Probability and Statistics
Coursework 4 - Jointly Distributed Random Variables

Question

Let X and Y be continuous random variables.

1. Show that $\mathrm{P}(X < Y + 1) = \int_{-\infty}^{\infty} \mathrm{P}(X < y + 1 | Y = y) f_Y(y) \mathrm{d}y$. Detail each step of your derivations[1].
2. Hence or otherwise, if X and Y are $\mathrm{Exp}(\lambda)$ and $\mathrm{N}(0,1)$, respectively, and independent, give an expression for the probability that $X < Y + 1$. Your answer needs to hold for any value of λ and show the details of the derivations.
3. Assume in this part that $\lambda = 1$, i.e. X and $Y + 1$ have the same mean. Show that this implies for the result obtained in part (b) that $\mathrm{P}(X < Y + 1) = \Phi(1) - \frac{1}{2\sqrt{e}}$.

Guidelines

- You can submit individually or in a group of two.
- Use only concepts, techniques, and known results seen in the module lectures, tutorials, and exercises.
- Clearly state the assumptions used in the derivations.
- Some marks may be taken off for submissions that are terse (lacking detail) or difficult to parse for the markers (untidy or hard to read).

Feedback

We aim to return to you feedback before the next coursework deadline.

[1] You cannot use techniques not included in the module syllabus, such as indicators functions.