Preparing for Industrial Input Shortages

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Abstract

Why do private incentives distort investment in resilience? I build on Grossman, Helpman and Lhuillier (2023) to model agents' investment in excess capacity of a homogeneous good, for example electricity, computing power, or semi-conductors. During shortages the marginal value of the good varies across agents, which distorts investment in excess capacity of the good. A secondary market for the homogeneous good would improve this misallocation; however, private incentives may cause the market price to be higher than would be socially optimal. I use this framework to evaluate policy solutions to create an efficient secondary market. In ongoing work, I will look for a mechanism to achieve the optimal allocation. This model provides a unified theory to compare the problem of optimal hospital surge capacity, bank liquidity buffers, and producer inventory. In Lewis-Hayre (2024), I apply the model to a novel situation: supply chain disruptions.

1 Introduction

Since the 10th century A.D., from the Iberian Peninsula to New Mexico, people have relied on communal irrigation systems called acequias. When water is abundant, acequias seem like any other irrigation system: its users extract water until the marginal value of water aligns with the marginal cost of using the water; however, during shortages, acequias allow a majordomo, a town official, to efficiently allocate water between residents. As such, this mechanism incentivizes cooperation to invest in acequias, which better capture snow melt, thereby promoting water adequacy. A drawback of acequias is that the majordomo must determine how to allocate water during shortages. How can policy incentivize these benefits of the acequia without the need for a majordomo?

A similar public goods problem arises with shortages of other homogeneous goods. The prospect of shortages of electricity, hospital equipment, cloud computing resources, and semi-conductors, has led governments to deem securing access to these goods a matter of national security. As such, governments have resorted to an industrial policy to secure access to critical goods. For example, the United States Government, between executive action, the Defense Production Act, and The CHIPS and Science Act, has invested in nuclear energy, personal protective equipment, cybersecurity, and semi-conductor factories.

Electricity, surgical masks, bank reserves, computing power, and semi-conductors, are mostly homogeneous like water. Yet in contrast with the example of acequias, private firms control access to these resources. Therefore, as emphasized in Grossman, Helpman, and Lhuillier (2023), "the question for governments is not whether shortages adversely affect households but whether firms' private incentives to avoid such shortages fall short of (or exceed) what is socially desirable." To determine whether policy intervention is needed and how to make such policy efficient if justified, policymakers must first understand: why do private incentives distort investment in resilience? Answering this question will provide a way to test the efficiency of current policy. In particular, this paper asks: how does firms' use of homogeneous resources during shortages affect investment in resilience?

In some cases this private control creates a system with a similar purpose to that of acequias. Utility companies sell excess energy on utility markets. Hospitals coordinate use of scarce medical equipment like ventilators, surgical masks, or ambulances. These examples have large literatures that study how to make these markets efficient in order to maximize use of scarce resources during shortages. Why are these examples similar? Why are such secondary markets beneficial?

In other cases, such cooperation between firms does not occur. Cloud computing companies do not share computing resources with one another to cope with cyberattacks. Producers do not sell each other semi-conductors when they are in short supply. Why do secondary markets form in some cases and not others? How do private incentives affect whether secondary markets form and how efficiently they operate?

In this paper, I build on Grossman, Helpman, and Lhuillier (2023) to model an agent's decision to set a target capacity of a homogeneous good in anticipation of potential shortages of the good. At first I assume agents cannot exchange goods on a secondary market in order to establish the importance of such a market. I show that other homogeneous goods behave as does water in the example of acequias. Under the assumption of supply frictions during shortages and under uncertainty in shocks, goods will be misallocated between agents during shortages. As a result, the marginal value of the good will vary across agents. If an agent with a lower value for the good uses the good for themselves, this consumption creates a negative externality.

How will this misallocation of goods affect private investment in resilience? This investment in resilience could take many forms. For example, an automobile producer could diversify their supply chain to mitigate the effects of semi-conductor shortages. A cloud computing company could invest in better cybersecurity. An energy company could invest in more dependable energy sources. In my model, I abstract away from how producers invest in resilience. Instead I model agents' decision to invest in resilience as an option to pay a cost to increase the availability of the good under various shocks.

