Preparing for Shortages through Efficient Use of Scarce Resources

Siddhartha Lewis-Hayre

November 2024

Click here for the latest vesrion.

Abstract

Why do private incentives cause an under-investment in resilience? I build on Grossman, Helpman and Lhuillier (2023) to model agents' investment in excess capacity of a homogeneous good, for example water, electricity, or computing power. During shortages the marginal value of the good varies across agents, causing an under-investment in excess capacity of the good. A secondary market for the homogeneous good would improve this misallocation; however, private incentives may cause the market price to be higher than would be socially optimal. I use this framework to evaluate policy solutions to create an efficient secondary market. In ongoing work, I will look for a mechanism to achieve the optimal allocation. This model provides a unified theory to compare the problem of optimal hospital surge capacity, bank liquidity buffers, and producer inventory. In Lewis-Hayre (2024), I apply the model to a novel situation: supply chain disruptions.

1 Introduction

Since the 10th century A.D., from the Iberian Peninsula to New Mexico, people have relied on communal irrigation systems called acequias. When water is abundant, acequias seem like any other irrigation system: its users extract water until the marginal value of water aligns with the marginal cost of using the water; however, during shortages, acequias allow a majordomo, a town official, to efficiently allocate water between residents. As such, this mechanism incentivizes cooperation to invest in acequias, which better capture snow melt, thereby promoting water adequacy. A drawback of acequias is that the majordomo must determine how to allocate water during shortages. How can policy incentivize these benefits of the acequia without the need for a majordomo?

A similar public goods problem arises with shortages of other homogeneous goods. The prospect of shortages of electricity, hospital equipment, cloud computing resources, and semi-conductors, has led governments to deem securing access to these goods a matter of national security. As such, governments have resorted to an industrial policy to secure access to critical goods. For example, the United States Government, between executive action, the Defense Production Act, and The CHIPS and Science Act, has invested in nuclear energy, personal protective equipment, cybersecurity, and semi-conductor factories.

Electricity, surgical masks, bank reserves, computing power, and semi-conductors, are mostly homogeneous like water. Yet in contrast with the example of acequias, private firms control access to these resources. Therefore, as emphasized in Grossman, Helpman, and Lhuillier (2023), "the question for governments is not whether shortages adversely affect households but whether firms' private incentives to avoid such shortages fall short of (or exceed) what is socially desirable." To determine whether policy intervention is needed and, if so, how to make such policy efficient, policymakers must first understand: why do private incentives distort investment in resilience? Answering this question will provide a way to test the efficiency of current policy. In particular, this paper asks: how does firms' use of homogeneous resources during shortages affect investment in resilience?

In some cases this private control creates a system with a similar purpose to that of acequias. Utility companies sell excess energy on utility markets. Hospitals coordinate use of scarce medical equipment like ventilators, surgical masks, or ambulances. These examples have large literatures that study how to make these markets efficient in order to maximize use of scarce resources during shortages. Why are these examples similar? Why are such secondary markets beneficial?

In other cases, such cooperation between firms does not occur. Cloud computing companies do not share computing resources with one another to cope with cyberattacks. Producers do not sell each other semi-conductors when they are in short supply. Why do secondary markets form in some cases and not others? How do private incentives affect whether secondary markets form and how efficiently they operate?

In this paper, I build on Grossman, Helpman, and Lhuillier (2023) to model an agent's decision to set a target capacity of a homogeneous good in anticipation of potential shortages of the good. At first I assume agents cannot exchange goods on a secondary market in order to establish the importance of such a market. I show that other homogeneous goods behave as does water in the example of acequias. Under the assumption of supply frictions during shortages and under uncertainty in shocks, goods will be misallocated between agents during shortages. As a result, the marginal value of the good will vary across agents. If an agent with a lower value for the good uses the good for themselves, this consumption creates a negative externality.

I show that if goods are not used efficiently during shortages, then agents will under-invest in resilience. Due to this misallocation of goods during shortages, the marginal value of the good will be lower than what is socially optimal, so agents will not have sufficient incentives to invest in resilience. This investment in resilience could take many forms. For example, an automobile producer could diversify their supply chain to mitigate the effects of semi-conductor shortages. A cloud computing company could invest in better cybersecurity. An energy company could invest in more dependable energy sources. In my model, I abstract away from how producers invest in resilience. Instead I model agents' decision to invest in resilience as an option to pay a cost to increase the availability of the good under various shocks. Regardless of the type of investment in resilience, the same argument shows that incentivizing sufficient preparation for shortages requires efficient use of goods during shortages.

