Modeling the Motion of a Mousetrap Car with Differential Equations

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Synopsis:

The purpose of this project was to experimentally determine a system of differential equations to model the motion of a mousetrap car.

Interest:

My interest in this topic stems from my Science Olympiad experience this year. I went through 4 different variations of a mousetrap car for the competition before finally arriving at the last one. At the competition, I guesstimated the amount of string I needed to go the distance traveled. I am interested in seeing whether I can use differential equations to model the movement of the car in order to more accurately achieve certain distances.

Topic:

Systems of Differential Equations (specifically the position-velocity system and the velocity-acceleration system).

Summary

I want to find out how exactly my mousetrap car accelerates in comparison to the distance it travels. Since the length of the string required to achieve this distance can be found from a simple calculation, I would then be able to predict how my car will move using a system of differential equations.

Methodology

1. Mark 8 meters of pavement on sidewalk
2. Wind mousetrap car string to fullest extent
3. Release mousetrap car
4. As moustrap car runs, record time it reaches each meter mark with stopwatch
5. Repeat steps 2 - 4 two more times
6. Calculate average velocity by calculating velocity between each two points (1 and 2, 2 and 3, and so on)
7. Use velocity calculations to calculate average acceleration between each two points (same as above)
8. Use final conditions to solve for differential equations (with respect to t)

Assumptions

1. I assume that the surface is constant throughout to simplify my calculations, whereas in reality, pavement is not smooth at all
2. I assume that the mousetrap will perform the exact same way every run
3. I assume that the mousetrap car goes perfectly straight

Analysis

Using the equations $v = \frac{\Delta d}{\Delta t}$ and $a = \frac{\Delta v}{\Delta t}$, we can calculate the average velocity and acceleration of the mousetrap car from this data.

Trial 1 Data

Points	Time (t)	Δt	Distance (d)	Δd	Velocity (v)	Δv	Acceleration
(0, 1)	3.13 s	3.13 s	1 m	1 m	0.319 $\frac{m}{s}$	0.319 $\frac{m}{s}$	0.102 $\frac{m}{s^2}$
(1, 2)	4.41 s	1.28 s	2 m	1 m	0.781 $\frac{m}{s}$	0.462 $\frac{m}{s}$	0.361 $\frac{m}{s^2}$
(2, 3)	5.44 s	1.03 s	3 m	1 m	0.971 $\frac{m}{s}$	0.19 $\frac{m}{s}$	0.184 $\frac{m}{s^2}$
(3, 4)	6.30 s	.86 s	4 m	1 m	1.163 $\frac{m}{s}$	0.192 $\frac{m}{s}$	0.223 $\frac{m}{s^2}$
(4, 5)	7.11 s	.81 s	5 m	1 m	1.235 $\frac{m}{s}$	0.072 $\frac{m}{s}$	0.089 $\frac{m}{s^2}$
(5, 6)	7.84 s	.73 s	6 m	1 m	1.369 $\frac{m}{s}$	0.134 $\frac{m}{s}$	0.184 $\frac{m}{s^2}$
(6, 7)	8.49 s	.65 s	7 m	1 m	1.538 $\frac{m}{s}$	0.169 $\frac{m}{s}$	0.260 $\frac{m}{s^2}$
(7, 8)	8.92 s	.43 s	8 m	1 m	2.326 $\frac{m}{s}$	0.788 $\frac{m}{s}$	1.833 $\frac{m}{s^2}$
Average	N/A	N/A	N/A	N/A	1.213 $\frac{m}{s}$	N/A	0.405 $\frac{m}{s^2}$

