Preparing for Supply Chain Disruptions: Using Scarce Resources

Siddhartha Lewis-Hayre

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Abstract

Secondary markets for common inputs improve welfare during shortages. Competition prevents these secondary markets from forming, causing an under-invest in inventory surplus. As such, shocks are more disruptive than they should be. I build on the framework of Grossman, Helpman and Lhuillier (2023) to consider producers' use of common inputs during shortages. I test the model empirically to show that, during shortages, auto-manufacturers' production follows a pecking order defined by prioritizing the production of models with the greatest markup in dollar terms. This test identifies the production adjustment costs arising from the reallocation of inputs required to follow the pecking order. I use this measurement of the production adjustment costs to calculate a counterfactual of COVID disruptions under the existence of a secondary market for inputs, the effect of which could also be achieved through a strategic reserve for critical inputs like semi-conductors.

1 Introduction

Starting in the 10th century A..D., Muslims in the Iberian Peninsula built communal irrigation systems called Acequias. The Spanish later adopted this technology and built it in their colonies, including in New Mexico. In both Spain and New Mexico, Acequias are still used today. When water is abundant, Acequias seem like any other irrigation system: its users extract water until the marginal value of water aligns with the marginal cost of using the water; however, Acequias have two main advantages when water scarcity leaves a village confronting a classic example of the public goods problem. Firstly, Acequias lessen the severity of shortages by more effectively capturing snowmelt runoff, thereby making shortages less acute. Secondly, town officials use Acequias to efficiently allocate the available water allowing a village to efficiently use the scarce resource. From an individual's perspective, neither of these features may seem particularly appealing. The villager with the best access to water should only be hurt by sharing. Furthermore, this villager may have enough water in all but the most extreme droughts so the additional cost of these Acequias will not benefit her. Nonetheless, together, these two features of Acequias create communal buy-in. Why does the combination of these two benefits make Acequias desirable?

Similar to the problem of water scarcity, COVID exposed that supply disruptions can create shortages of critical goods. As such, governments have deemed access to critical inputs a matter of national security. To secure access to critical inputs, governments have mostly resorted to policies to diversify their supply chains. The CHIPS and Science Act looks to secure access to critical inputs by boosting domestic manufacturing of inputs like semi-conductors. [TALK ABOUT SUPPLY CHAIN FLEXIBILITY] One particularly intriguing solution is a nascent plan by the Biden Administration to create a sovereign wealth fund which could potentially buy futures to secure access to commodities in times of stress.

Why should the government intervene in securing critical inputs? As emphasized in Grossman, Helpman and Lhuillier (2023): "the question for governments is not whether shortages adversely affect households but whether firms' private incentives to avoid such shortages fall short of (or exceed) what is socially desirable."

As with water shortages, in normal times, critical inputs are not in short supply, so the marginal cost of a good should align with its marginal benefit as the social planner would want. In crisis, however, scarcity creates a public goods problem. Firms that have disproportionate access to critical goods, will use their critical goods for less valuable aims than would another firm. Therefore, using the critical good for a less valuable project creates a negative externality. This misallocation between firms is likely to occur in times of stress because of supply frictions and the unpredictability of shocks. Shocks happen quickly, so supply cannot immediately adjust. Furthermore, shocks are difficult to predict and have varying impact on different firms, making it unlikely that firms can correctly anticipate the supply of inputs they need under different crisis scenarios.

One solution to this misallocation of resources between firms in times of stress could be for firms to sell each other critical goods at a market price. Since firms compete for long-term market share, even though selling scarce inputs in such a secondary market may be profitable in the short term, firms are unlikely sell one-another scarce inputs in order to avoid losing market share to their competition. Such incentives create a wedge between the market solution and the social planners', which governments should address.

If resources are not used efficiently in times of stress, then firm's will under-invest in excess inventory of the resources.

As in the example of Acequias, policy that focuses on will naturally encourage crisis preparation.

I test this ...

2 Model of Excess Capacity as Insurance

Crises happen suddenly while crisis response is slow. As such, individuals, companies, and governments prepare for crises by investing in excess capacity of critical goods. This excess capacity serves as insurance.

On one hand, this insurance could protect against demand shocks. For example, hospitals have surge capacity to prepare for an influx of patients. Banks keep capital buffers in case their assets under-perform. Cloud computing companies build excess computing power in case demand for computational resources spike. Countries stockpile vaccines to prepare for outbreaks. In times of peace, militaries have more soldiers than they actively use in order to prepare for war.

On the other hand, this insurance could also protect against supply shocks. For hospitals, the same affliction affecting patients could also impair the hospital's operations, be it a climate disaster, a disease, or a war. For banks, consumers could withdraw their deposits. For cloud computing, a cyber-attack could compromise computing resources. For vaccines, the vaccines could prove less effective than anticipated. For militaries, soldiers could desert once war begins.

When the good is needed the most, it is in short supply because shocks happen more suddenly than supply can adjust. This friction creates a need for excess capacity. How much excess capacity is optimal? How does this amount of investment in excess capacity compare to what is socially optimal? I propose a simple model where agents choose a target for how much of a critical good they want to have given uncertainty in the good's availability and given uncertainty in the value of the good.

2.1 Model Setup

Agents use a critical good to advance valuable projects. First each agent chooses a target quantity of the critical good under uncertainty in the value of the good and whether she will have access to the full quantity targeted. Next the state of the world is revealed to all agents. Finally, given the state of the world, each agent allocates her available quantity of the critical good between her projects to maximize their total value.