A secondary market for the good could fix this misallocation during shortages. If during shortages, agents sold each other goods on a secondary market and the market clearing price was equal to the social planners' marginal value of the good, then the arising allocation would coincide with the social optimum. Private incentives, however, are likely to distort the secondary market. In the case of cloud computing, in the long term, companies could steal market share by letting their competitor fail to meet demand. In the example of firms using the good as an input to their production processes, giving inputs to competitors allows competitors to increase supply which weakens demand for the output of the first firm. On the other hand, the competitor will also be willing to pay a premium to take the input away from competition, so the effect on the secondary market will depend on consumer demand. In ongoing work, I fully develop this example which makes my model a direct extension of Grossman, Helpman, and Lhuillier (2023) to include homogeneous goods. Regardless of the form of private incentives, the relevant question is: under subsidies that achieve the social optimal allocation if a secondary market were not feasible, how would private incentives affect the market price of the secondary market? I create a framework to understand such distortions. I show that in the specific example where agents are hoarders, the price on the secondary market is artificially high, so agents may be unwilling to pay to create a secondary market even if it is socially optimal.

How could policy intervention help foster an efficient secondary market? A common solution to this problem is a reserve, but to achieve the social optimum with a reserve, agents would have to lower their targets so they never receive more of the good than is socially optimal. Another solution is a subsidy on transactions in the secondary market that depends on the state of the world. I show that this policy requires a larger than necessary expected subsidy. A better alternative is a policy which pays agents to enter into a smart contract requiring them to sell a predetermined number of goods for a given price depending on the state. Both of these policies would be difficult to implement since the policymaker must know the social benefit of trading in different states in order to determine the size of the subsidy or the optimal smart contract. In ongoing work, I search for an incentive compatible mechanism to establish efficient secondary markets, which would make it optimal for agents to report the true values of their projects. I will also consider combining a subsidy with a reserve assuming a penalty that is increasing with the size of the reserve.

This paper builds on three strands of literature. The first strand focuses on supply chains. I build on the supply chain literature by showing a new effect of the rigidity of supply chain networks. As opposed to previous literature which focuses on how shocks propagate vertically in the supply chain, I consider how disruptions in the supply chain create an unharnessed value from relationships between firms at the same level of the supply chain. I also build on the novel policy analysis in Grossman, Helpman, and Lhuillier (2023). Another major contribution of Grossman, Helpman, and Lhuillier (2023) is modeling the nuances of homothetic demand beyond CES utility functions. I abstract away from these complexities of demand to make the problem more transferable to situations like governments investing in disaster relief resources, or a manager allocating tasks to workers. Other supply chain literature focuses on forming relationships between producers and their suppliers. Grossman, Helpman, and Sabal (2023) considers the externalities in supply chains arising endogenously through sequential bargaining. Meanwhile Acemoglu and Tahbaz-Salehi (2021) focuses on amplification of shocks through a supply chain.

The second strand focuses on production networks. In this literature the prices of a firm's inputs adjust dynamically with shocks. As a result, the market price for inputs is efficient. I show that adding rigidities to production networks disrupts this power of flexible pricing and creates a misalignment in the value of inputs between firms. Acemoglu et al. (2012) shows how such networks transform idiosyncratic shocks into aggregate fluctuations. Liu and Tsyvinski (2024) studies production adjustment costs in input-output networks. Liu (2019) considers why these production networks create a need for industrial policy. Pellet and Tahbaz-Salehi (2023) introduces rigidities to production networks by modeling firms that must make their production decisions for some inputs under uncertainty. This leaves the possibility for heterogeneity in a firm's value for a given input. However, this paper focuses on the effects to the equilibrium and does not consider how firms could try to overcome this friction through a secondary market.

The third strand focuses on shortages. This literature spans many topics including energy markets, cloud computing resource sharing, hospital surge capacity, and interbank lending. I provide a common framework to unite a central problem in these literatures. The model provides an economic explanation for why cooperation between firms may be lower than what is socially optimal. The model also provides a way to test policy solutions to confront shortages, the principles of which can be applied to any of these industries. Of these literature on shortages, the work on interbank lending is the most expansive. Allen and Gale (2000) study how banks' can manage the risk of deposit outflows through interbank markets. Allen, Carletti, and Gale (2009) show the importance of a central bank to create sufficient liquidity in such markets. In energy markets, Hogan (2013) proposes operating reserve to correct pricing distortions during shortages.