I show that if goods are not used efficiently during shortages, the market solution may either over or under invest in resilience. This is because the misallocation of goods during shortages creates two competing forces. On one hand, when compared to social optimum, the market solution will use an extra good inefficiently because during shortages, an extra good may not go to the agent who would get the most value from an extra good. Less efficient use of the marginal good, will push agents to under-invest in resilience. On the other hand, in the market solution, the goods allocated before this marginal good may also be misallocated. This will cause some agents to have a very high value for an extra good, pushing them to over-invest in resilience. If the agents are symmetric, I show that the convexity of the marginal value of a project determines wether the market solution over or under invests in resilience. This conclusion that depending on the projects' value functions, the market solution may either cause an over or under investment in resilience parallels a similar conclusion in Grossman, Helpman, and Lhuillier (2023), which shows that whether firms under or over invest in resilience depends on the elasticity of consumer demand.

A secondary market for the good could fix this misallocation during shortages. If during shortages, agents sold each other goods on a secondary market and the market clearing price was equal to the social planners' marginal value of the good, then the arising allocation would coincide with the social optimum. Private incentives, however, are likely to distort the secondary market. In the case of cloud computing, in the long term, companies could steal market share by letting their competitor fail to meet demand. In the example of firms using the good as an input to their production processes, giving inputs to competitors allows competitors to increase supply which weakens demand for the output of the first firm. On the other hand, the competitor will also be willing to pay a premium to take the input away from competition, so the effect on the secondary market will depend on consumer demand. In ongoing work, I fully develop this example which makes my model a direct extension of Grossman, Helpman, and Lhuillier (2023) to include homogeneous goods. Regardless of the form of private incentives, the relevant question is: under subsidies that achieve the social optimal allocation, if a secondary market were not feasible, how would private incentives affect the market price of the secondary market? I create a framework to understand such distortions. I show that in the specific example where agents are hoarders, the price on the secondary market is artificially high, so agents may be unwilling to pay to create a secondary market even if it is socially optimal.

How could policy intervention help foster an efficient secondary market? A common solution to this problem is a reserve, but to achieve the social optimum with only a reserve, agents would have to lower their targets so that they never receive more of the good than is socially optimal. Another solution is a subsidy on transactions in the secondary market that depends on the project and the state of the world. I show that this policy requires a larger than necessary expected subsidy. A better alternative is a policy which pays agents to enter into a smart contract requiring them to sell a predetermined number of goods for a given price depending on the state. Both of these policies would be difficult to implement since the policymaker must know the social benefit of trading in different states in order to determine the size of the subsidy or the optimal smart contract. In ongoing work, I search for an incentive compatible mechanism to establish efficient secondary markets, which would make it optimal for agents to report the true values of their projects. I will also consider combining a subsidy with a reserve assuming a penalty that is increasing with the size of the reserve.

This paper builds on three strands of literature. The first strand focuses on supply chains. I build on the supply chain literature by showing a new effect of the rigidity of supply chain networks. As opposed to previous literature which focuses on how shocks propagate vertically in the supply chain, I consider how disruptions in the supply chain create an unharnessed value from relationships between firms at the same level of the supply chain. I also build on the novel policy analysis in Grossman, Helpman, and Lhuillier (2023). Another major contribution of Grossman, Helpman, and Lhuillier (2023) is modeling the nuances of homothetic demand beyond CES utility functions. I abstract away from these complexities of demand to make the problem more transferable to situations like governments investing in disaster relief resources and situations like cloud computing where consumers buy a subscription to a service that could be disrupted. Other supply chain literature focuses on forming relationships between producers and their suppliers. Grossman, Helpman, and Sabal (2023) considers the externalities in supply chains arising endogenously through sequential bargaining. Meanwhile Acemoglu and Tahbaz-Salehi (2021) focuses on amplification of shocks through a supply chain.

The second strand focuses on production networks. In this literature the prices of a firm's inputs adjust dynamically with shocks. As a result, the market price for inputs is efficient. I show that adding rigidities to production networks disrupts this power of flexible pricing and creates a misalignment in the value of inputs between firms. Acemoglu et al. (2012) shows how such networks transform idiosyncratic shocks into aggregate fluctuations. Liu and Tsyvinski (2024) studies production adjustment costs in input-output networks. Liu (2019) considers why these production networks create a need for industrial policy. Pellet and Tahbaz-Salehi (2023) introduces rigidities to production networks by modeling firms that must make their production decisions for some inputs under uncertainty. This leaves the possibility for heterogeneity in a firm's value for a given input. However, this paper focuses on the effects to the equilibrium and does not consider how firms could try to overcome this friction through a secondary market.

