^4}{4!} = \frac{0.008229 * 530.8}{24} = 0.1820$$

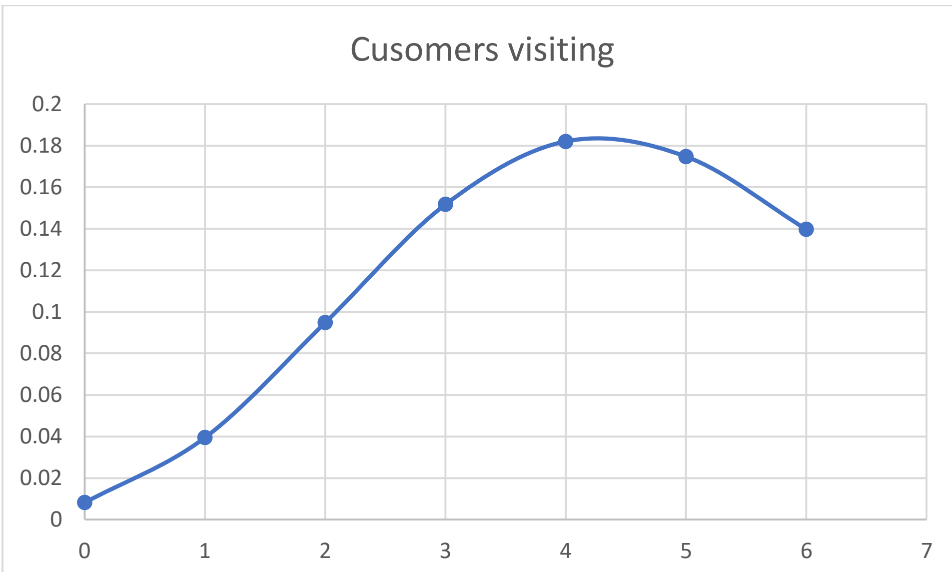
X=5

$$\frac{e^{(-4.8)} * (4.8)^5}{5!} = \frac{0.008229 * 2548.09}{120} = 0.1747$$

X = 6

$$\frac{e^{(-4.8)} * (4.8)^6}{6!} = \frac{0.008229 * 12230.59}{720} = \mathbf{0.1397}$$

P(xi>3) = 0.1820+0.1747+ 0.1397= 0.4964



Problem Statement 4:

I work as a data analyst in Aeon Learning Pvt. Ltd. After analyzing data, I make reports, where I have the efficiency of entering 77 words per minute with 6 errors per hour. What is the probability that I will commit 2 errors in a 455-word financial report?

What happens when the no. of words increases (in case of 1000 words) or decreases (255 words)? How is the λ affected? How does it influence the PMF? Give a pictorial representation of the same to validate your answer.

Solution:

6 errors per hour

Per hour words 77*60 =4620

1 error = $\frac{6}{4620} = \frac{1}{770}$

M = $\frac{455}{770}$ **= 0.591**

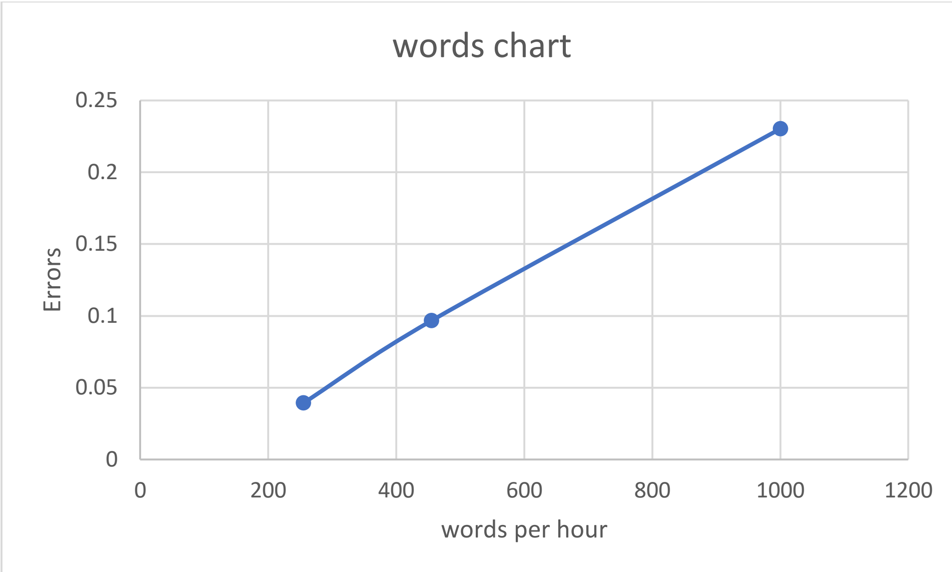
(pxi=2) = $\frac{e^{(-0.591)} * (0.591)^2}{2!} = \frac{0.55377 * 0.349281}{2}$ **= 0.09671**

M = $\frac{1000}{770}$ **= 1.2987**

(pxi=2) = $\frac{e^{(-1.2987)} * (1.2987)^2}{2!} = \frac{0.2731 * 1.6866}{2}$ **= 0.2303**

M = $\frac{255}{770}$ **= 0.3311**

(pxi=2) = $\frac{e^{(-0.3311)} * (0.3311)^2}{2!} = \frac{0.7181 * 0.1962}{2}$ **= 0.0393621**



Problem Statement 5:

The current measured in a copper wire is modelled by a continuous random variable X. X is in milliamperes. Assume that the range of X is [0, 20 mA]. The probability density function is given by, f(x) = 0.05 for 0 ≤ x ≤ 20. What is the probability that a current measurement is less than 10 milliamperes? Draw the PDF and the CDF diagrams as well.

f(x)= 0.05 0 ≤ x ≤ 20

$\int_0^{10} f(x).dx = \int_0^{10} 0.05 = [0.05]_0^{10} = 0.5 - 0 = 0.5$