2 Model of Excess Capacity

Agents use a critical good to advance valuable projects. First each agent chooses a target quantity of the critical good under uncertainty in the value of the good and whether she will have access to the full quantity targeted. Next the state of the world is revealed to all agents. Finally, given the state of the world, each agent allocates her available quantity of the critical good between her projects to maximize their total value. For now I assume there are no secondary markets so agents cannot trade each other the critical good.

This decision under uncertainty can be interpreted as a supply friction that prevents availability of the good to adjust to shocks. Private incentives come from the fact that agents only get value from some projects. As such, the social planner is an agent that gets value from all projects.

One way to interpret the model is a firm deciding how much excess inventory of inputs it would like to target. When taken in this context, this model follows a similar framework as Grossman, Helpman, and Lhuillier (2023) except I abstract the details of demand to give the problem applicability beyond the case of a producer. As shown in their paper, to understand firms incentives to invest in resilience a model must remove the distortions of market power. Their paper uses optimal subsidies to overcome the wedge between the market solution and the social optimum created by producer's market power. Instead, I assume the producer's value is equal to the social optimum. One micro-foundation for this assumption is that producers practice first-degree and consumers are only willing to buy one good. Alternatively, this assumption could be interpreted as the social planner has already implemented optimal project and state dependent consumption subsidies.

2.1 Model Setup

There is a discrete set of agents denoted I. There are a discrete set of projects denoted J. An agent $i \in I$ invests in a set of projects denoted $J_i \subseteq J$. I assume only one agent can invest in each project, so $\{J_i\}_{i \in I}$ partitions J. After the agents learn the state of the world $\omega \in \Omega$, which is realized with probability $f(\omega)$, agent i allocates her available units of the critical good, denoted q_i, between the projects J_i. Denote q_{ij} as the amount of the critical good that agent i allocates to good j. So we have that:

$$\sum_{j \in J_i} q_{ij} \leq q_i$$

The value that the agent receives from the project depends on the quantity of the critical good that the agent allocates to the project, q_{ij}, and the state of the world, $\omega \in \Omega$. When deciding how to allocate the critical good between projects, the agent knows the state of the world; therefore, given quantity of the critical good, q_i, agent i solves:

$$
\begin{aligned}
v_i(q_i, \omega) = \max_{\{q_{ij}\}_{j \in J_i}} & \sum_{j \in J_i} v_{ij}(q_{ij}, \omega) \\
\text{s.t.} & \sum_{j \in J_i} q_{ij} \leq q_i
\end{aligned}
\tag{1}
$$

Where $v_i(q_i, \omega)$ denotes the total value of agent i and $v_{ij}(q_{ij}, \omega)$ denotes the value agent i receives from project j. Assume v_{ij} is increasing and concave, so the agents gain a decreasing marginal benefit from investing an additional unit of the critical good into project i.

Before the agent learns the state of the world, she must choose a target of how much critical good she wants; however, she might not receive as much of the good as she wanted. In particular, if agent i sets a target of θ_i units of the critical good, then she receives $q_i(\vec{\theta}, \omega)$ in state ω where $\vec{\theta} = \{\theta_i\}_{i \in I}$. I assume $q_i(\vec{\theta}, \omega)$ is weakly increasing in θ_i. Agents solve the following problem to prepare for potential crises:

$$
V_i = \max_{\theta_i} V_i(\vec{\theta}) = \max_{\theta_i} \mathbb{E}[v_i(q_i(\theta_i))|\vec{\theta}_{-i}] - c_i(\theta_i) = \max_{\theta_i} \int_{\Omega} v_i(q_i(\vec{\theta}, \omega), \omega) f(\omega)\, d\omega \; - c_i(\theta_i)
$$

Where $V_i(\vec{\theta})$ is the expected value of agent i given targets $\vec{\theta}$, where $\vec{\theta}_{-i} = \{\theta_l\}_{l \in I \setminus \{i\}}$ is the targets of other agents, and where $c_i(\theta_i)$ is the cost of setting the critical good target. Assume $c_i(\theta_i)$ is weakly convex in θ_i.

In the case of a price-discriminating producer selling differentiated goods that use a common input, then the critical good represents the common input and $v_i(q_{ij}, \omega)$ represents the profits from using q_{ij} units of inputs to produce final goods and selling those final goods at an implicit price.