The third strand focuses on shortages. This literature spans many topics including energy markets, cloud computing resource sharing, hospital surge capacity, and interbank lending. I provide a common framework to unite a central problem in these literatures. The model provides an economic explanation for why cooperation between firms may be lower than what is socially optimal. The model also provides a way to test policy solutions to confront shortages, the principles of which can be applied to any of these industries. Of these literature on shortages, the work on interbank lending is the most expansive. Allen and Gale (2000) study how banks' can manage the risk of deposit outflows through interbank markets. Allen, Carletti, and Gale (2009) show the importance of a central bank to create sufficient liquidity in such markets. The central bank serves as a reserve for liquidity. In energy markets, Hogan (2013) proposes operating reserve to correct pricing distortions during shortages.

2 Model of Excess Capacity

Agents use a critical good to advance valuable projects. First each agent chooses a target quantity of the critical good under uncertainty in the value of the good and whether she will have access to the full quantity targeted. Next the state of the world is revealed to all agents. Finally, given the state of the world, each agent allocates her available quantity of the critical good between her projects to maximize their total value. For now I assume there are no secondary markets so agents cannot trade each other the critical good.

This decision under uncertainty can be interpreted as a supply friction that prevents the availability of the good to adjust to shocks. Private incentives come from the fact that agents only get value from some projects. As such, the social planner is an agent that gets value from all projects.

One way to interpret the model is a firm deciding how much excess inventory of inputs it would like to order from its suppliers. When taken in this context, my model follows a similar framework as Grossman, Helpman, and Lhuillier (2023) except I abstract the details of demand to give the problem applicability beyond the case of a producer. As shown in their paper, to understand firms incentives to invest in resilience a model must remove the distortions of market power. Their paper uses optimal subsidies to overcome the wedge between the market solution and the social optimum created by producer's market power. Instead, I assume the producer's value is equal to the social optimum. One micro-foundation for this assumption is that producers practice first-degree and consumers are only willing to buy one good. Alternatively, this assumption could be interpreted as only considering the problem after social planner has already implemented optimal project and state dependent consumption subsidies.

2.1 Model Setup

There is a discrete set of agents denoted I. There are a discrete set of projects denoted J. An agent $i \in I$ invests in a set of projects denoted $J_i \subseteq J$. I assume only one agent can invest in each project, so $\{J_i\}_{i \in I}$ partitions J. After the agents learn the state of the world $\omega \in \Omega$, which is realized with probability $f(\omega)$, agent i allocates her available units of the critical good, denoted

q_i, between the projects J_i. Denote q_{ij} as the amount of the critical good that agent i allocates to good j. So we have that:

$$\sum_{j \in J_i} q_{ij} \leq q_i$$

The value that the agent receives from the project depends on the quantity of the critical good that the agent allocates to the project, q_{ij}, and the state of the world, $\omega \in \Omega$. When deciding how to allocate the critical good between projects, the agent knows the state of the world; therefore, given a quantity of the critical good, q_i, agent i solves:

$$\begin{aligned} v_i(q_i, \omega) = \max_{\{q_{ij}\}_{j \in J_i}} & \sum_{j \in J_i} v_{ij}(q_{ij}, \omega) \\ \text{s.t.} & \sum_{j \in J_i} q_{ij} \leq q_i \end{aligned} \tag{1}$$

Where $v_i(q_i, \omega)$ denotes the total value of agent i and $v_{ij}(q_{ij}, \omega)$ denotes the value agent i receives from project j. Assume v_{ij} is strictly increasing and strictly concave, so the agents gain a decreasing marginal benefit from investing an additional unit of the critical good into project i.

