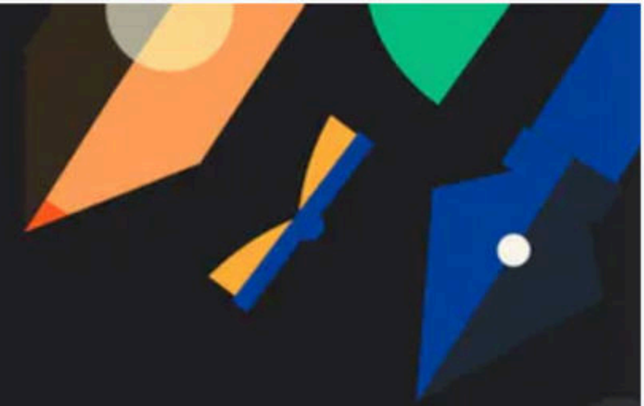


addition
minimum
k query
p2uery

Square Root Decomposition on Arrays

Course on Square Root Decomposition

2ueries
Trees (Bonus)
MO's algorithm

Utkarsh Gupta • Lesson 2 • June 23, 2021

B

we choose the best B

$$O\left(B + \frac{N}{B}\right)$$

Best value of B, if $T = O\left(B + \frac{N}{B}\right)$

A. $N^{1/2}$

B. $N^{1/3}$

C. $N^{1/4}$

D. N

→ Calculus

→ AM = GM

$B = \frac{N}{B}$

$\Rightarrow B = \sqrt{N}$

Best value of B, if $T = O\left(B^2 + \frac{N}{B}\right)$