2.1.1 Social Planner's Problem

The social planner is equivalent to an agent that gets value from all projects. In other words, if $J_i = J$, we get the social planner. A delicate part of the social planner's problem is how to aggregate the cost and quantity functions. This aggregation does not necessarily benefit the social planner over the market solution in order to emphasize the real mechanism which is the inability of supply to adjust quickly. The full details of the social planner's problem can be found in the appendix (A.1).

3 Comparing the Market Solution with the Social Optimum

In the market solution, critical good targets are lower than what is socially optimal. To see why, I first consider what happens in crisis. Given this behavior in crisis, I consider how agents will set their targets to prepare for crisis. In crisis, an agent's investments of the critical good follow a pecking order defined by the value of extra investment into her projects. Agents' prioritization of their own projects silos this pecking order, preventing the critical good from being optimally utilized. Unless the critical good is perfectly allocated between agents, the market allocation of the critical good will be inefficient. The lack of a secondary market where agents can trade the critical good causes this distortion. If agents could trade the critical good at a market price, the market allocation in crisis would be efficient. Without a secondary market, there is insufficient preparation for crisis. This is because when deciding critical good targets, agents anticipate a lower value for the critical good than would a social planner.

3.1 Using Scarce Resources

Given a fixed quantity of the critical good, an agent invests the critical good following a pecking order of the value of advancing her projects until the agent runs out of the critical good; therefore, how the agents partition the projects determines the social welfare. Consider agent i who invests in projects J_i and has q_i units of the essential good in state ω. In this subsection everything is implicitly a function of the state ω. The first order condition is:

$$v'_{ij}(q_{ij}) = \lambda_i \quad \forall j \in J_i$$

Where λ_i is agent i's the shadow price of the critical good. The first order condition says that the value of the agent's final good allocated to each project should be the same.

To better understand this decision problem, consider the following dual problem. Since v_{ij} is increasing we can denote q^*_{ij} as the inverse of v'_{ij}. More specifically:

$$q^*_{ij}(\lambda) = \{q : \lambda = v'_{ij}(q)\}$$

$q^*_{ij}(\lambda)$ represents the quantity of goods that the agent can allocate to project j and still have the last unit be worth at least λ. Let $q^*_i(\lambda) = \sum_{j \in J_i} q^*_{ij}(\lambda)$. Then agent i solves the following problem:

$$\min_{\lambda_i} \lambda_i$$
$$\text{s.t. } q_i^*(\lambda_i) \leq q_i \tag{2}$$

The interpretation is that the agent allocates the critical good unit-by-unit to the most valuable available project until the agent runs out of the critical good. The optimization problem asks: following this pecking order, what is the least beneficial use of the critical good an agent can make before running out of the critical good?

Now I turn to the social planners problem. It is the same as the agent's problem if one agent were to invest in all projects (i.e. $J_i = J$). It requires:

$$v'(q_{ij}) = \lambda \quad \forall j \in J$$

Where $q = \sum_{j \in J} q_{ij}$, $v(q) = \sum_{j \in J} v_{ij}(q_{ij})$, and λ is the social planner's shadow price for the critical good. As such, the market solution coincides with the social optimum if and only if $\lambda_i = \lambda$ for all $i \in I$.

The dual problem highlights the knife-edge nature of this condition. Denote $q^*(\lambda) = \sum_{j \in J} q_{ij}^*(\lambda)$. Given λ which solves $q^*(\lambda) = q$, then we have the following proposition on when the market solution is socially optimal.

Proposition 1. *The market solution is socially optimal if and only if $q_i^*(\lambda) = q_i$ for all $i \in I$.*

Since $q_i^*(\lambda)$ is increasing in λ, then there is a unique shadow price that solves this equation for each agent. This shadow price must be the same across agents. Note q_{ij}^* is derived exclusively from v_{ij} which is exogenously given, so for arbitrary q_i, this equality will almost never hold.

Of course q_i is not arbitrary. In fact, if a secondary market existed for critical good, the price λ would clear the market and remove this inefficiency. In other words, this inefficiency hinges on the fact that agents find themselves in crisis with the wrong quantity of the critical good and cannot buy and sell the critical good to rectify this misallocation. Section 4, I will investigate why a secondary market may or may not form during shortages.