Before the agent learns the state of the world, she must choose a target of how much critical good she wants; however, she might not receive as much of the good as she wanted. In particular, if agent i sets a target of θ_i units of the critical good, then she receives $q_i(\vec{\theta}, \omega)$ in state ω where $\vec{\theta} = \{\theta_i\}_{i \in I}$. I assume $q_i(\vec{\theta}, \omega)$ is weakly increasing in θ_i. Agents solve the following problem to prepare for potential crises:

$$V_i = \max_{\theta_i} V_i(\vec{\theta}) = \max_{\theta_i} \mathbb{E}[v_i(q_i(\theta_i))|\vec{\theta}_{-i}] - c_i(\theta_i) = \max_{\theta_i} \int_{\Omega} v_i(q_i(\vec{\theta}, \omega), \omega) f(\omega)\, d\omega \ - c_i(\theta_i)$$

Where $V_i(\vec{\theta})$ is the expected value of agent i given targets $\vec{\theta}$, where $\vec{\theta}_{-i} = \{\theta_l\}_{l \in I \setminus \{i\}}$ is the targets of other agents, and where $c_i(\theta_i)$ is the cost of setting the critical good target. Assume $c_i(\theta_i)$ is weakly convex in θ_i.

In the case of a producer selling differentiated goods that use a common input, then the critical good represents the common input and $v_i(q_{ij}, \omega)$ represents the profits from using q_{ij} units of inputs to produce final goods and selling those final goods at an implicit price. The equivalence between the producer's profit and the social welfare could arise from first-degree price discrimination where producers do not compete with one another or from optimal consumption subsidies. The critical good target represents the producer's order of an input. A supply disruption could cause the quantity of goods delivered to be lower than the producer's order.

2.1.1 Social Planner's Problem

The social planner is equivalent to an agent that gets value from all projects. In other words, if $J_i = J$, we get the social planner. Denote $v(q)$ as the value of the social planner given q units of the critical good. A delicate part of the social planner's problem is how to aggregate the cost and quantity functions. This aggregation does not necessarily benefit the social planner over the market solution in order to emphasize the real mechanism which is the inability of agents to adjust their supply to shocks. The full details of the social planner's problem can be found in Appendix Section (A.2).

3 Comparing the Market Solution with the Social Optimum

The market solution distorts critical good targets when compared with the social optimal. To see why, I first consider what happens during shortages. Given this behavior during shortages, I consider how agents will set their targets to prepare for shortages. During shortages, an agent's investments of the critical good follow a pecking order defined by the value of extra investment into her projects. Agents' prioritization of their own projects silos this pecking order, preventing the critical good from being optimally utilized. Unless the critical good is perfectly allocated between agents so agents' marginal value of the critical good aligns, the market allocation of the critical good will be inefficient. The lack of a secondary market where agents can trade the critical good causes this distortion. If agents traded the critical good and the arising market price was the same as the social planner's marginal value of the good, then the allocation would be efficient. Without a secondary market, the market solution skews preparation for crisis. I show that in the case of symmetric agents, the direction of this skew depends on the convexity of a project's marginal value.

3.1 Using Scarce Resources

Given a fixed quantity of the critical good, an agent invests the critical good following a pecking order of the value of advancing her projects until the agent runs out of the critical good; therefore, how the agents partition the projects determines the social welfare. Consider agent i who invests in projects J_i and has q_i units of the essential good in state ω. In this subsection everything is implicitly a function of the state ω. The first order condition is:

$$v'_{ij}(q^m_{ij}) = \lambda_i \quad \forall j \in J_i$$

Where q^m_{ij} is agent i's allocation of critical goods to project j in the market solution, and λ_i is agent i's the shadow price of the critical good. The first order condition says that the value of the agent's final good allocated to each project should be the same.

I define $\lambda_i(q_i)$ to be the shadow price of the critical good for agent i given that the agent has q_i units of the critical good. One way to think about $\lambda_i(q_i)$, is that the agent sorts each unit allocated to its projects by the marginal value of that unit, and allocates the q_i goods to the projects in this order until she runs out of the critical good. In this way, the agents follow a pecking order. I elaborate on this pecking order in Appendix Section A.4, where I show that this λ_i can be derived directly from the functional form of $\{v_{ij}\}_{j \in J_i}$. In fact, $\lambda_i(q_i)$ is strictly increasing in q_i, so their is a unique quantity that leaves the agent with a given shadow price.

The social planner's problem is the same as an agent's problem except the social planner gets value from all the projects. As such, the first order conditions are:

$$v'_{ij}(q^*_{ij}) = \lambda \quad \forall j \in J$$

Where q^*_{ij} is the social planner's optimal allocation of the critical good to project j. As before, I define $\lambda(q)$ to be the marginal value of the good to a social planner that has q units of the good. This function is strictly increasing and can be derived from the functions $\{v_{ij}\}_{j \in J}$. This leads to the following proposition:

Proposition 1. *The market solution is socially optimal if and only if $\lambda_i(q_i) = \lambda(q)$ for all $i \in I$.*

The proposition says that the shadow price of all agents must align. Since $\lambda_i(q_i)$ is increasing in q_i, then each agent must have exactly the correct number of goods to achieve the social optimum. Since λ_i is derived exclusively from $\{v_{ij}\}_{j \in J_i}$ which is exogenously given, this equality will almost never hold for arbitrary q_i.