There is a discrete set of agents denoted I. There are a discrete set of projects denoted J. An agent $i \in I$ invests in a set of projects denoted $J_i \subseteq J$. I assume only one agent can invest in each project, so $\{J_i\}_{i \in I}$ partitions J. After the agents learn the state of the world $\omega \in \Omega$, which is realized with probability $f(\omega)$, agent i allocates her available units of the critical good, denoted q_i, between the projects J_i. Denote q_{ij} as the amount of the critical good that agent i allocates to good j. So we have that:

$$\sum_{j \in J_i} q_{ij} \leq q_i$$

The value that the agent receives from the project depends on the quantity of the critical good that the agent allocates to the project, q_{ij}, and the state of the world, $\omega \in \Omega$. When deciding how to allocate the critical good between projects, the agent knows the state of the world; therefore, given quantity of the critical good, q_i, agent i solves:

$$\begin{aligned} v_i(q_i, \omega) = \max_{\{q_{ij}\}_{j \in J_i}} & \sum_{j \in J_i} v_{ij}(q_{ij}, \omega) \\ \text{s.t.} & \sum_{j \in J_i} q_{ij} \leq q_i \end{aligned} \tag{1}$$

Where $v_i(q_i, \omega)$ denotes the total value of agent i and $v_{ij}(q_{ij}, \omega)$ denotes the value agent i receives from project j. Assume v_{ij} is increasing and concave, so the agents gain a decreasing marginal benefit from investing an additional unit of the critical good into project i.

Before the agent learns the state of the world, she must choose a target of how much critical good she wants; however, she might not receive as much of the good as she wanted. In particular, if agent i sets a target of θ_i units of the critical good, then she receives $q_i(\vec{\theta}, \omega)$ in state ω where $\vec{\theta} = \{\theta_i\}_{i \in I}$. I assume $q_i(\vec{\theta}, \omega)$ is weakly increasing in θ_i. Agents solve the following problem to prepare for potential crises:

$$\max_{\theta_i} V_i(\vec{\theta}) = \max_{\theta_i} \mathbb{E}[v_i(q_i(\theta_i))|\vec{\theta}_{-i}] - c_i(\theta_i) = \max_{\theta_i} \int_{\Omega} v_i(q_i(\vec{\theta}, \omega), \omega) f(\omega)\, d\omega \; - c_i(\theta_i)$$

Where $V_i(\vec{\theta})$ is the expected value of agent i given targets $\vec{\theta}$, where $\vec{\theta}_{-i} = \{\theta_l\}_{l \in I \setminus \{i\}}$ is the targets of other agents, and where $c_i(\theta_i)$ is the cost of setting the critical good target. Assume $c_i(\theta_i)$ is weakly convex in θ_i.

In the case of a producer selling differentiated goods that use a common input, then the critical good represents the common input and $v_i(q_{ij}, \omega)$ represents the profits from using q_{ij} units of inputs to produce final goods and selling those final goods at an implicit price which is a function of q_{ij}. To capture substitution patterns, v_{ij} should be a function of the quantities of all the goods, but for now I ignore this possibility. I will more fully describe this application of a producer selling differentiated goods in the empirical section where I extend this model to capture a producer's decision on inventory surplus.

2.1.1 Social Planner's Problem

The social planner is equivalent to an agent that gets value from all projects. In other words, if $J_i = J$, we get the social planner. The only delicate part of the social planner is how to aggregate the cost and quantity functions. This aggregation should not benefit the social planner over the market solution in order to emphasize the real mechanism which is the inability of supply to adjust quickly. The full details of the social planner's problem can be found in the appendix (A.1).

2.2 Comparing the Market Solution with the Social Optimum

In the market solution, critical good targets are lower than what is socially optimal. To see why, I first consider what happens in crisis. Given this behavior in crisis, I consider how agents will set their targets to prepare for crisis. In crisis, an agent's investments of the critical good follow a pecking order defined by the value of extra investment into her projects. Agents' prioritization of their own projects silos this pecking order, preventing the critical good from being optimally utilized. Unless the critical good is perfectly allocated between agents, the market allocation of the critical good will be inefficient. The lack of a secondary market where agents can trade the critical good causes this distortion. If agents could trade the critical good at a market price, the market allocation in crisis would be efficient. Without a secondary market, there is insufficient preparation for crisis. This is because when deciding critical good targets, agents anticipate a lower value for the critical good than would a social planner.

2.2.1 Using Scarce Resources

Given a fixed quantity of the critical good, an agent invests the critical good following a pecking order of the value of advancing her projects until the agent runs out of the critical good; therefore, how the agents partition the projects determines the social welfare. Consider agent i who invests in projects J_i and has q_i units of the essential good in state ω. In this subsection everything is implicitly a function of the state ω. The first order condition is:

$$u'_{ij}(q_{ij}) = \lambda_i \quad \forall j \in J_i$$

Where λ_i is agent i's the shadow price of the critical good. The first order condition says that the value of the agent's final good allocated to each project should be the same.

To better understand this decision problem, consider the following dual problem. Since v_{ij} is increasing we can denote q^*_{ij} as the inverse of u'_{ij}. More specifically:

$$q^*_{ij}(\lambda) = \{q : \lambda = u'_{ij}(q)\}$$

$q^*_{ij}(\lambda)$ represents the quantity of goods that the agent can allocate to project j and still have the last unit be worth at least λ. Let $q^*_i(\lambda) = \sum_{j \in J_i} q^*_{ij}(\lambda)$. Then agent i solves the following problem:

$$\begin{aligned} &\min_{\lambda_i} \lambda_i \\ &\text{s.t. } q^*_i(\lambda_i) \leq q_i \end{aligned} \tag{2}$$

The interpretation is that the agent allocates the critical good unit-by-unit to the most valuable available project until the agent runs out of the critical good. The optimization problem asks: following this pecking order, what is the least beneficial use of the critical good an agent can make before running out of the critical good?

