Coprime Subsequence

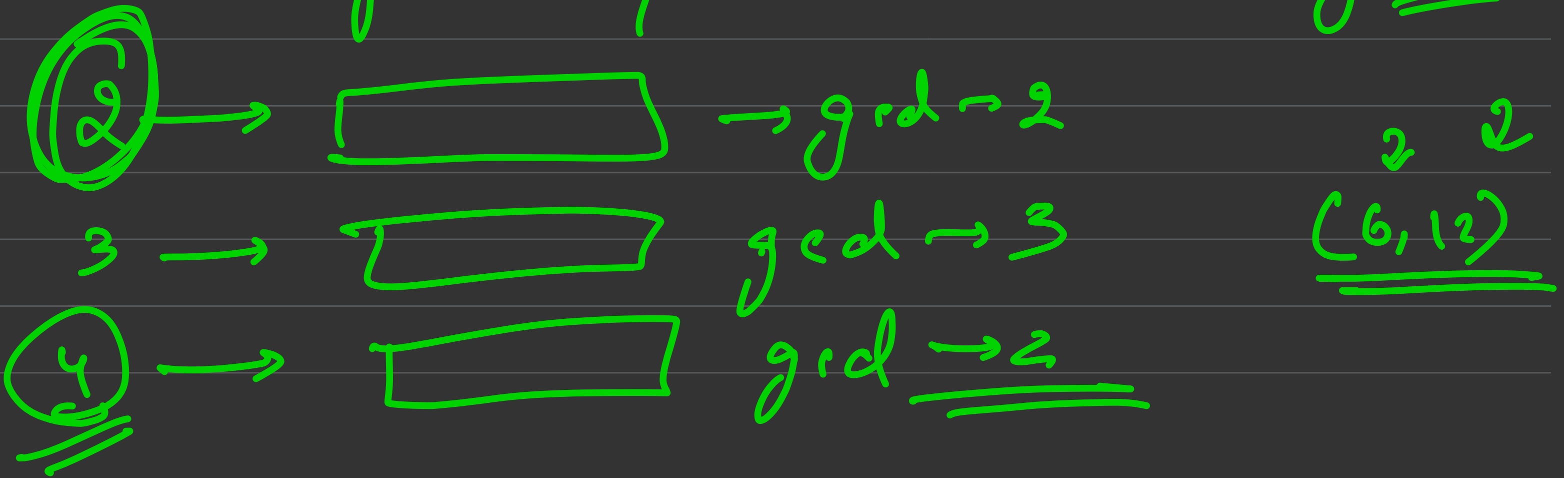
$n \to 10^5$ → 1) compute all subsequence xxx

↳ Instead of calc. all subsequence with gcd 1

↳ We will calc all other subsequence with gcd > 1 & then remove them from all possible subsequence.

n → 2 − 1 → (all other subseq with gcd>1)

Let's say we've a number x, we need to calc no. of subsequences divisible by x.



$2\ 2$

$T = (2) - (3) - (5) + (6) - (7) + (10)$

$- (11) + (13) + (14) + (15) - - - - - -$

$T = \mu(x) \times (2^{f(x)} - 1)$ → log

ans

prev

$n \log\log n + n \log n$

Q→ Given 2 no.'s n, m such that m divides n, prove that f_m divides f_n when f_x denotes fibonacci

↳ $m | n$ → m divides n

To prove → $f(m) | f(n)$ given $m | n$

$$f_n = f_{n-1} + f_{n-2}$$

$m \mid n$ → then $\boxed{n = mx}$

$\boxed{\text{PMI}}$

↳ assum that for any value $\underline{\underline{k}}$

$f(ka)$ is divisible by $\underline{f(a)}$ as $\underline{a \mid ka}$

$f_{(k+1)a}$ is divisible by f_a → To Prove

we can directly say $\underline{a \mid (k+1)a}$

$$\boxed{f_{m+n} \rightarrow f_{m+1} f_n + f_m f_{n-1}}$$ ✓

$$f_{a+ak} = f_{a+1} f_{ak} + f_a f_{ak-1}$$

$$\underline{f_{(a+1)k}} = \underbrace{f_{ak} f_{a+1} + f_a f_{ak-1}}_{\text{↳ divisible by } f_a}$$

$\underline{HP}$

$$\boxed{f_{m+n} \rightarrow f_{m+1} f_n + f_m f_{n-1}}$$

$f_{x+1} = f_{x+1} \times f_1 + f_x \times f_0$ $\boxed{\begin{matrix} f_1 = 1 \\ f_0 = 0 \end{matrix}}$

This holds true → Base case

assume it's true for

$f_{x+k} = f_{x+1} f_k + f_x f_{k-1}$

$$\underline{\underline{f_{x+k+1}}} = f_{x+k} + f_{x+k-1} \qquad \underline{\underline{\text{(fibonacci)}}}$$

$$= \left(f_{x+1} f_k + f_x f_{k-1}\right) + \left(f_{x+1} f_{k-1} + f_x f_{k-2}\right)$$

$$= f_{x+1}\left(f_k + f_{k-1}\right) + f_x\left(f_{k-1} + f_{k-2}\right)$$

$$= f_{x+1} f_{k+1} + f_x f_k$$

$$\underline{\underline{\text{HP}}}$$

Q. Prove that consecutive fibonacci are co-prime.

$$\gcd(f_n, f_{n+1}) = 1$$

$$\gcd(f_1, f_2) = 1$$

assume → $\gcd(f_k, f_{k+1}) = 1$

To proove — $\underline{\underline{\gcd(f_{k+1}, f_{k+2}) = 1}}$

$$\gcd(f_{k+1}, f_{k+2}) = \gcd(f_{k+1}, f_{k+1} + f_k)$$

$$\underline{\gcd(a+b, a)} = \underline{\gcd(a, b)} \quad \checkmark\checkmark$$

$$\underbrace{\gcd(f_{k+1}, f_{k+1}+f_k)}_{\boxed{HP}} = \gcd(f_{k+1}, f_k) = \underline{\underline{1}}$$

1) $\rightarrow \gcd(f_k, f_{k+1}) = 1$

2) $f_{m+n} = f_{m+1} f_n + f_m f_{n-1}$

3) $m \mid n \longrightarrow \underline{\underline{f_m \mid f_n}}$

$\rightarrow$ we know this

Q. Prove that

$$\underline{\underline{\gcd(f_m, f_n) = f_{\gcd(m,n)}}}$$

$$\gcd(f_m, f_n) = f_{\gcd(m,n)}$$ //

↳ $n = qm + r$ for $\underline{\underline{n > m}}$

$$\gcd(f_m, f_{qm+r}) = f_{\gcd(m,n)}$$

$$\gcd(f_m, f_{qm+1} f_r + f_{qm} f_{r-1})$$

$m \mid qm$

↳ $\underline{\underline{f_m \mid f_{qm}}}$

$$\gcd(f_m, f_{qm+1} f_r)$$

↳ $\gcd(f_m, f_r)$

$\underline{r = n \bmod m}$

$$\gcd(f_n, f_m) = \gcd(f_m, f_{n\%m})$$

$$\gcd(a, b) = \underline{\underline{\gcd(b, a\%b)}}$$

$$\gcd(f_{100}, f_{80}) = \gcd(f_{80}, f_{20}) = \gcd(f_{20}, f_0)$$

$$= \underline{\underline{f_{20}}}$$

$$\underline{\underline{\gcd(f_n, f_m)}} = \gcd(f_m, f_{n\%m})$$

$$= \gcd(f_{\gcd(m,n)}, f_0)$$

$$= \underline{\underline{f_{\gcd(m,n)}}}$$