x

Trial 2 Data

Points	Time (t)	Δt	Distance (d)	Δd	Velocity (v)	Δv	Acceleration
(0, 1)	2.66 s	2.66 s	1 m	1 m	0.376 $\frac{m}{s}$	0.376 $\frac{m}{s}$	0.141 $\frac{m}{s^2}$
(1, 2)	3.79 s	1.13 s	2 m	1 m	0.885 $\frac{m}{s}$	0.509 $\frac{m}{s}$	0.450 $\frac{m}{s^2}$
(2, 3)	4.78 s	.99 s	3 m	1 m	1.011 $\frac{m}{s}$	0.126 $\frac{m}{s}$	0.127 $\frac{m}{s^2}$
(3, 4)	5.75 s	.97 s	4 m	1 m	1.031 $\frac{m}{s}$	0.020 $\frac{m}{s}$	0.021 $\frac{m}{s^2}$
(4, 5)	6.54 s	.79 s	5 m	1 m	1.266 $\frac{m}{s}$	0.235 $\frac{m}{s}$	0.297 $\frac{m}{s^2}$
(5, 6)	7.27 s	.73 s	6 m	1 m	1.369 $\frac{m}{s}$	0.103 $\frac{m}{s}$	0.141 $\frac{m}{s^2}$
(6, 7)	7.70 s	.43 s	7 m	1 m	2.326 $\frac{m}{s}$	0.957 $\frac{m}{s}$	2.226 $\frac{m}{s^2}$
(7, 8)	8.20 s	.42 s	8 m	1 m	2.381 $\frac{m}{s}$	0.055 $\frac{m}{s}$	0.131 $\frac{m}{s^2}$
Average	N/A	N/A	N/A	N/A	1.213 $\frac{m}{s}$	N/A	0.442 $\frac{m}{s^2}$

Trial 3 Data

Points	Time (t)	Δt	Distance (d)	Δd	Velocity (v)	Δv	Acceleration
(0, 1)	3.12 s	3.12 s	1 m	1 m	0.321 $\frac{m}{s}$	0.321 $\frac{m}{s}$	0.103 $\frac{m}{s^2}$
(1, 2)	4.29 s	1.17 s	2 m	1 m	0.855 $\frac{m}{s}$	0.534 $\frac{m}{s}$	0.456 $\frac{m}{s^2}$
(2, 3)	5.39 s	1.10 s	3 m	1 m	0.909 $\frac{m}{s}$	0.319 $\frac{m}{s}$	0.290 $\frac{m}{s^2}$
(3, 4)	6.20 s	.81 s	4 m	1 m	1.235 $\frac{m}{s}$	0.326 $\frac{m}{s}$	0.402 $\frac{m}{s^2}$
(4, 5)	6.93 s	.73 s	5 m	1 m	1.369 $\frac{m}{s}$	0.134 $\frac{m}{s}$	0.184 $\frac{m}{s^2}$
(5, 6)	7.60 s	.67 s	6 m	1 m	1.493 $\frac{m}{s}$	0.124 $\frac{m}{s}$	0.185 $\frac{m}{s^2}$
(6, 7)	8.15 s	.55 s	7 m	1 m	1.818 $\frac{m}{s}$	0.325 $\frac{m}{s}$	0.591 $\frac{m}{s^2}$
(7, 8)	8.58 s	.43 s	8 m	1 m	2.326 $\frac{m}{s}$	0.508 $\frac{m}{s}$	1.181 $\frac{m}{s^2}$
Average	N/A	N/A	N/A	N/A	1.291 $\frac{m}{s}$	N/A	.424 $\frac{m}{s^2}$

Some of the data does not add up, likely due to random error when I was operating the stopwatch.

Now we can solve the following equations (replacing the variable y with distance)

$$\frac{dy}{dt} = v$$

$$\frac{dv}{dt} = mv + ny + c$$

First, we have that at the point y = 8, v = 1.239 (on average)

$$\frac{dy}{dt} = v = 1.239$$

$$\frac{dv}{dt} = m * 1.239 + n * 8 = .424$$

If we guess n = 1, we can get m = -6.115. We then have the following equations, from which we can now solve for a function of y with respect to t.

$$\frac{dy}{dt} = v$$

$$\frac{dv}{dt} = -6.115v + y$$

First, substitute the first equation into the second equation to get

$$\frac{dv}{dt} = -6.115\frac{dy}{dt} + y$$

$\frac{dv}{dt}$ can be expressed as $\frac{dy^2}{d^2t}$, so substitute and move terms to get a damped harmonic oscillator.

$$\frac{dv}{dt} + 6.115\frac{dy}{dt} - y = 0$$

Using the guessing method, we find the solutions to this equation to be

$$y = e^{.159t} \text{ or } y = e^{-6.27t}$$

Conclusion

From the above calculations and data, we see that it is indeed possible to generate an equation for the distance a mousetrap car will travel with respect to time. Furthermore, due to the part where I guessed the value of n, I conjecture that there are in fact an infinite number of differential equations that can explain this system. They might lead to the same solutions for y, but the systems themselves are infinite.

Extension

I could use the research in this project to build a better mousetrap car, one that maximizes distance and speed. I could also use the equation I found to better improve the accuracy of my current mousetrap car by extending the equation to include the length of the winding string.