Now I turn to the social planners problem. It is the same as the agent's problem if one agent were to invest in all projects (i.e. $J_i = J$). It requires:

$$u'(q_{ij}) = \lambda \quad \forall j \in J$$

Where $q = \sum_{j \in J} q_{ij}$, $u(q) = \sum_{j \in J} v_{ij}(q_{ij})$, and λ is the social planner's shadow price for the critical good. As such, the market solution coincides with the social optimum if and only if $\lambda_i = \lambda$ for all $i \in I$.

The dual problem highlights the knife-edge nature of this condition. Denote $q^*(\lambda) = \sum_{j \in J} q^*_{ij}(\lambda)$. Given λ which solves $q^*(\lambda) = q$, then we have the following proposition on when the market solution is socially optimal.

Proposition 1. *The market solution is socially optimal if and only if $q^*_i(\lambda) = q_i$ for all $i \in I$.*

Since $q^*_i(\lambda)$ is increasing in λ, then there is a unique shadow price that solves this equation for each agent. This shadow price must be the same across agents. Note q^*_{ij} is derived exclusively from v_{ij} which is exogenously given, so for arbitrary q_i, this equality will almost never hold.

Of course q_i is not arbitrary. In fact, if a secondary market existed for critical good, the price λ would clear the market and remove this inefficiency. In other words, this inefficiency hinges on the fact that agents find themselves in crisis with the wrong quantity of the critical good and cannot buy and sell the critical good to rectify this misallocation.

Whether a secondary market emerges depends on whether agents can reallocate goods quickly and cheaply enough to improve the allocation. It also depends on the agents' incentives. If there are large enough positive externalities from selling critical goods to other agents, then a secondary market should emerge. Some examples include: banks lend each other liquidity in times of stress to prevent contagion, governments ship each other vaccines in outbreaks to prevent the spread of disease, militaries share munitions and soldiers in times of war to defeat a common enemy. However if there are negative externalities from selling critical goods to other agents, then a secondary market may not form. Google is unlikely to give Amazon access to its spare computational capacity when Amazon is out of computational power because Google might benefit if Amazon fails to meet demand as Google could win over some of Amazon's customers. In these cases where a secondary market does not emerge, critical goods are under-utilized in crisis.

2.2.2 Preparing for Crisis

How should agents set their targets of the critical good? Agents can only choose one critical good target. Since the target is not state dependent, different agents find themselves with different values for the critical good in different states. As such, the agents do not use the critical good optimally so they have less value for it; therefore, agents' targets are lower than what is socially optimal.

We want to compare θ to $\hat{\theta}$, where $\hat{\theta} = \sum_{i \in I} \theta_i$. Denote $\lambda(\omega) = u'(q(\theta, \omega), \omega)$. In other words, $\lambda(\omega)$ is the social planner's shadow price of the critical good in state ω. Then we have the following proposition:

Proposition 2. *If $q_i^*(\lambda(\omega)) \neq q_i(\theta_i, \omega)$ for some $\omega \in \Omega$ and for some $i \in I$, then $\hat{\theta} < \theta$.*

In other words, unless the value from the different projects perfectly align to make every producers' final unit of the critical good equally valuable in all states, the market solution will have lower orders than is socially optimal.

Even if crises severity is perfectly correlated between agents, the market solution does worse than the social optimum. If one agent faces crisis and other agents do not, then it make sense that helping the agent in crisis could improve social welfare. For example, if a hurricane hits Florida, California should use its relief resources to help Florida. This theory goes further to say that even if every state in the country is hit by natural disaster at the same time, relief should be allocated proportionally so that the last bit of relief received by every state has the same value. Due to the uncertainty in the severity and timing of disasters, relief will not be properly distributed between states before the disaster hits, so a reallocation will be required.

3 Empirical Methodology

To test this model of excess capacity as insurance, I extend it to capture a producer's dynamic decision to order inputs and choose which final goods to produce with the available inputs. I use this model to test whether, during shortages, automobile manufacturers follow a pecking order that prioritizes cars with the greatest markups in dollar terms. Following this pecking order requires reallocating inputs between models, which creates production adjustment costs. The test of the pecking order identifies the production adjustment costs. I use this measurement of the production adjustment costs to calculate a counterfactual welfare under the existence of a secondary market.

3.1 Model of Producer Inventory Surplus

To prepare for crisis producers must decide how much inventory surplus to hold in order to prepare for supply chain disruptions and demand shocks. I expand the model of excess capacity to answer: what is the optimal amount of inventory surplus for a producer? If an input is not a commodity, there is no secondary market for it. Instead producers must order these inputs under uncertainty as to whether the parts will arrive. If the parts do not arrive, often the producer is simply out of luck. The model shows that producers will not fully utilize parts during stress, so inventory surpluses will be smaller than is optimal.

The producer model builds on the base model in the following ways. There are a countable number of periods $t \in \mathbb{N}$, where each time period behaves similarly to the base model. The state of the world $\omega_t \in \Omega$ is revealed at the start of each period. Agents are producers and projects are the goods that the producers produce. Instead of a single critical good, there are now a discrete set of inputs denoted K. Producers' value is profits denoted π_{it}. A producers' critical input target is its order for each input. Producers make orders for the next period. Denote θ_{ikt} to be producer i's order of input k scheduled to be delivered in period t but chosen in period $t-1$. Furthermore, producers must sell good j at p_{ijt} which is exogenous. Grossman, Helpman and Lhuillier (2023) focus on these pricing dynamics, whereas I focus on the impact of having common inputs. As such, I simplify away these pricing dynamics. [1]

Consumers are very simple. Each period a fixed number of consumers $s_{ijt}(\omega)$ are willing to pay p_{ijt} for good j. Without changing the producer's problem, this share could depend on the price and availability of the goods produced by other consumers, but for now we ignore this possibility. As such, revenues are:

$$\mathcal{R}_{ijt}(\varsigma_{ijt}) = p_{ijt} \min\{\varsigma_{ijt}, s_{ijt}(\omega)\}$$

Where ς_{ijt} is the quantity of good j sold in period t.