3.2 Preparing for Crisis

How should agents set their targets of the critical good? Agents can only choose one critical good target. Since the target is not state dependent, different agents find themselves with different values for the critical good in different states. As such, the agents do not use the critical good optimally so they have less value for it; therefore, agents' targets are lower than what is socially optimal.

We want to compare θ^* to $\hat{\theta}$, where $\hat{\theta} = \sum_{i \in I} \theta_i$. Denote $\lambda(\omega) = v'(q(\theta^*, \omega), \omega)$. In other words, $\lambda(\omega)$ is the social planner's shadow price of the critical good in state ω. Then we have the following proposition:

Proposition 2. *If $q_i^*(\lambda(\omega)) \neq q_i(\theta_i, \omega)$ for some $\omega \in \Omega$ and for some $i \in I$, then $\hat{\theta} < \theta^*$.*

In other words, unless the value from the different projects perfectly align to make every producers' final unit of the critical good equally valuable in all states, the market solution will have lower orders than is socially optimal.

Even if crisis severity is perfectly correlated between agents, the market solution does worse than the social optimum. If one agent faces crisis and other agents do not, then it make sense that helping the agent in crisis could improve social welfare. For example, if a hurricane hits Florida, California should use its relief resources to help Florida. This theory goes further to say that even if every state in the country is hit by natural disaster at the same time, relief should be allocated proportionally so that marginal relief received by every state has the same value. Due to the uncertainty in the severity and timing of disasters, relief will not be properly distributed between states before the disaster hits, so a reallocation will be required.

4 Formation of Secondary Markets

Why do secondary markets form in some instances of this problem and not in others? Perhaps the up-front cost of implementing such a secondary market is prohibitively large. For example, it is a difficult technical challenge to get the computer systems of Amazon and Google to connect their cloud computing networks. Once systems exist to facilitate a secondary market, there can still be transaction costs or coordination problems. Even banks struggle to borrow from the discount window during crises despite the relative ease of transferring money compared to transacting tangible resources. Such costs would exist regardless of private incentives.

However, private incentives could also distort secondary markets, causing an inadequate (or superfluous) investment in the secondary market. If a firm sells a scarce input to a competitor and the competitor uses this resource to boost output and sales, this could dilute the value of the first firm's products. On the other hand, the competitor firm will over-value extra units of the good. Depending on demand, this may cause an under (or over) reliance on secondary markets.

I investigate this example more thoroughly in ongoing work. In the example of cloud computing, by the time computational shortages arise, consumer may have already chosen their providers, but providers could still be unwilling to sell their additional resources on secondary markets to win future contracts with their competitors clients.

Although the direction of the distortion is unclear, in all these examples private incentives cause a distortion in willingness to transact on the secondary market, which means the total private value of the secondary market is either insufficient or excessive. The gap between the private and socially optimal value of the secondary market means secondary markets may not form even though they should, or vice versa. To isolate the impact of these incentives, I consider: how would private incentives affect the price on the secondary market given a policy that would be optimal if the secondary market did not exist?

4.1 Private Incentives in a Secondary Market

I show that if agents have private incentives that distort their willingness to sell critical good to other agents during shortages, then even the existence of a secondary market will not be enough to align the market solution with the social optimum. I assume that the value of the projects of a given agent depends on other agents' allocations. Since an agent making her own allocation decision takes the other agent's allocations as given, this can be captured by making an agent's value from her projects a function of the amount of the critical good she sells (buys) on the secondary market. In particular, now in state ω, agent i solves the problem:

$$\begin{aligned} \pi_i(\vec{q}, p, \omega) = \max_{\{q_{ij}\}_{j \in J_i}, q_i^\tau} & \sum_{j \in J_i} \pi_{ij}(q_{ij}, q_i^\tau, \omega) + p(\omega) q_i^\tau \\ \text{s.t. } & q_i^\tau + \sum_{j \in J_i} q_{ij} \leq q_i \end{aligned} \tag{3}$$

Where q_i^τ is the quantity that firm i sells (buys) on the secondary market at price $p(\omega)$. Furthermore $\pi_{ij}(q_{ij}, q_i^\tau, \omega)$ is agent i's value of project j, and $\pi_i(\vec{q}, p, \omega)$ is agent i's total value under the allocation $\vec{q}$. The market clearing condition is $\sum_{i \in I} q_i^\tau = 0$. As before, I denote $v_{ij}(q_{ij}, \omega)$ as the social welfare from project j if q_{ij} goods are allocated to project j.