A. $N^{1/2}$

B. $N^{1/3}$ ✓✓

C. $N^{1/4}$

D. $(N \log N)^{1/2}$

E. $N^{1/2}(\log N)$

$B^2 = \frac{N}{B}$

$B = \sqrt[3]{N}$

Best value of B, if $T = O\left(B + \frac{N \log N}{B}\right)$

A. $N^{1/2}$

B. $N^{1/3}$

C. $N^{1/4}$

D. $(N \log N)^{1/2}$

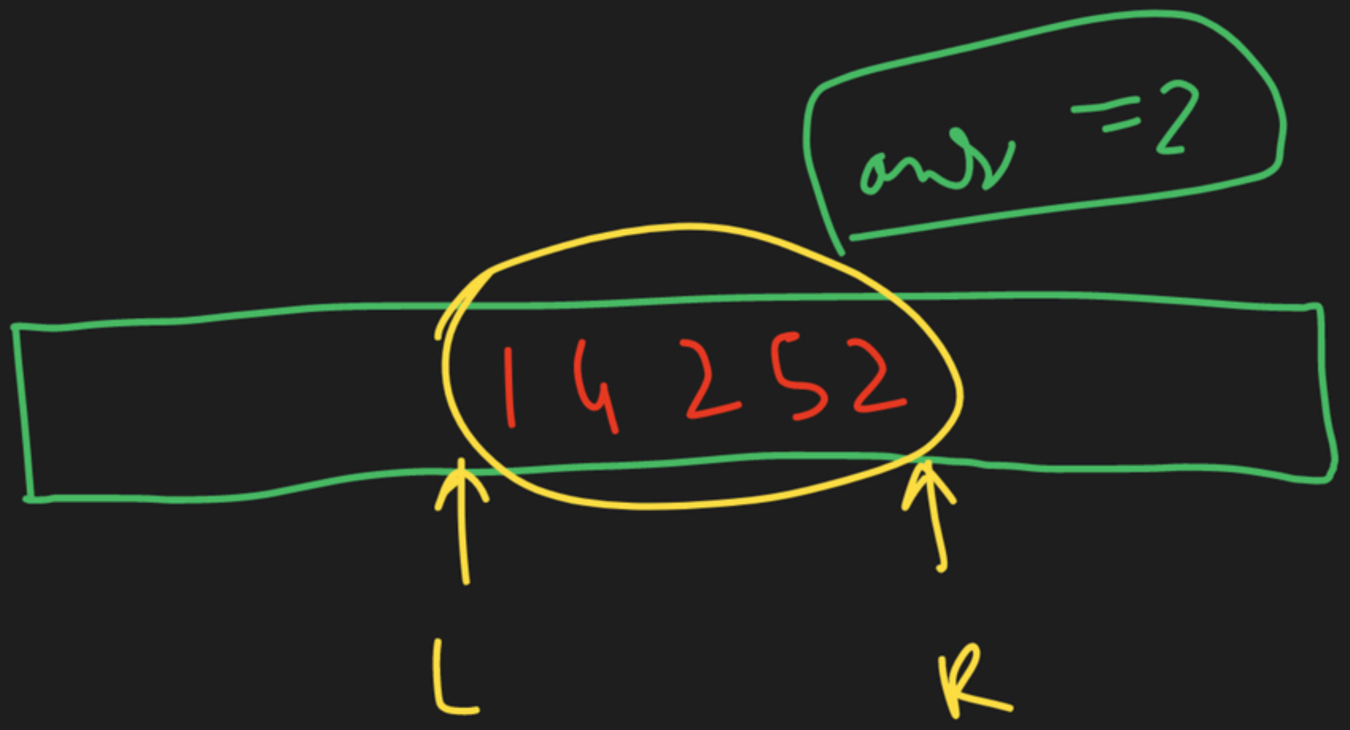
E. $N^{1/2}(\log N)$

$K = N \log N$

$O\left(B + \frac{K}{B}\right)$

$B = \sqrt{K}$

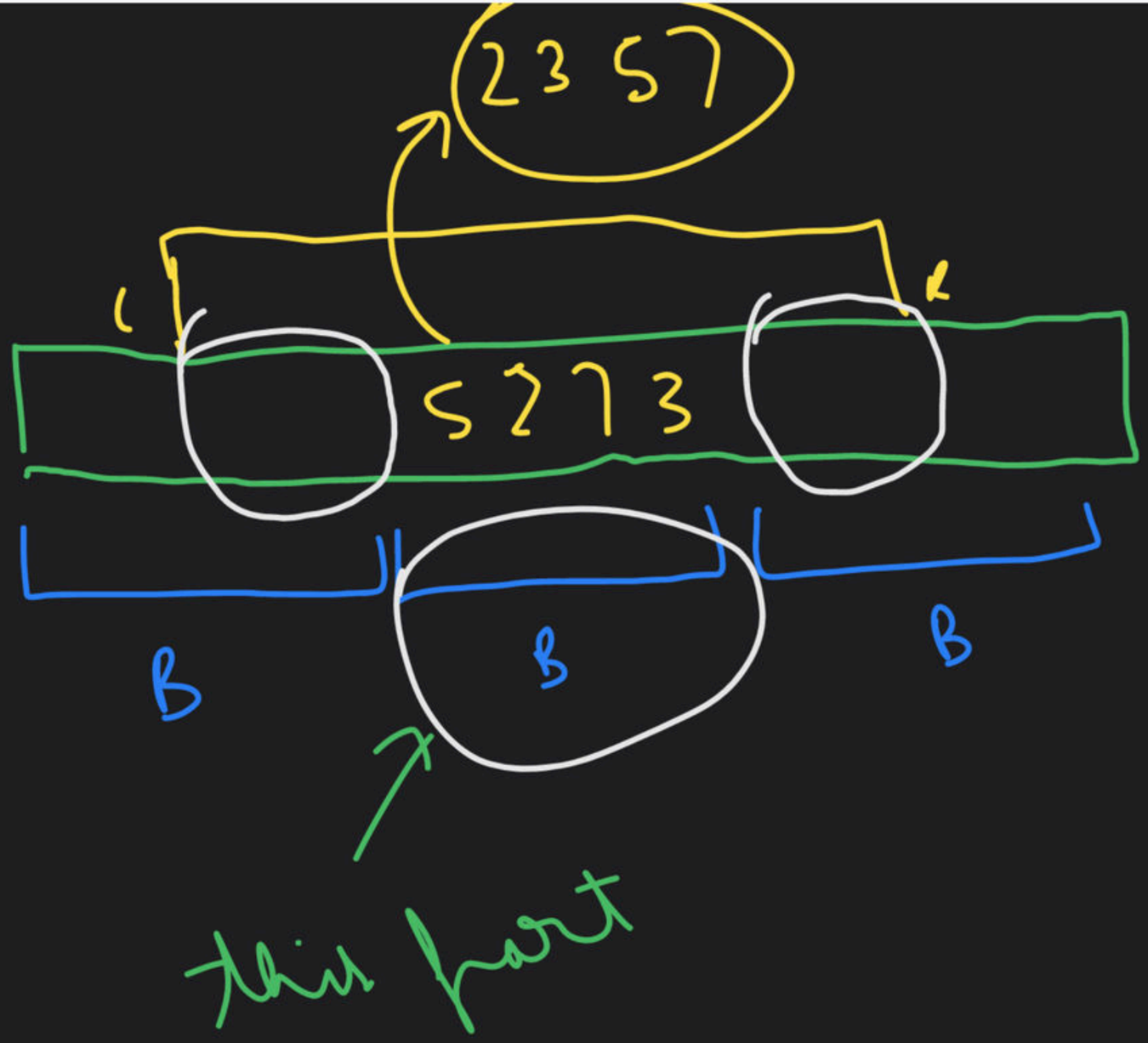
$= \sqrt{N \log N}$



(L, R, x)

3

update → (idx, val)