Producer's operate a Leontief production function for each good. In particular:

$$y_{ijt} = \min_{k \in K_j} \alpha_{ijk} x_{ijkt}$$

Where y_{ijt} is the amount of good j produced at time t, $K_j \subseteq K$ are the inputs used to produce

[1] For the empirical case of automobiles, static pricing also aligns with the stylized fact that MSRP does not much change over the course of the year. Even changes in loan interest rates are mostly explained by the yield curve. Although cash subventions fall close to zero in times of stress, they are small relative to the price of the car.

good j, x_{ijkt} is the quantity of input k required to produce one unit of good j, and α_{ijk} for $j \in K_j$ are the coefficients of the production function.

A producer faces costs from ordering inputs and running the production function. It also pays production adjustment costs. The production adjustment costs represent that it is difficult to change production plans. They take the form:

$$C(y_{ijt}) = \frac{\eta}{2}(\frac{y_{ijt} - \bar{y}_{ij}}{\bar{y}_{ij}} - \epsilon_{ijt})^2$$

where $\bar{y}_{ij}$ is an exogenously given optimal production level and ϵ_{ijt} is an i.i.d random shock to production costs. The interpretation is that producers prepare for a target level of production and deviations from this optimal level incur costs. [ADD REFERENCES TO PRODUCTION ADJUSTMENT COSTS.] [2] The total cost of producing good j is:

$$\mathcal{C}_{ijt}(\theta_{ikt}, y_{ijt}) = \sum_{k \in K_j} c_k \theta_{ikt} + \sum_{j \in J_i} c_{ij} y_{ijt} + \sum_{i \in I_j} C(y_{ijt})$$

Where c_k is the cost of ordering good k, and c_{ij} is the cost of running the production function for good j. As such producer i's profits from good j take the form:

$$\pi_{ijt} = \mathcal{R}_{ijt}(\varsigma_{ijt}) - \mathcal{C}_{ijt}(\theta_{ikt}, y_{ijt})$$

And total profits are:

$$\pi_{it} = \sum_{j \in J_i} \pi_{ijt}$$

The last addition is that producers can carry over inputs and final goods to future periods. In particular let ι_{ijt} denote producer i's inventory of good j. Let ι_{ijt} denote producer i's inventory of input k. Then we have the following inventory constraints:

$$\iota_{ikt+1} \leq \iota_{ikt} + q_{ikt}(\vec{\theta}_t, \omega_t) - x_{ikt}$$

$$\iota_{ijt+1} \leq \iota_{ijt} + y_{ijt} - \varsigma_{ijt}$$

Where $x_{ikt} = \sum_{j \in J_i} x_{ijkt}$. All the variables must be non-negative. I leave the producer's full

[2] A generalization of the model would let producers dynamically choose the target level; however, for my empirical purposes, the observation period is short enough to assume that frictions to adjustment keep the target fixed.

decision problem for the appendix.

3.2 Testing the Pecking Order

To what extent do producers follow the pecking order and reallocate inputs to their most profitable goods when facing input shortages? For a secondary market to improve welfare, the benefit of reallocating inputs to the profitable goods must exceed the costs of changing production decisions. Therefore, to test the pecking order, I will search for an identification strategy to measure the production adjustment costs and show that they are not prohibitively large, so producers reallocate inputs to follow the pecking order.

I test the pecking order using the auto industry. The automobile industry is of particular interest because automobiles are an essential consumer good whose production relies heavily on semi-conductors, which are central to the discussion about securing critical inputs for crises. I can test the hypothesis of a reallocation of inputs with the auto industry because producers produce multiple models of cars, and different models use some of the same parts.

For empirical identification, I rely on the supply shortages of 2021, during the COVID pandemic. In March 2020, production and sales froze causing a drop in inventories. Despite the disruptions, production and sales rebounded very quickly. Starting in February 2021, however, producers inventories took a second hit. Figure 1 shows that starting in February 2021, inventories plummeted by more than in March 2020. During this period of February 2021 to March 2022, inventory of more expensive cars fell less sharply. As a result, the average price of a vehicle in inventory shot up during 2021 despite a relatively flat trajectory before and after that year as shown in Figure 2. Figure 3 shows that at least part of the reason for the increase in the average price of an car in inventory was that production of the less expensive cars fell by more than that of expensive cars relative to pre-COVID levels. Why did higher priced cars' production fall by less during inventory shortages?

One explanation could be producers' demand expectations. Perhaps producers wrongly expected large decreases in demand for less expensive cars in 2021 because they predicted that strain on middle-income household balance sheets would remain for longer than it did. Another explanation could be that supply chain disruptions were more severe for cars with a lower price. Extending Grossman, Helpman and Lhuillier (2023) to consider goods with different market share functions would show that producers should more heavily invest in the resilience of the supply chains of goods with lower demand elasticity. For this reason, perhaps supply chains are more resilient for cars with the largest markups in dollar terms, which also likely have higher prices.