Of course q_i is not arbitrary. In fact, if a secondary market existed for critical good, the price $\lambda(q)$ would clear the market and remove this inefficiency. In other words, this inefficiency hinges on the fact that agents find themselves in a shortage with the wrong quantity of the critical good and cannot buy and sell the critical good to rectify this misallocation. In Section 4, I will investigate how private incentives that distort an agent's value function could cause this secondary market to be inefficient.

3.2 Preparing for Shortages

How should agents set their targets of the critical good? Agents can only choose one critical good target. Since the target is not state dependent, different agents find themselves with different values for the critical good in different states. As such, the agents do not use the critical good optimally. This means the total targets in the market solution does not align with that of the social optimal.

As before let the superscript m denote the market solution and $*$ denote the social optimum. We want to compare θ^* to θ^m, where $\theta^m = \sum_{i \in I} \theta_i^m$. As before, denote $\lambda(\omega)$ as the social planner's shadow price of the good in state ω.

A given agent's first order condition is:

$$\int_\Omega \lambda_i(q_i(\vec{\theta}^{\,m}, \omega), \omega) q_i'(\vec{\theta}^{\,m}, \omega) f(\omega)\, d\omega \ = c_i'(\theta_i^m)$$

While the social planner's first order condition is:

$$\int_\Omega \lambda(q(\theta^*, \omega), \omega) q'(\theta^*, \omega) f(\omega)\, d\omega \ = c'(\theta^*)$$

Given these first order conditions, Proposition 1 shows that except for very specific functional forms of the quantity and value functions, the total targets in the market solution will not equal the targets in the social planner's optimum. Unless in every state the value from the different projects perfectly align to make the final unit of the critical good equally valuable across agents, the market solution will have a different total target than is socially optimal. If agents cannot trade goods, there is no reason to expect this condition to hold. Agents only have one degree of freedom in that they get to choose θ_i^m, meanwhile q_i is exogenously given, so except in degenerate cases, this equality will not hold in all states. In ongoing work, I show that a slight perturbation to any of these functions ensures that the market's target does not equal the social planner's target.

If in a given state, an agent receives less (more) of the critical good than is socially optimal, then the agent's marginal value of the good will be higher (lower) than in the social optimum. In every state, some agents will have too high a marginal value for the critical good, and others will have too low a marginal value. How these dependencies in marginal value average out across agents and over periods, determines whether the market solution will over- or under-invest in resilience. To highlight these dynamics I consider a case where agents are symmetric. This allows me to pin down the direction of the distortion in the investment in resilience.

3.2.1 Symmetric Agents Example

Are critical good targets in the market solution higher or lower than in the social optimum? I show this depends upon the functional form of the value of the projects.

To simplify this illustration, I consider an example where all projects have the same value function. Furthermore, to equate the social planner's cost of setting a target with the agent's collective cost, I assume a linear cost function and symmetric shocks with an idiosyncratic and a systematic component.

Assumption 1. *All agents have a single project, all of which have the same value function v_0. Furthermore, given the state space $\omega = (\omega_I, \omega_S) \in \Omega = \Omega_I \times \Omega_S$, assume $c_i(\theta) = c_0\theta$ and $q(\vec{\theta}, \omega) = \theta_i \gamma_i(\omega_I)\delta(\omega_S)$ for some $\gamma_i(\cdot)$ and $\delta(\cdot)$ where $\sum_{i \in I} \gamma_i(\omega_I) = 1$, $\int_{\Omega_I} \gamma_i(\omega_I)\, d\omega = 1$, and $\gamma_i(\omega_I)$ is not constant.*

The idiosyncratic component comes from Ω_I and the systematic component comes from Ω_S. These assumptions lead to the following proposition, the proof of which is in Appendix Section A.4.1:

Proposition 2. *If the marginal value function is strictly convex (concave) in all states then the target in the market solution is lower (higher) than the target in the social optimum.*

There are two competing forces that determine whether their is excessive or insufficient investment in resilience in the market solution. On one hand, in the market solution, agents would use an extra goods less efficiently than would the social planner. This force pushes the market solution to under-invest in resilience. On the other hand, the market solution also causes the first goods to be inefficeintly allocated to projects. This means some agents might have a very high marginal value for an extra good. As such, this force pushes the market solution to over-invest in resilience.