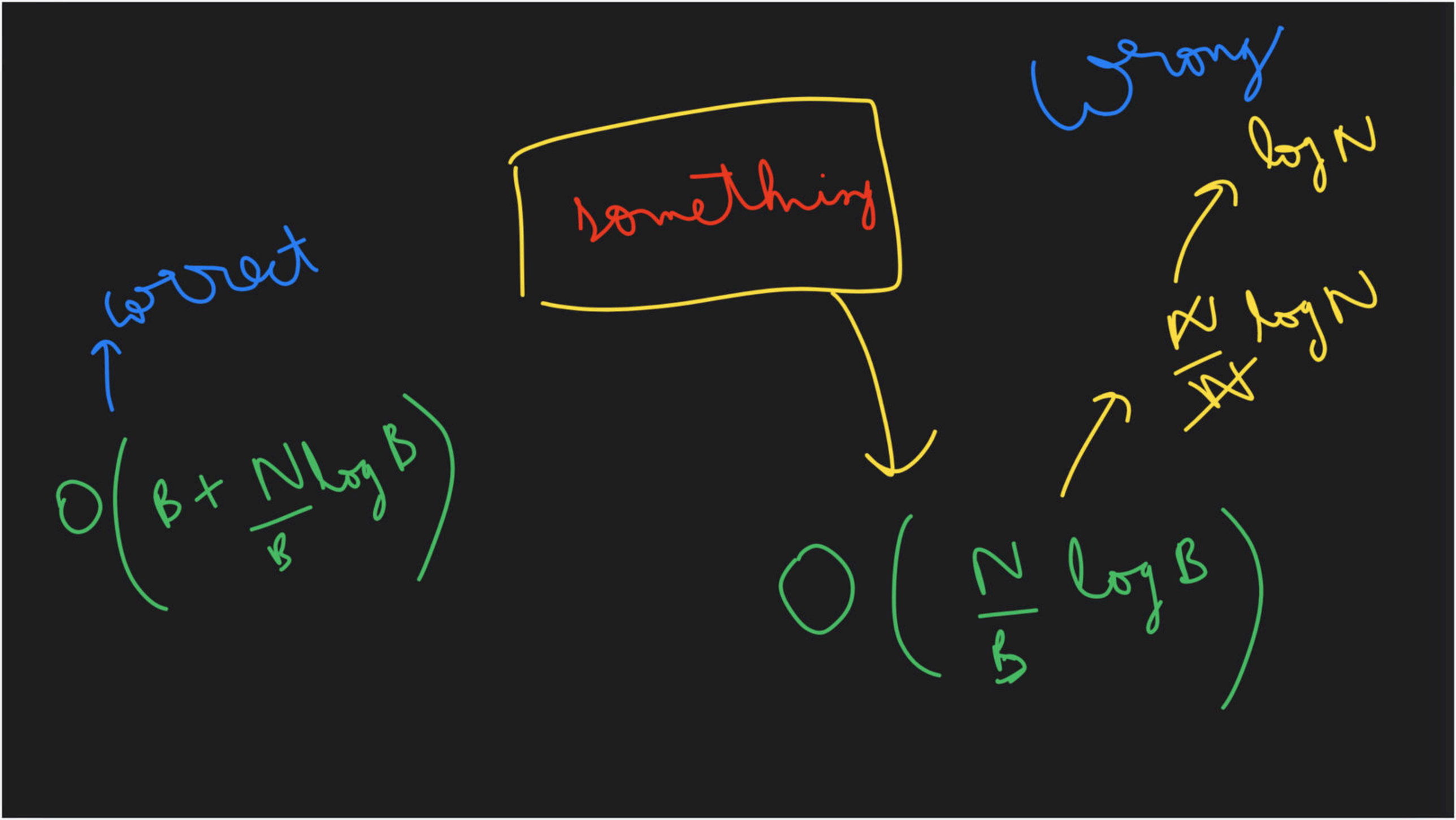
$> \quad \geq$

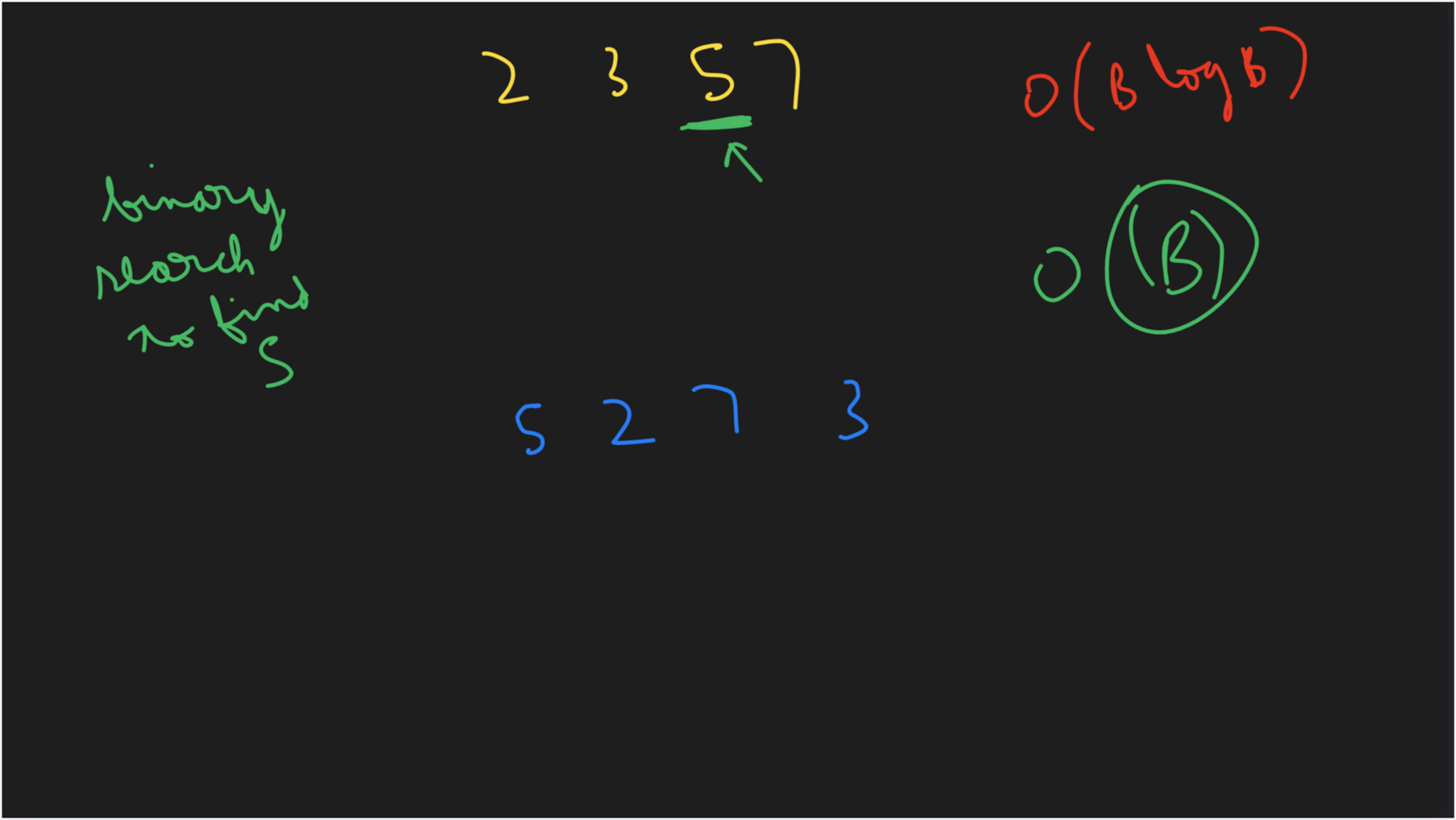
$= \quad =$

$< \quad <$

$O(B)$ unknown constant

$O\left(\frac{N}{B} \cdot \log B\right)$


correct
$O\left(B + \frac{N}{B} \log B\right)$
something
$O\left(\frac{N}{B} \log B\right)$
wrong
$\log N$
$\not{N} \log N$


2 3 5 7
O(B log B)
binary search to find 5
O(B)
5 2 7 3

$O(B)$

$O\left(B + \frac{N}{B} \log B\right)$ ← SES

queries only !!

$$O\left(B + \frac{N}{B}\log B\right) \approx O\left(B + \frac{N}{B}\log\sqrt{N}\right)$$

Approx best value of B !