Another possibility is that producers followed the pecking order and reallocated inputs to produce their most profitable goods. Such a reallocation could arise either from supply shocks or incorrect demand expectations. The subsequent analysis is agnostic on whether demand or supply caused the shock. An identification strategy to test the reallocation argument will have to show that the two alternative explanations of the data are not driving the results. To show that producer expectations for demand does not explain the results, I need to accurately measure producer's demand expectations. To confront the possibility that producers invest more in supply chain resilience for profitable cars, I need to level cars on the degree to which supply chain disruptions impacted their production. I propose robustness tests to deal with these two possibilities.

Even if reallocation occurred, at what stage of the production process does this reallocation happen? Four potential levels for input reallocation are: producer, plant, platform, and model. At the producer level, producers may truck delivered shipments of inputs to factories that make the more profitable cars. At the plant level, producers may strategically use the parts delivered to the plant to build the most profitable cars. At the platform level, production lines may prioritize profitable models. At the model level, producers may focus on different trims when faced with shortages. [For now I only test the producer and plant level, the other two levels are a work in progress.] I test the pecking order at each of these four levels to understand producers' use of scarce inputs in times of crisis.

3.2.1 Adjustment Cost Identification Strategy

To test the pecking order, I use the first order conditions for the production of a given good. The first order conditions reveal an identification strategy to measure the production adjustment costs associated with the pecking order. The identification strategy asks: how much extra do producers prioritize expensive cars when supply disruptions become more severe? I look to disprove the null hypothesis that production adjustment costs are prohibitively large.

The first order condition for y_{ijt} in the producer's problem is:

$$0 = \xi_{ijt} + \mu_{ijt} - \sum_{k \in K_j} \xi_{ikt} - \sum_{k \in K} \lambda_{ik} - C'_{ij}(y_{ijt}) - c_j \tag{3}$$

Where ξ_{ijt} is the value of extra inventory of good j today (or the Lagrange multiplier for the non-negativity constraint on ι_{ijt+1}, the inventory of good j); μ_{ijt} is the value of saving inventory of good j until tomorrow (or the Lagrange multiplier for the inventory constraint of good j); ξ_{ikt} is the value of extra inventory of input k today (or the Lagrange multiplier on the non-negativity constraint on ι_{ijkt+1}, the inventory of part k); and λ_{ijk} is the value of saving an extra unit of input k (or the Lagrange multiplier on λ_{ijk}, which equals δc_k).

What is the value of having an extra unit of car i? It is represented by the terms $\xi_{ijt} + \mu_{ijt}$. The terms' interpretation is that the value of an extra car is either the discounted value of a car tomorrow, or the revenues from selling the car.

In other words, if we denote $\upsilon_{ijt} = \mathbb{1}_{\iota_{ijt+1}>0}$, then we get that the value of an extra unit of car i is:

$$(1 - \upsilon_{ijt})p_{ijt} + \upsilon_{ijt}\delta\mathbb{E}_t[V'(\iota_{ijt+1})]$$

I approximate that:

$$p_{ijt}\mathbb{E}_t[\delta^{\tau_{ijt}}] = (1 - \upsilon_{ijt})p_{ijt} + \upsilon_{ijt}\delta\mathbb{E}_t[V'(\iota_{ijt+1})]$$

Where τ_{ijt} is the number of months until it takes to sell the marginal unit of good j produced in time t. In other words, the value of an extra car is its discounted expected revenue.

Denote $\sigma_{ijt} = \mathbb{E}_t[\delta^{\tau_{ijt}}]$ and $\hat{y}_{ijt} = \frac{y_{ijt}}{\bar{y}_{ij}}$. Then the first-order-conditions become:

$$\hat{y}_{ijt} = \frac{1}{\eta}(p_{ijt}\sigma_{ijt} - \sum_{k\in K_j}\xi_{ikt} - \sum_{k\in K_j}\lambda_{ik} - c_{ij}) - \bar{y}_{ij} + \epsilon_{ijt}$$

This gives the empirical specification:

$$\hat{y}_{ijt} = \beta_1 p_{ijt}\sigma_{ijt} + \sum_{k\in K_j}\gamma_{ikt}\chi_{ikt} + \sum_{k\in K_j}\gamma_k + \gamma_{ij} + \epsilon_{ijt} \tag{4}$$

Where $\chi_{ikt} = \mathbb{1}_{\iota_{ikt+1}=0}$ is an indicator specifying whether part k is scarce to producer i at time t, and ϵ_{ijt} is the error term. The coefficient of interest is β_1, and the null hypothesis is that $\beta_1 = 0$. The specification has model fixed effects, γ_{ij}, which represent the marginal cost of the car; input fixed effects, γ_k, which represent the cost of an input; and input-producer-time fixed effects, γ_{ikt}, which are only activated when a part is scare. The scarcity fixed effects represent the shadow cost (above the normal cost) of good j at time t to producer i. I refer to $\hat{y}_{ijt}$ as "Production (%)"

Since the expected months inventory of the marginal car, σ_{ijt}, and the scarcity of an input,

χ_{ikt}, are not directly observable, I will approximate them. All information about demand enters through these two variables, so accurately measuring them could alleviate endogeneity concerns caused by producers' demand expectations. I will now present different specifications of this model of production to measure adjustment costs at the platform, plant, and producer levels and to attack identification challenges.

3.2.2 Critical Input Specification

To begin, I assume that every car requires a single critical input. In other words, every producer can make a certain number of cars each period, and the producer decides how to allocate this production budget across cars. The empirical specification simplifies to:

$$\hat{y}_{ijt} = \beta_1 p_{ijt}\sigma_{ijt} + \beta_2\sigma_{ijt} + \beta_3 p_{ijt} + \gamma_{it}\chi_{it} + \gamma_{ij} + \epsilon_{ijt} \tag{5}$$

This specification is an extreme case where the only part in short supply is an identical semiconductor that is used in every car. The control σ_{ijt} is a first step at leveling cars on how severely their supply chains were disrupted. The control p_{ijt} absorbs the effect of the price of a model changing when a new generation is released.