4 Formation of Secondary Markets

Why do secondary markets form in some instances of this problem and not in others? Perhaps the up-front cost of implementing such a secondary market is prohibitively large. For example, it is a difficult technical challenge to get the computer systems of Amazon and Google to connect their cloud computing networks. Once systems exist to facilitate a secondary market, there can still be transaction costs or coordination problems. Even banks struggle to borrow from the discount window during crises despite the relative ease of transferring money compared to transacting tangible resources. Such costs would exist regardless of private incentives.

However, private incentives could also distort secondary markets, causing an inadequate (or superfluous) investment in the secondary market. If a firm sells a scarce input to a competitor and the competitor uses this resource to boost output and sales, this could dilute the value of the first firm's products. On the other hand, the competitor firm will over-value extra units of the good. Depending on demand, this may cause an under (or over) reliance on secondary markets. I investigate this example more thoroughly in ongoing work. In the example of cloud computing, by the time computational shortages arise, consumer may have already chosen their providers, but providers could still be unwilling to sell their additional resources on secondary markets to win future contracts with their competitors' clients.

Although the direction of the distortion is unclear, in all these examples private incentives cause a distortion in willingness to transact on the secondary market, which means the total private value of the secondary market is either insufficient or excessive. The gap between the private and social value of the secondary market means secondary markets may not form even though they should, or vice versa. To isolate the impact of these incentives, I consider: how would private incentives affect the price on the secondary market given a policy that would be optimal if the secondary market did not exist?

4.1 Private Incentives in a Secondary Market

I show that if agents have private incentives that distort their willingness to sell critical good to other agents during shortages, then even the existence of a secondary market will not be enough to align the market solution with the social optimum. I assume that the private value of a project to a given agent depends on other agents' allocations. I call this private value the agent's profit. Since an agent making her own allocation decision takes the other agent's allocations as given, this can be captured by making an agent's profit a function of the amount of the critical good she buys (sells) on the secondary market. In particular, now in state ω, agent i solves the problem:

$$\pi_i(\vec{q},p,\omega) = \max_{\{q_{ij}\}_{j\in J_i}, q_i^\tau} \sum_{j\in J_i} \pi_{ij}(q_{ij}, q_i^\tau, \omega) - p(\omega) q_i^\tau$$
$$\text{s.t. } q_i^\tau + \sum_{j\in J_i} q_{ij} \leq q_i \tag{2}$$

Where q_i^τ is the quantity that firm i buys (sells) on the secondary market at price $p(\omega)$. Furthermore $\pi_{ij}(q_{ij}, q_i^\tau, \omega)$ is agent i's profit of project j, and $\pi_i(\vec{q},p,\omega)$ is agent i's total profit under the allocation $\vec{q}$. The market clearing condition is $\sum_{i\in I} q_i^\tau = 0$. As before, I denote $v_{ij}(q_{ij},\omega)$ as the social welfare from project j if q_{ij} goods are allocated to project j.

The social optimum is achieved if the market price $p(\omega)$ follows $p(\omega) = \lambda(\omega)$ for all $\omega \in \Omega$. The first order conditions for an agent i that buys or sells goods on the secondary markets becomes:

$$\frac{\partial}{\partial q_{ij}} \pi_{ij}(q_{ij}^m, q_i^{\tau\, m}, \omega) + \frac{\partial}{\partial q_i^\tau} \pi_{ij}(q_{ij}^m, q_i^{\tau\, m}, \omega) = p(\omega)$$

Whereas the social planners' first order conditions are the same as before:

$$v'_{ij}(q_{ij}^*, \omega) = \lambda(\omega)$$

So depending on the form of the private incentives they can cause a distortion in the secondary market resulting in a misallocation of goods.

To understand how these private incentives distort secondary markets, the relevant question is: how would private incentives distort a secondary market under a policy that would be optimal if the secondary market did not exist? As in the original analysis this means assuming that private profits coincide with social value. I maintain this assumption.