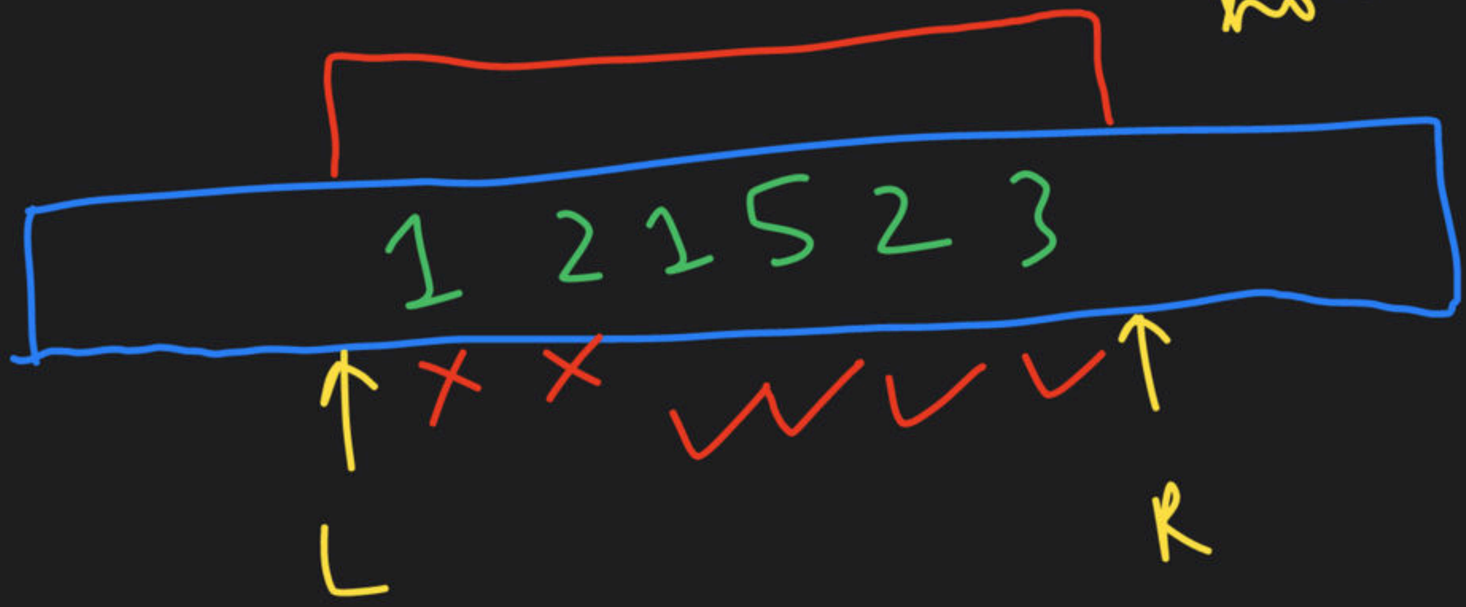
B = slightly $> \sqrt{N}$

$B \approx \sqrt{N}$

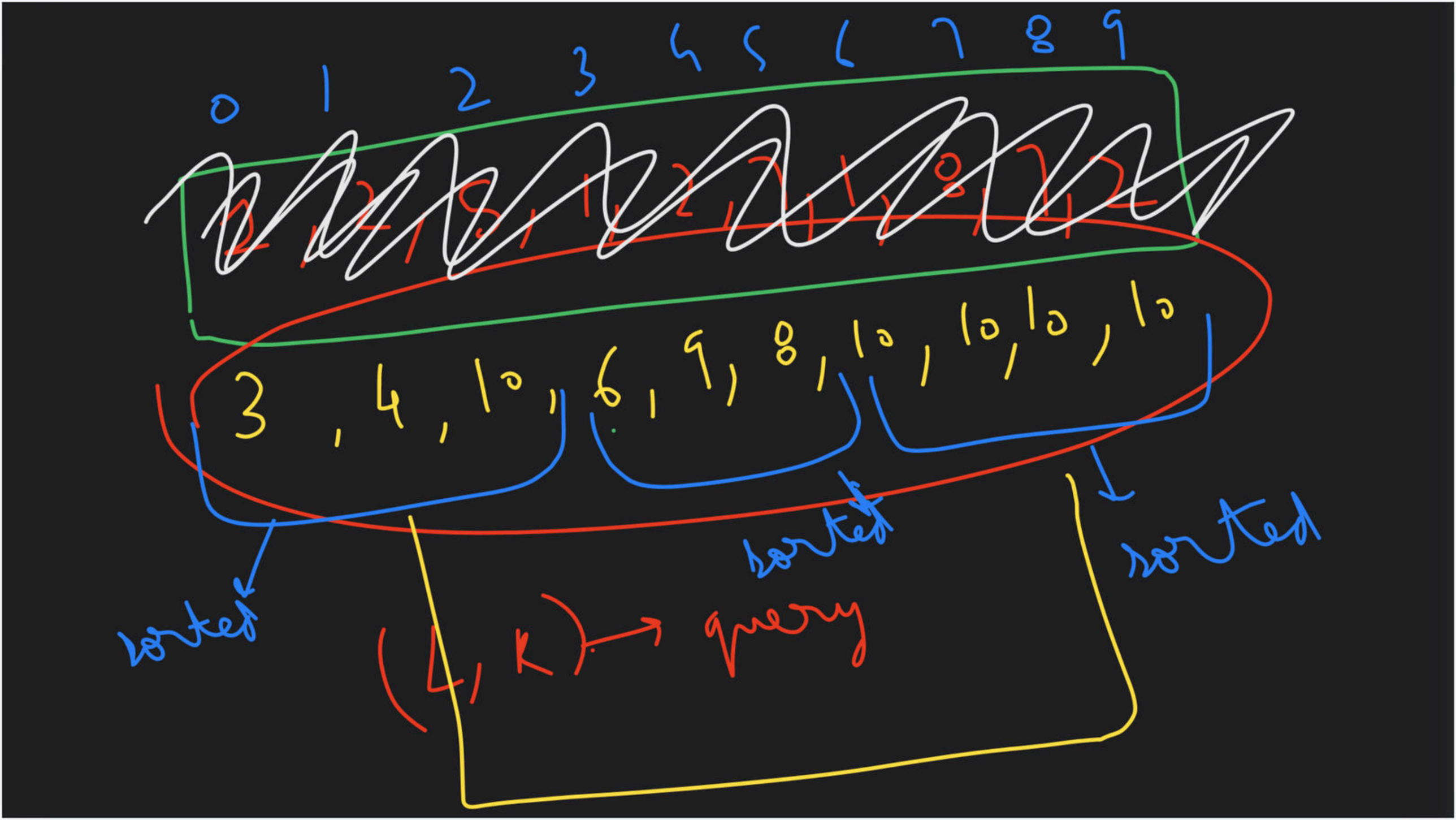
$$O\left(B + \frac{N \log \sqrt{N}}{B}\right)$$