In this specification, I use inventory as a proxy for the expected number of months to sell the marginal car. In particular, σ_{ijt} is inventory of car j in time t normalized by a baseline inventory level. I call this variable "Inventory (%)". Without normalization, this variable would not be comparable across cars since producers' optimal level of months inventory varies significantly between cars. The assumption behind this normalization is that the optimal level of inventory for each car remains constant. The stress indicator χ_{ikt} is whether inventory has fallen more than 50% from its baseline level.

3.2.3 Producer Expectation Robustness

This normalized inventory is an imperfect proxy for a producer's expected months inventory of the marginal car. One reason is that a producer's optimal inventory level could change due to shifts in a producer's expectations of consumer demand. As such, I repeat the critical input specification using two different proxies for a producer's value of an extra car.

The first proxy, "Used Price (%)", is the average price of a used car for a given model as a fraction of a baseline used-car price. If a model's used car price is higher, presumably an additional new unit of that car model is more likely to sell. For example, used car prices spiked during COVID due to inventory shortages. One shortcoming of this proxy is that not all used cars are equally good substitute for their new counterpart. Since the used version of a less expensive make is probably a better substitute for the new version of the car, this should make the measure conservative. Another shortcoming is that manufacturers value of an extra car includes information about the future availability of the car which may not be priced in to used car markets.

For the second proxy, I similarly define "Subvention (%)" as the ratio of the subvention rate of a given car to a baseline subvention rate. Manufacturers lower subvention rates when inventory is low. For example, the subvention rate fell to nearly zero at the height of the shortages in 2021. As such, this metric holds information on how manufacturers view their inventory levels relative to anticipated demand. One shortcoming of this measure is that manufacturers all use subventions for different purposes. As such, baseline subvention rates vary significantly. Many baseline subvention rates are close to zero making any change in the subvention rate unobservable. Furthermore, the disparate reasons for subventions mean that manufacturers might also decrease subventions for reasons other than shortages of a given vehicle. [cite scott-morton?]

To do better than these two proxies for producers' value of an extra car, I would need to structurally estimate demand in the vein of Berry, Levinsohn, and Pakes (1995) [Also include Econometrica paper on markups].

3.2.4 Missed Deliveries Specification [Results Forthcoming]

To level cars on the degree of supply chain disruptions, I will compare cars that are missing the same parts using data on missed deliveries. I will estimate equation 4 in its full form, where the scarcity indicator, χ_{ikt} will be an indicator specifying whether a producer i missed a shipment of input k at time t. Liu, Smirnyagin and Tsyvinski (2024) propose a buyer-seller disruptions indicator. I will extend their methodology to make a supplier-producer-part disruption indicator. Focusing on fully missed shipments implies a disruption and not that producers are merely lowering sales expectations.

Individual parts will have a different input-producer-time fixed effect γ_{ikt}. As such, this specification measures producers' value of a given part (which can be substituted between a certain subset of a producers' cars). Although there are a huge number of parts, the part-specific fixed effects can be grouped across parts if the parts all fit into the same set of cars. Furthermore, the number of disruptions should be small enough that most of these fixed effects are ignored as the scarcity indicator will be turned off.

Since disruptions are at the part level, this specification removes the confounding factor that suppliers might strategically fortify supply chains for the most important parts which would likely support production of the more expensive cars. By honing in on the source of the disruption, this specification asks: if a shock affects two models equally, does the shock cause producers to reallocate parts to the more expensive of the two models?

3.2.5 Pecking Order Levels

Thus far the methodology has focused on the make level analysis; however I use each specification to test the four levels of reallocation. The four levels of reallocation correspond to partitioning the agents' set of projects at different granularity. In terms of the specifications, this means different ways of grouping observations and specifying inputs fixed effects. At the most aggregated level, the producer level, quantities are by model and input fixed effects vary by producer. At the plant level, quantities are by model and plant while input fixed effects vary by plant. At the platform level, quantities are also by plant and model but input fixed effects vary by platform. At the model level, prices and production vary by trim and input fixed effects are at the model level.

3.3 Data

I use monthly production and inventory data from Ward's Intelligence. Ward's provides U.S. production data for a given model and plant. Furthermore it includes U.S. inventory by model. Although the data begins in 1990, in order to focus on the COVID period I limit the sample period to start in 2018. The sample ends in June 2024. This gives enough observations before COVID to establish a baseline period. I define this baseline period to be January 2018 to February 2020. Additionally, I remove models whose production averages less than 50 cars per months over the sample.

I use the SEC's ABS-EE data on auto loans to get an average invoice price on new cars for each generation of a given model.[3] I also use the ABS-EE dataset to get monthly data on the average used car price for a given model, and the cash subvention rate for new cars. After combining these two data sets, I am left with 125 models and 10 thousand observations.

[3]Since Wards does not break down production by model generation, I infer model release dates from ABS-EE and change the model's price upon the release of a new generation.

3.3.1 Measuring Production and Producer Expectations

I measure production and producer expectations as a fraction of a baseline level. I define the baseline level to be the respective variable's average value during the baseline period (January 2018 to February 2020). This applies to the variables "Production (%)", "Inventory (%)", "Used Car (%)", and "Subvention (%)". In the case of production, the baseline represents the producers' target level of production. The assumption's interpretation is that producers designed plants to operate at an optimal capacity before COVID and were unable to adapt this optimal level of production during COVID. In the case of the variables measuring producer expectation, this normalization allows comparison across models. The assumption is that the optimal level of these variables does not change overtime.