Assumption 2. *If $q_i^\tau = 0$ then*

$$\pi_{ij}(q_{ij}, q_i^\tau, \omega) = v_{ij}(q_{ij}, \omega)$$

To apply this framework to a given application, the challenge is identifying the functional form of $\pi_{ij}(q_{ij}, q_i^\tau, \omega)$. In ongoing work, I compute this function in the case where agents are firms producing different varieties of an output. This example is the direct extension of Grossman, Helpman, and Lhuillier (2023) to include homogeneous inputs.

To highlight the implications of distortions in private incentives, I give an example of agents with a particular profit functiont. In particular, I consider the case where agents have a private incentive to hoard goods.

4.1.1 Hoarder Example

Assuming agents hoard goods during shortages, how would this affect the secondary market? I make the following assumption to isolate the impact of such private incentives.

Assumption 3.

$$\pi_{ij}(q_{ij}, q_i^\tau, \omega) = v_{ij}(q_{ij}, \omega) - \phi_{ij}(q_i^\tau)$$

Where $\phi_{ij}(\cdot)$ is strictly decreasing and strictly convex when $q_i^\tau < 0$. Furthermore $\phi_{ij}(\cdot)$ is identically zero when $q_i^\tau \geq 0$.

The interpretation of this assumption is that selling goods on the secondary market decreases an agent's value for her projects. As a result, agents get disproportionate value from using their goods on their own projects. I also assume away the degenerate case, so assume that in the social optimum there is some reallocation of goods between agents in every state.

Since agents overvalue their own goods, then they will be reluctant to sell their goods causing the secondary market to be smaller than would be socially optimal. As a result, the price on the secondary market will be larger than optimal. In particular, we have the following proposition:

Proposition 3. $p(\omega) > \lambda(\omega)$

Corollary 1. $\sum_{i \in I} |q_i^{\tau\, m}| < \sum_{i \in I} |q_i^{\tau\, *}|$

Corollary 2. $\sum_{i \in I} \pi_i(\vec{q}^{\,m}, p, \omega) < \sum_{i \in I} v_i(\vec{q}^{\,m}, \omega) < \sum_{i \in I} v_i(\vec{q}^{\,*}, \omega)$.

Where $q_i^{\tau\, *}$ is the transfer to agent i in the social optimum.

The interpretation of the corollaries is that (i) the total amount of trade will be smaller than is socially optimal; (ii) there will be a misallocation of goods in the market solution.

4.2 Preparing for Shortages

In the market solution, an agent's expected profits with a secondary market is:

$$\max_{\theta_i} \Pi_i(\vec{\theta}) = \max_{\theta_i} \int_\Omega \pi_i(q_i(\vec{\theta}^*, \omega), p(\vec{\theta}^*, \omega), \omega) f(\omega)\, d\omega \; - c_i(\theta_i^*)$$

The first order conditions are:

$$\int_\Omega \pi_i'(q_i(\vec{\theta}^{\,m}, \omega), p(\vec{\theta}^{\,m}, \omega), \omega) q_i'(\vec{\theta}^{\,m}, \omega) f(\omega)\, d\omega \; = c_i'(\theta_i^m)$$

Depending on the function form of π_{ij}, this could lead θ_i^m to be larger or smaller than is socially optimal.

4.2.1 Preparing for Shortages: Symmetric Hoarders Example

Now I combine the examples of symmetric and hoarding agents to give an example of a scenario where the market solution's target is different from that the socially optimal target. We can extend the logic from 2 and use Corollary A3 of Proposition A3 to get the following proposition.

Proposition 4. *In the case of hoarding agents, if the marginal value function is strictly convex (concave) in all states then the target in the market solution is lower (higher) than the target in the social optimum.*

This proposition says that if agents hoard the good, which causes an inefficient secondary market, then targets of the good will be lower than is socially optimal. This propositions shows how the argument in 2 is very general. As long as there is some misallocation during shortages, the convexity of the social planner's marginal value of a project determines whether the market solution over or under invests in targets of the critical good.

4.3 Funding a Secondary Market

As discussed before, a secondary market may require upfront costs to set it up. I show how the wedge created by private incentives can cause agents to under or over value the existence of the secondary market when compared to the social planner. As a result, a secondary market may not form even if it is socially optimal or vice versa.