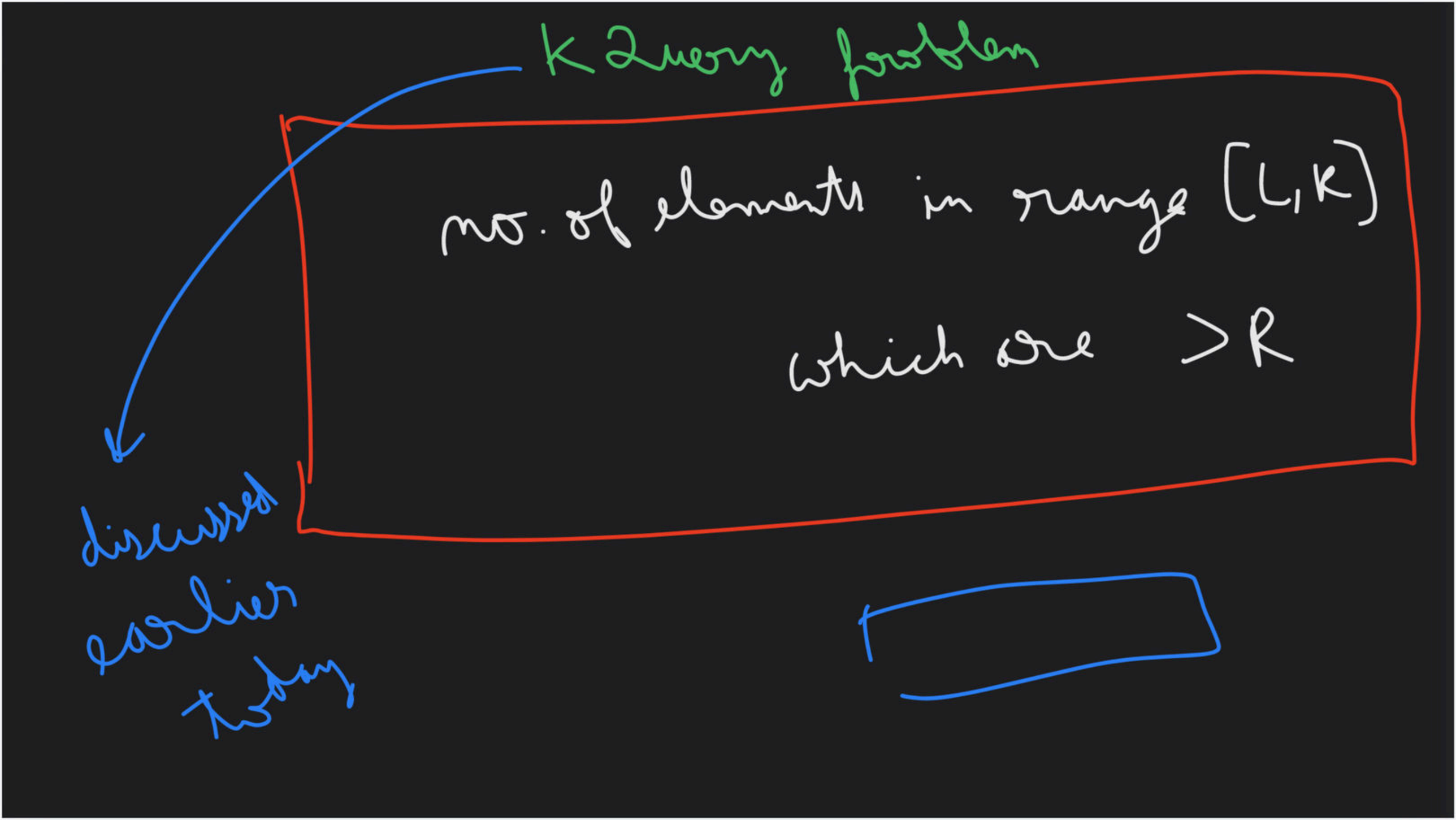
$$B \simeq \sqrt{N \log \sqrt{N}} \simeq \sqrt{\frac{N \log N}{2}}$$

D2 query → Sliding

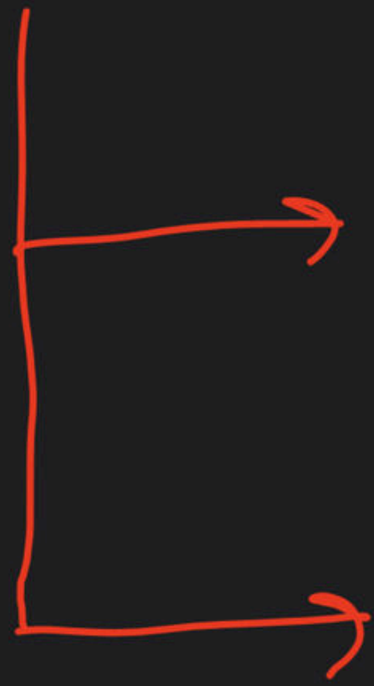


how many distinct elements


0 1 2 3 4 5 6 7 8 9
3 , 4 , 10 , 6 , 9 , 8 , 10 , 10 , 10 , 10
sorted
sorted
sorted
(L, R) → query