The social optimum is achieved if the market price $p(\omega)$ follows $p(\omega) = \lambda(\omega)$ for all $\omega \in \Omega$. The first order conditions for an agent i that sells goods on the secondary markets becomes:

$$\frac{\partial}{\partial q_{ij}}\pi_{ij}(q_{ij}, q_i^\tau, \omega) + \frac{\partial}{\partial q_i^\tau}\pi_{ij}(q_{ij}, q_i^\tau, \omega) = p(\omega)$$

Whereas the social planners' first order conditions are the same as before:

$$v'_{ij}(q_{ij}, \omega) = \lambda(\omega)$$

So depending on the form of the private incentives they can cause a distortion in the secondary market resulting in a misallocation of goods.

To understand how these private incentives distort secondary markets, the relevant question is: how would private incentives distort a secondary market under a policy that would be optimal if the secondary market did not exist? As in the original analysis where I assume private profits coincide with social value, I maintain this assumption.

Assumption 1. *If* $q_i^\tau = 0$ *then*

$$\pi_{ij}(q_{ij}, q_i^\tau, \omega) = v_{ij}(q_{ij}, \omega)$$

To apply this framework to a given application, the challenge is identifying the functional form of $\pi_{ij}(q_{ij}, q_i^\tau, \omega)$. In ongoing work, I compute this function in the case where agents are firms producing different varieties of an output. This example is the direct extension of Grossman, Helpman, and Lhuillier (2023) to include homogeneous inputs.

In the following analysis, I assume a particular functional form to highlight the implications of distortions to private incentives. In particular, I consider the case where agents have a private incentive to hoard goods.

4.1.1 Hoarder Example

Assuming agents hoard goods during shortages, how would this affect the secondary market? I make the following assumption to isolate the impact of such private incentives.

Assumption 2.

$$\frac{\partial}{\partial q_i^\tau}\pi_{ij}(q_{ij}, q_i^\tau, \omega) \begin{cases} < 0 & \text{if } q_i^\tau \geq 0^+ \\ = 0 & \text{if } q_i^\tau \leq 0^-. \end{cases}$$

The interpretation of this assumption is that selling goods on the secondary market decreases an agent's value for her projects. As a result, agents get disproportionate value from using their goods on their own projects.

Since agents overvalue their own goods, then they will be reluctant to sell their goods causing the secondary market to be smaller than would be socially optimal. As a result, the price on the secondary market will be larger than optimal. In particular, we have the following proposition:

Proposition 3. $p(\omega) > \lambda(\omega)$

Corollary 1. $|q_i^{\tau}| \leq |q_i^{*\,\tau}|$ *with equality if and only if* $q_i^{*\,\tau} = 0$.

Corollary 2. $\sum_{i\in I} \pi_i(\vec{q}, p, \omega) < \sum_{i\in I} v_i(\vec{q}, \omega) < \sum_{i\in I} v_i(\vec{q}^{\,*}, \omega)$.

The interpretation of the corollaries is that (i) the total amount of trade will be smaller than is socially optimal; (ii) goods will be misallocated; (iii) the marginal value of an extra good will be lower than in the social optimum.

4.2 Preparing for Crisis

In the market solution, an agent's expected profits with a secondary market is:

$$\max_{\theta_i} \Pi_i(\vec{\theta}) = \max_{\theta_i} \int_{\Omega} \pi_i(q_i(\vec{\theta}, \omega), p(\vec{\theta}, \omega), \omega) f(\omega)\, d\omega \;\; - c_i(\theta_i)$$

The first order conditions are:

$$\int_{\Omega} \pi_i'(q_i(\vec{\theta}, \omega), p(\vec{\theta}, \omega), \omega) q_i'(\vec{\theta}, \omega) f(\omega)\, d\omega \;\; = c_i'(\theta_i)$$

Depending on the function form of π_{ij}, this could lead θ_i to be larger or smaller than is socially optimal.

4.2.1 Preparing for Crisis: Hoarder Example

Since the agents' marginal value of the good will be lower in each state, then their total targets of the good will be lower than is socially optimal. From Corollary 2 of Proposition 3 we get that:

$$\sum_{i\in I} \frac{\partial}{\partial \theta_i} \Pi_i(\vec{\theta}) < \sum_{i\in I} \frac{\partial}{\partial \theta_i} V_i(\vec{\theta})$$

This gives the following proposition:

Proposition 4. $\theta^m < \theta^*$.