3.3.2 Measuring Missed Deliveries [Results Forthcoming]

To measure missed deliveries for the missed deliveries specification, I will use S&P Panjiva Bill of Lading data, which includes shipment-level data on seaborn imports. The data includes the company name of the exporter and the importer, the port of lading and unlading, and details about the shipment's contents. S&P imputes 6-digit HS-Codes which specify the contents of the shipment. Liu, Smirnyagin and Tsyvinski (2024) use the same data to make a buyer-seller disruption measure. I will extend this measure to be at the HS-Code level.

To compliment this disruptions measure, I will use part interchangeability data to determine which models share which parts. Since the S&P data classifies parts at the level of 6-digit HS-Codes, I will also classify interchangeability at this level of granularity. For each 6-digit HS-Code, I will partition cars based on whether they use the same part. Each subset within the partition of each 6-digit HS-Code corresponds to a part indexed by k. If a disruption happens to a given 6-digit HS-Code, then the stress indicator will activate for all parts within that 6-digit HS-Code. As a result, cars will be compared against other cars that use the same part of the type of the disrupted 6-digit HS-Code.

3.4 Results

3.4.1 Results: Critical Input Specification

Table 1 displays the results from the critical input specification (Equation 5). Column (1) shows the results at the make level. The coefficient on inventory shows that greater inventory is correlated with greater production, meaning that this inventory control is serving its purpose of leveling cars on how badly they were hit by shortages. The coefficient on the interaction between price and inventory is -0.00606 (-16.06). Its interpretation is that the production of more expensive cars increases by more in response to decreases in inventory. The magnitude of the coefficient implies that if one car has a price of 10 thousand dollars more than another car and both cars' inventories collapse to zero, the more expensive car's production will fall by 5% less of its pre-COVID baseline production. In COVID, many cars' inventory did collapse to zero, so this 5% is significant in economic magnitude. I will give a better interpretation of the magnitude of this coefficient by considering the counterfactual of a secondary market.

Column (2) shows the result at the plant level. Many observations are dropped since many factories only produce one car, so the number of observations is actually lower than in the first column. Despite comparing production within factories as opposed to within makes, the coefficient on the interaction of production with inventory stays at a similar magnitude at -0.00406 (-5.25).This suggests much of the differential response of production to changes in inventory by car price can be explained by decision-making within factories.

3.4.2 Results: Secondary Market Counterfactual [Work in Progress]

3.5 Figures and Tables

Figure 1: Inventory by Vehicle Price Bucket

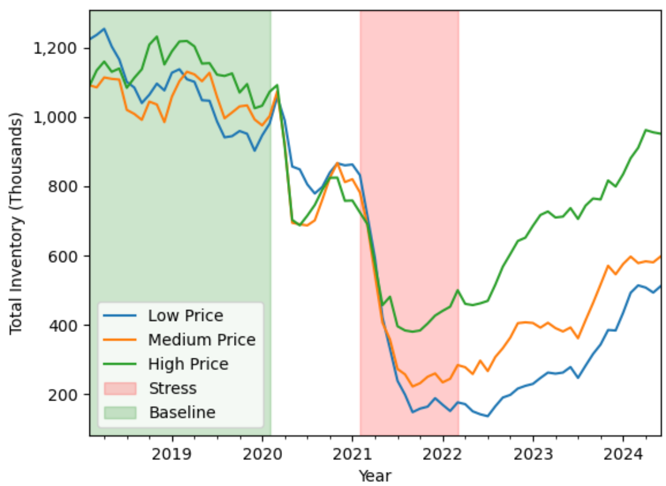


Figure 2: Average Price of Vehicle in Inventory

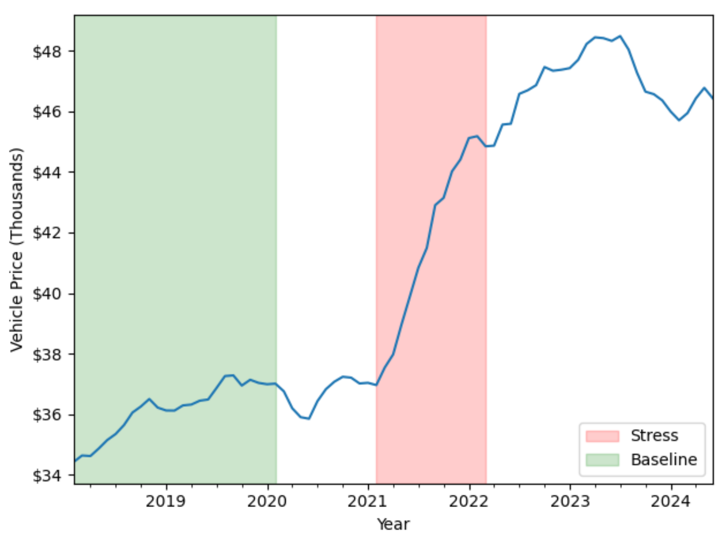
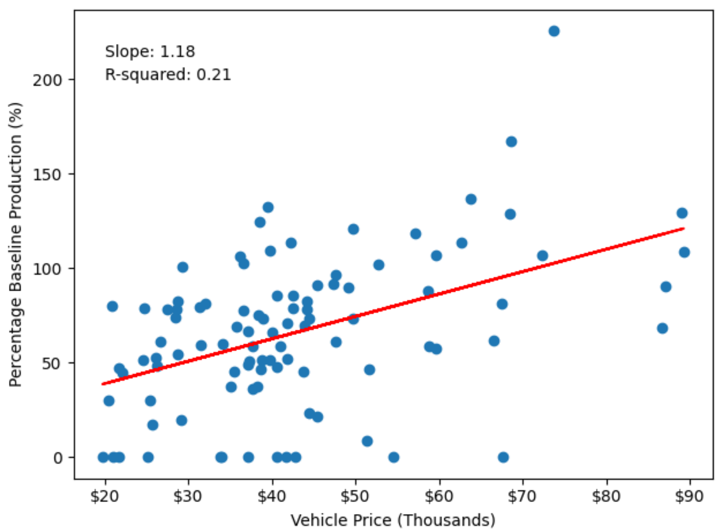