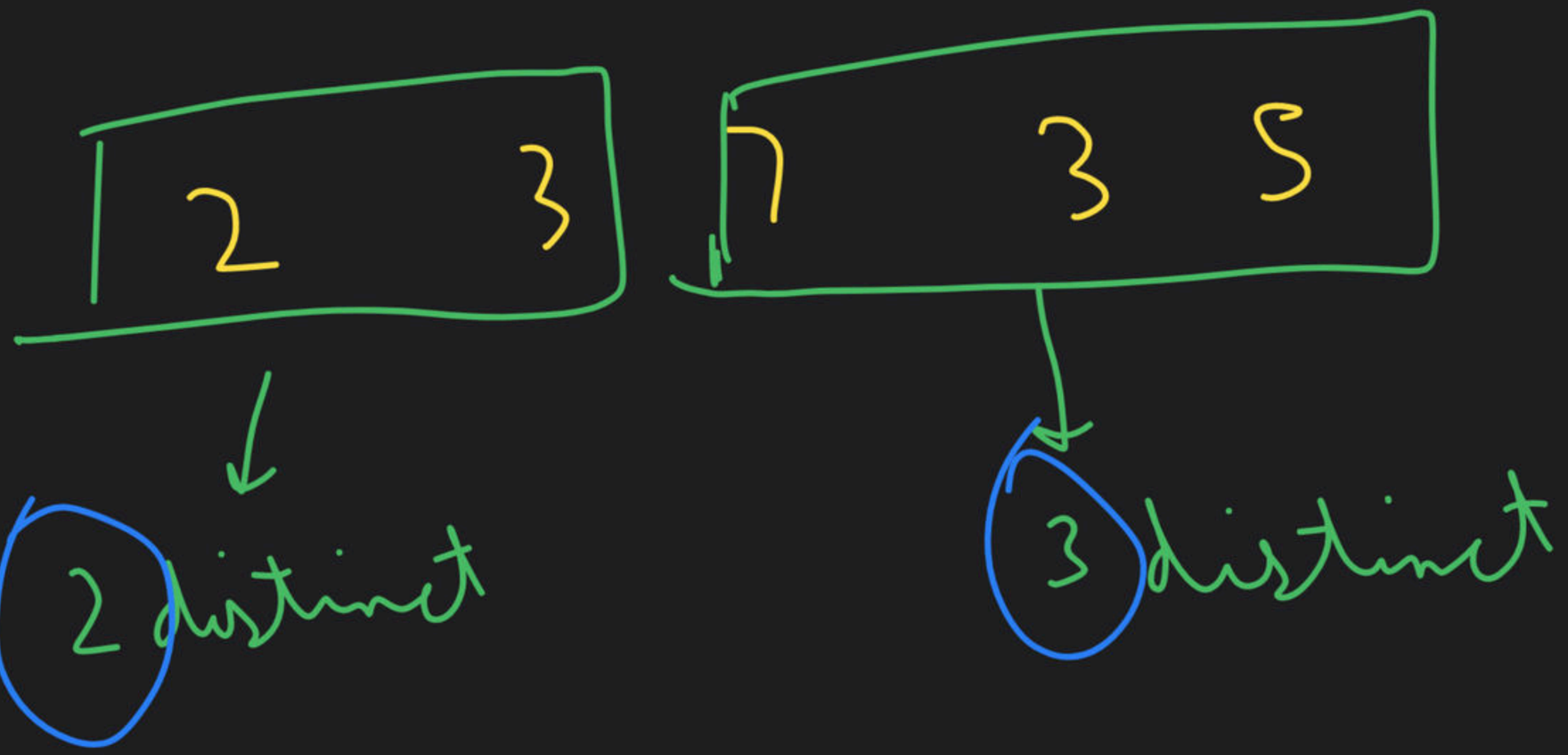

k2query problem
no. of elements in range [L,R]
which are > K
discussed earlier today

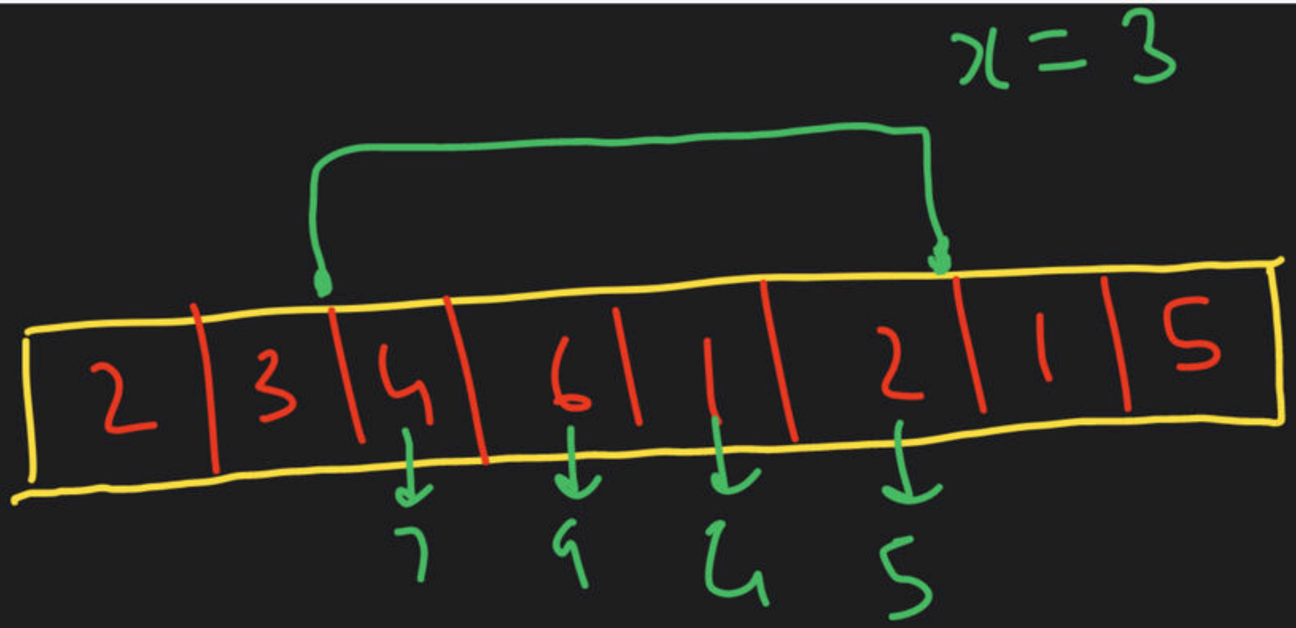
prefix sum



combinable
(made up term)

inversion
(add ↔ subtract)