This proposition says that if agents hoard the good, which causes an inefficient secondary market, then targets of the good will be lower than is socially optimal.

4.2.2 Funding a Secondary Market

As discussed before, a secondary market may require upfront costs to set it up. I show how the wedge created by private incentives can cause agents to under (over) value the existence of the secondary market when compared to the social planner. As a result, a secondary market may not form even if it is socially optimal or vice versa.

To model the upfront cost of the secondary market, I assume that before setting their targets agents must collectively decide whether or not to build a secondary market. To build the secondary market they must collectively pay a fixed cost. I do not consider how they split this cost, I just assume the secondary market will be built if the agents have the collective willingness to pay.

As such, the relevant comparison is between total profits in the market solution, and total value in the social planners problem. If the fixed cost C follows:

$$\sum_{i\in I}\Pi_i(\vec{\theta}) < C < \sum_{i\in I}V_i(\vec{\theta})$$

Then the inefficiency in the secondary market will cause it not to form even though an efficient secondary market would be worth the start-up costs. However, if:

$$\sum_{i\in I}V_i(\vec{\theta}) < C < \sum_{i\in I}\Pi_i(\vec{\theta})$$

Then a secondary market will form even though it is not socially optimal.

4.2.3 Funding a Secondary Market: Hoarder Example

Returning to the example of agents that hoard the critical good, how would this hoarding tendency impact the formation of a secondary market?

From Corollary 2 of Proposition 3:

$$\sum_{i\in I}\Pi_i(\vec{\theta}) < \sum_{i\in I}V_i(\vec{\theta})$$

This means that if the fixed cost of forming a market, C, follows:

$$\sum_{i\in I}\Pi_i(\vec{\theta}) < C < \sum_{i\in I}V_i(\vec{\theta})$$

Then a secondary market will not form even though it would be socially optimal to pay for the secondary market if the critical good were used efficiently during shortages.

4.3 Policy Solutions [Work In Progress]

How can policy incentivize the formation of secondary markets when such markets are socially optimal? As throughout this paper, I assume that a policy is already in place to remove any distortions that would arise if a secondary market were infeasible. What additional policy could help facilitate efficient secondary markets? Although the magnitude and direction of these policies depend on the application, the model provides a framework to think about the relative effectiveness of different policies.

4.3.1 Reserve

4.3.2 Optimal Subsidy

Another way to achieve the social optimum would be to subsidize (or tax) the secondary market where the tax is conditional on the state ω. Denote the subsidy as $\kappa(\omega)$. The optimal subsidy is:

$$\kappa(\omega) = p(\omega) - \lambda(\omega)$$

The total cost of this subsidy would be:

$$S = \int_{\Omega} \kappa(\omega)\, d\omega$$

4.3.3 Smart Contract

Is there a more efficient way to achieve the social optimum? This optimal subsidy may pay a given agent in one state even though that agent may get rent in another state. Netting this benefit over states would make the total size of the intervention smaller.

In particular, to incentivize such a contract, the policy maker could subsidize the agents to enter into the contract by paying the agents an amount equal to $V - \Pi$. If allocated correctly between agents, all agents would be indifferent to entering into such a contract.

I show that subsidizing this contract would be cheaper than the optimal subsidy. In particular:

Proposition 5. $|V - \Pi| \leq |S|$

4.3.4 Implementation Limitations

A drawback of this analysis is that to implement these policies, the social planner must know the functional forms of the firms' value functions and the firms' private incentives. Ultimately, the social planner would want to create a mechanism that is incentive compatible so that firms' accurately report their own value functions and private incentives. In ongoing work I am looking for such a mechanism. For now, I only explain the difficulty of creating such a mechanism.

References

Acemoglu, Daron and Alireza Tahbaz-Salehi (2021). "Firms, Failures, and Fluctuations: The Macroeconomics of Supply Chain Disruptions". In: *SSRN Electronic Journal*. DOI: 10.2139/ssrn.3658861.

Acemoglu, Daron et al. (2012). "The Network Origins of Aggregate Fluctuations". In: *SSRN Electronic Journal*. DOI: 10.2139/ssrn.1947096.

Allen, Franklin, Elena Carletti, and Douglas Gale (2009). "Interbank market liquidity and central bank intervention". In: *Journal of Monetary Economics* 56(5). ISSN: 03043932. DOI: 10.1016/j.jmoneco.2009.04.003.