Figure 3: Percentage of Baseline Production by Vehicle Price



Cirtical Input Specification

	(1)	(2)
Dependent variable: Production (%)		
Level:	Manufacturer	Plant
Vehicle Price*Inventory (%)	-0.00606***	-0.00406***
	(-16.06)	(-5.250)
Inventory (%)	0.845***	0.689***
	(17.17)	(28.41)
Vehicle Price	2.23***	1.34***
	(10.17)	(7.130)
Constant	-60.2***	-16.3*
	(-4.733)	(-1.715)
Observations	7,296	6,337
R-Squared	0.629	0.636
Model FE	YES	YES
Manufacturer*Time FE	YES	
Plant*Time FE		YES

Robust t-statistics in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Appendix

A Model of Excess Capacity as Insurance

A.1 Social Planner's Problem

The social planner can be thought of as an agent that gets value from all projects. The only subtlety is how to aggregate costs and quantities across agents.

If the social planner targets the same quantity of the critical good as all the agents combined, the social planner should receive at least the same quantity of the critical good as all the agents combined in each state. Denote θ as the social planner's target, $q(\theta, \omega)$ as the quantity of the critical good the social planner receives, and $c(\theta)$ as the cost of investing in the social planner's critical good target. Then:

$$\begin{aligned} c(\theta) = \min_{\vec{\theta}} \sum_{i \in c} c_i(\theta_i) \\ \text{s.t.} \sum_{i \in I} \theta_i \leq \theta \end{aligned} \tag{6}$$

and for all $\omega \in \Omega$:

$$\begin{aligned} q(\theta, \omega) = \max_{\vec{\theta}} \sum_{i \in c} q_i(\theta_i, \omega) \\ \text{s.t.} \sum_{i \in I} \theta_i \leq \theta \end{aligned} \tag{7}$$

The social planner does not necessarily do strictly better than the agents' aggregate costs and quantities. An example where the social planner's costs and quantities align with the agents' aggregate costs and quantities for any vector of targets, $\vec{\theta}$, is when $\Omega = [0, 1]$, $c_i(\theta_i) = c_0 \theta_i$, and $q_i(\vec{\theta}, \omega) = \gamma(\omega)\theta_i$ for all $i \in I$ and for some c_0 and $\gamma(\cdot) : [0, 1] \to [0, 1]$. Even in this case the market solution does not align with the social optimum.

A.2 Preparing for Crisis

Agent i's first order conditions for θ_i is:

$$\int_{\Omega} v_i'(q_i(\vec{\theta}, \omega), \omega) q_i'(\vec{\theta}, \omega) f(\omega)\, d\omega \ = c_i'(\theta_i)$$

where $q_i'(\vec{\theta}, \omega) = \frac{\partial}{\partial \theta_i} q_i(\vec{\theta}, \omega)$. Meanwhile the social planner solves:

$$\int_{\Omega} u'(q(\theta, \omega), \omega) q'(\theta, \omega) f(\omega)\, d\omega \ = c'(\theta)$$

We want to compare θ to $\hat{\theta}$, where $\hat{\theta} = \sum_{i \in I} \theta_i$. By equation (7), we have that:

$$q_i'(\vec{\theta}, \omega) \geq \sum_{i \in I} q_i'(\vec{\theta}, \omega) \quad \forall \omega \in \Omega$$

$$c_i'(\vec{\theta}) \leq \sum_{i \in I} c_i'(\vec{\theta})$$

$$u'(q(\theta, \omega), \omega) \leq \sum_{i \in I} v_i'(q_i(\vec{\theta}, \omega), \omega)$$

Where the last inequality follows by proposition (1).

B Model of Producer Inventory Surplus

B.1 Producer Problem

The producer i maximizes expected discounted profits:

$$\begin{aligned} \max_{\{\{\{\theta_{ikt+1}\}_{k \in K_j}\}, \{y_{ijt}, \varsigma_{ijt}\}_{i \in I_j}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \mathbb{E}_0[\delta^t \pi_{it}] \\ \text{s.t. } y_{ijt} = \min_{k \in K_j} \alpha_{ijk} x_{ijkt} \\ \iota_{ikt+1} \leq \iota_{ikt} + q_{ikt}(\vec{\theta}_t, \omega_t) - x_{ikt} \quad \forall k \in K_i \\ \iota_{ijt+1} \leq \iota_{ijt} + y_{ijt} - \varsigma_{ijt} \quad \forall j \in J_i \\ \iota_{ikt+1} \geq 0, x_{ikt} \geq 0, \theta_{ikt+1} \geq 0 \quad \forall k \in K_i \\ \iota_{ijt+1} \geq 0, y_{ijt} \geq 0, \varsigma_{ijt} \geq 0 \quad \forall j \in J_i \end{aligned} \tag{8}$$

Where θ_{ik0} and ι_{ij0} are given $\forall k \in K_j$, and ι_{ij0} is given $\forall i \in I_j$. And δ is the discount factor.