2
3
7
3
5
2 distinct
3 distinct

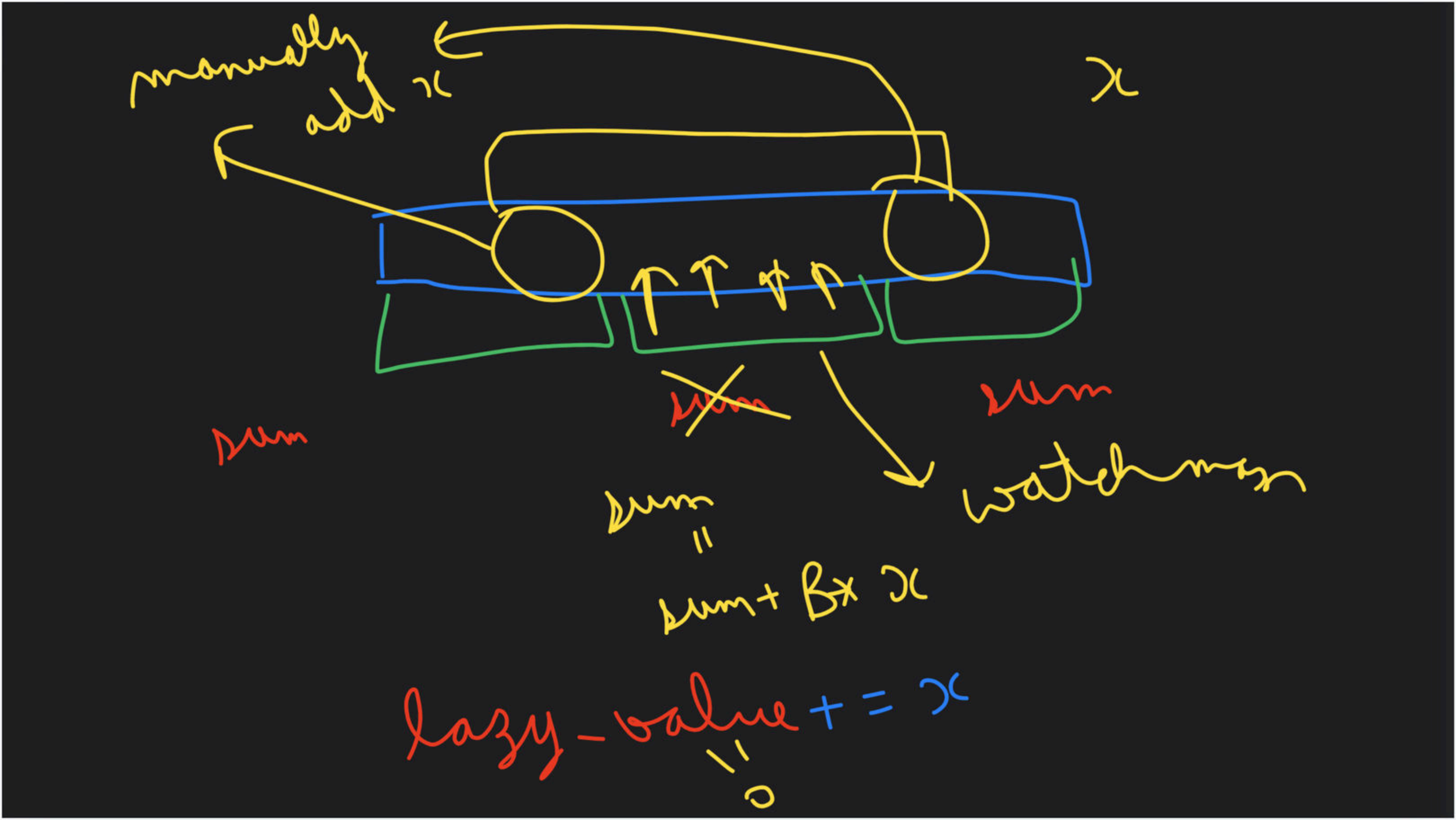


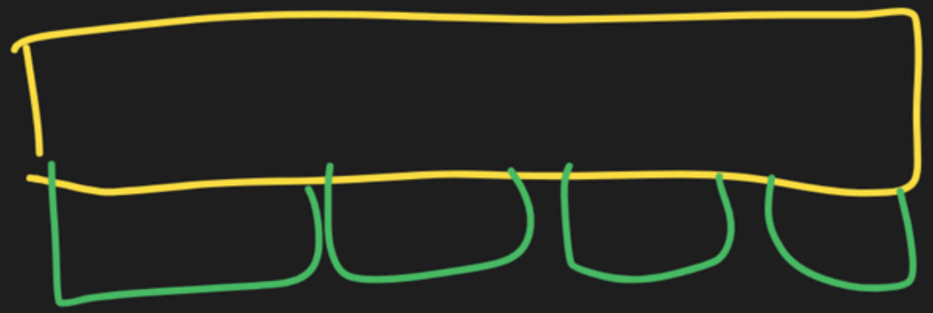
using sqrt decomposition

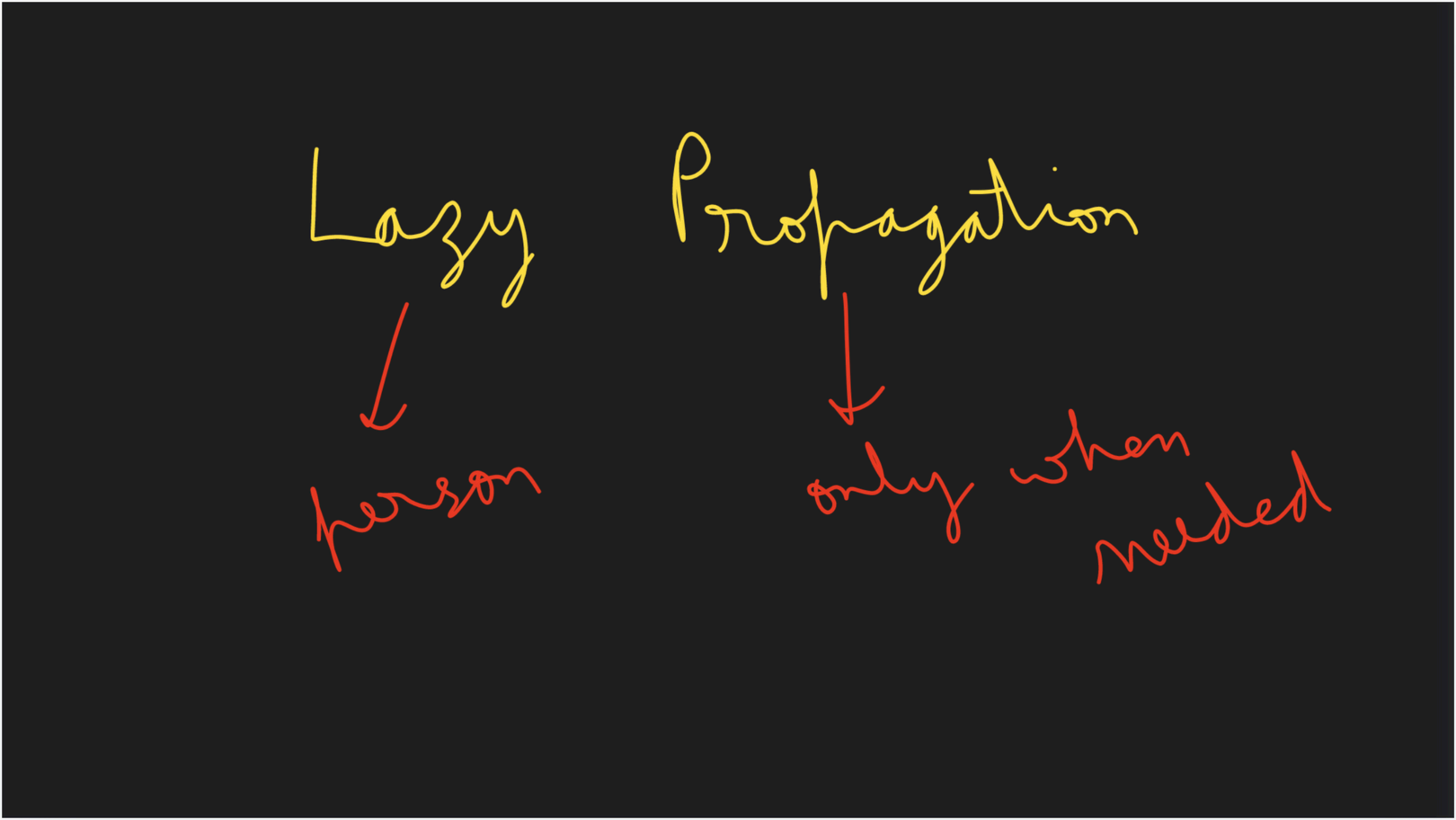
Q

1. $[L, R]$ → sum of elements

2. $[L, R, x]$ → add x to all elements in range $[L \ldots R]$