Allen, Franklin and Douglas Gale (2000). "Financial contagion". In: *Journal of Political Economy* 108(1). ISSN: 00223808. DOI: 10.1086/262109.

Grossman, Gene M., Elhanan Helpman, and Hugo Lhuillier (2023). "Supply Chain Resilience: Should Policy Promote International Diversification or Reshoring?" In: *Journal of Political Economy* 131(12). ISSN: 1537534X. DOI: 10.1086/725173.

Grossman, Gene M., Elhanan Helpman, and Alejandro Sabal (2023). "Resilience in Vertical Supply Chains". In: *SSRN Electronic Journal*. DOI: 10.2139/ssrn.4589701.

Hogan, William W. (2013). "Electricity scarcity pricing through operating reserves". In: *Economics of Energy and Environmental Policy*. Vol. 2. 2. DOI: 10.5547/2160-5890.2.2.4.

Liu, Ernest (2019). "Industrial policies in production networks". In: *Quarterly Journal of Economics* 134(4). ISSN: 15314650. DOI: 10.1093/qje/qjz024.

Liu, Ernest and Aleh Tsyvinski (2024). "A Dynamic Model of Input–Output Networks". In: *Review of Economic Studies*. ISSN: 0034-6527. DOI: 10.1093/restud/rdae012.

Pellet, Thomas and Alireza Tahbaz-Salehi (2023). "Rigid production networks". In: *Journal of Monetary Economics* 137. ISSN: 03043932. DOI: 10.1016/j.jmoneco.2023.05.008.

Appendix

A Model of Excess Capacity

A.1 Social Planner's Problem

The social planner can be thought of as an agent that gets value from all projects. The only subtlety is how to aggregate costs and quantities across agents.

If the social planner targets the same quantity of the critical good as all the agents combined, the social planner should receive at least the same quantity of the critical good as all the agents combined in each state. Denote θ as the social planner's target, $q(\theta, \omega)$ as the quantity of the critical good the social planner receives, and $c(\theta)$ as the cost of investing in the social planner's critical good target. Then:

$$\begin{aligned} c(\theta) = \min_{\vec{\theta}} & \sum_{i \in c} c_i(\theta_i) \\ \text{s.t.} & \sum_{i \in I} \theta_i \leq \theta \end{aligned} \tag{4}$$

and for all $\omega \in \Omega$:

$$\begin{aligned} q(\theta, \omega) = \max_{\vec{\theta}} & \sum_{i \in c} q_i(\theta_i, \omega) \\ \text{s.t.} & \sum_{i \in I} \theta_i \leq \theta \end{aligned} \tag{5}$$

The social planner does not necessarily do strictly better than the agents' aggregate costs and quantities. An example where the social planner's costs and quantities align with the agents' aggregate costs and quantities for any vector of targets, $\vec{\theta}$, is when $\Omega = [0, 1]$, $c_i(\theta_i) = c_0\theta_i$, and $q_i(\vec{\theta}, \omega) = \gamma(\omega)\theta_i$ for all $i \in I$ and for some c_0 and $\gamma(\cdot) : [0, 1] \to [0, 1]$. Even in this case the market solution does not align with the social optimum.

A.2 Preparing for Crisis

Agent i's first order conditions for θ_i is:

$$\int_\Omega v_i'(q_i(\vec{\theta}, \omega), \omega) q_i'(\vec{\theta}, \omega) f(\omega)\, d\omega \ = c_i'(\theta_i)$$

where $q_i'(\vec{\theta}, \omega) = \frac{\partial}{\partial \theta_i} q_i(\vec{\theta}, \omega)$. Meanwhile the social planner solves:

$$\int_\Omega v'(q(\theta, \omega), \omega) q'(\theta, \omega) f(\omega)\, d\omega \ = c'(\theta)$$

We want to compare θ to $\hat{\theta}$, where $\hat{\theta} = \sum_{i \in I} \theta_i$. By equation (5), we have that:

$$q_i'(\vec{\theta}, \omega) \geq \sum_{i \in I} q_i'(\vec{\theta}, \omega) \quad \forall \omega \in \Omega$$

$$c_i'(\vec{\theta}) \leq \sum_{i \in I} c_i'(\vec{\theta})$$

$$v'(q(\theta, \omega), \omega) \leq \sum_{i \in I} v_i'(q_i(\vec{\theta}, \omega), \omega)$$

Where the last inequality follows by proposition (1).