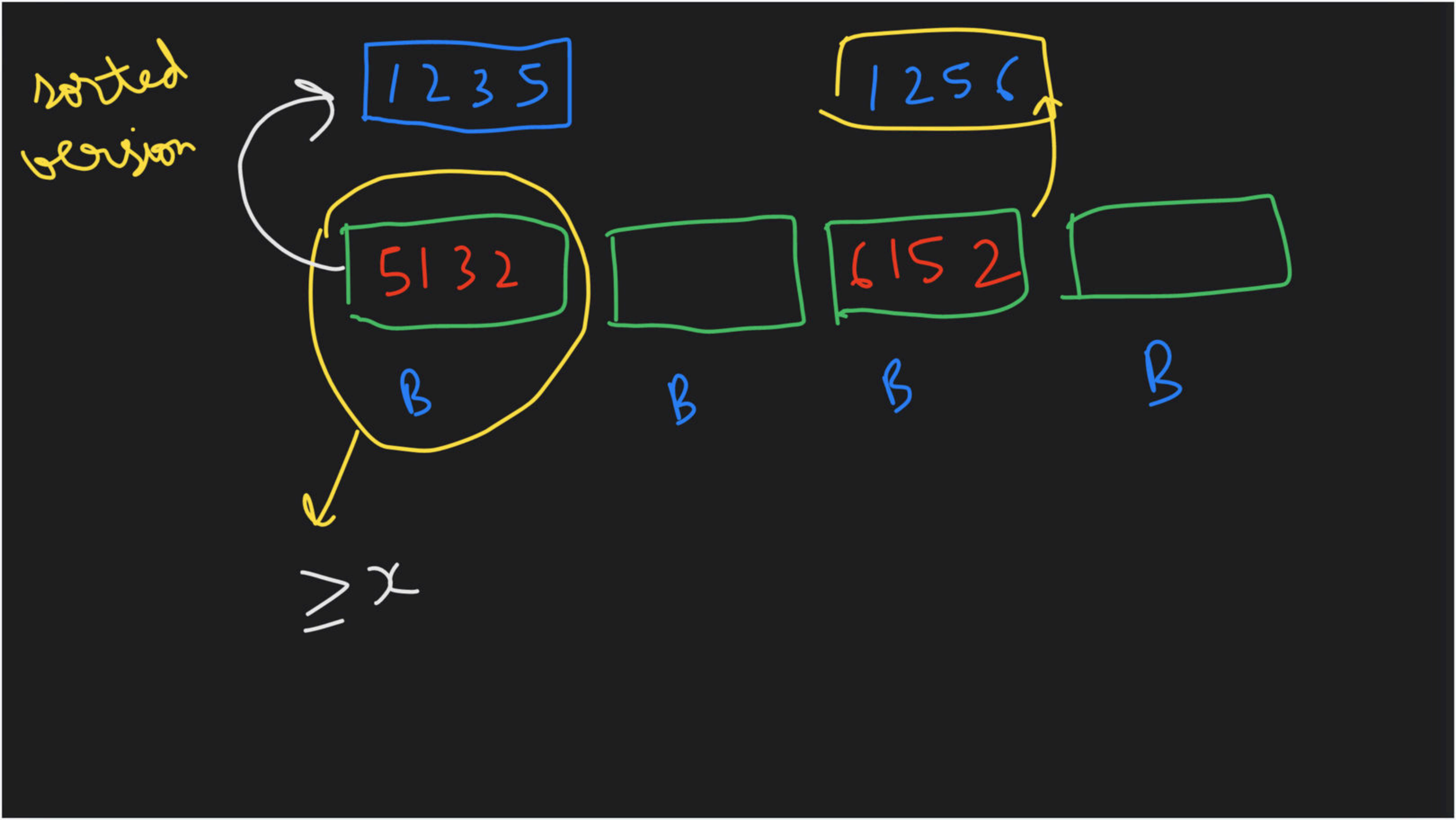

manually add x
x
sum
sum
sum
watchman
sum
=
sum + B * x
lazy_value += x
=




Lazy
Propagation
person
only when needed

greater → upper bound

>= → lower bound


sorted version
1235
1256
5132
6152
B
B
B
B
≥ x