To model the upfront cost of the secondary market, I assume that before setting their targets agents must collectively decide whether or not to build a secondary market. To build the secondary market they must collectively pay a fixed cost. I do not consider how they split this cost, I just assume the secondary market will be built if the agents have the collective willingness to pay.

As such, the relevant comparison is between total profits in the market solution, and total value in the social planners problem. Denote V_i^* as an agent's value under the social planner's allocation. If the fixed cost C follows:

$$\sum_{i \in I} \Pi_i < C < \sum_{i \in I} V_i^*$$

Then the inefficiency in the secondary market will cause it not to form even though an efficient secondary market would be worth the start-up costs. However, if:

$$\sum_{i \in I} V_i^* < C < \sum_{i \in I} \Pi_i$$

Then a secondary market will form even though it is not socially optimal.

4.3.1 Funding a Secondary Market: Hoarder Example

Returning to the example of agents that hoard the critical good, how would this hoarding tendency impact the formation of a secondary market?

From Corollary 2 of Proposition A3:

$$\sum_{i \in I} \Pi_i < \sum_{i \in I} V_i^*$$

This means that if the fixed cost of forming a market, C, follows:

$$\sum_{i \in I} \Pi_i < C < \sum_{i \in I} V_i^*$$

Then a secondary market will not form even though it would be socially optimal to pay for the secondary market if the good were used efficiently during shortages.

4.4 Policy Solutions [Work in Progress]

How can policy incentivize the formation of secondary markets when such markets are socially optimal? As throughout this paper, I assume that a policy is already in place to remove any distortions that would arise if a secondary market were infeasible. What additional policy could help facilitate efficient secondary markets? Although the magnitude and direction of these policies depend on the application, the model provides a framework to think about the relative effectiveness of different policies.

4.4.1 Reserve

One way to achieve the social optimal is through a reserve. The policymaker could buy extra critical goods to align the total targets of the critical good with the social optimum. One example of such a reserve is the U.S. government's Strategic Petroleum Reserve, which aims to prevent gasoline prices from rising too high. The government looks to buy petroleum when it is relatively cheap and sell it when it is expensive, so if executed correctly, the reserve is profitable. As such, a private entity could be incentivize to operate such a reserve.

Although the reserve boosts welfare, I show that it has the adverse effect of causing agents to reduce their targets and rely heavily on the reserve. I consider the case where agents can buy extra goods from the reserve but there is no secondary market. I rule out a secondary market becuase if the policymaker only has the reserve at her disposal, then the secondary market will not help shrink the size of the optimal reserve. On one hand, if the secondary market is efficient, then there is no need for a reserve. On the other hand, if private incentives cause the secondary market to be inefficient, there will have to be no transactions on the secondary market if the reserve is to acheive the social optimum.

For a reserve to achieve the optimal allocation without a secondary market, since agents are unable to give up goods, agents' targets must be low enough to ensure that there is no state in which they receive more of the critical good than they receive in the socially optimal allocation. This means that the following must hold:

$$q_i(\vec{\theta}, \omega) \leq q_i^*(\omega) \quad \forall \omega \in \Omega$$

If we denote:

$$\begin{aligned} \bar{\theta} = \max_{\vec{\theta}} &\sum_{i \in I} \theta_i \\ \text{s.t. } & q_i(\vec{\theta}, \omega) \leq q_i^*(\omega) \\ & \forall \omega \in \Omega \quad \forall i \in I \end{aligned} \tag{3}$$

Then we have that the the target of the reserve θ^r must follow:

$$\theta^r \geq \theta^* - \bar{\theta}$$

Any such θ^r will achieve the social optimum if private incentives do not cause a distortion in the market for reserves. In this case, $\lambda(\omega)$ clears the market for reserves.

Although the reserve has the potential to achieve the social optimum, it comes at the cost of lowering each agent's target in favor of a larger reserve. In ongoing work, I consider how the reserve would not be optimal if there were transaction costs, and I consider the value of combining a reserve with other policy tools like a subsidy.

4.4.2 Optimal Subsidy

Another way to achieve the social optimum would be to subsidize (or tax) the secondary market where the tax is conditional on the state and agent. Denote the subsidy as $\kappa_i(\omega)$. The optimal subsidy is:

The total cost of this subsidy would be:

$$S = \sum_{j \in J} \int_{\Omega} q_i^{\tau *} \kappa_i(\omega) \, d\omega$$

Since such a subsidy would have to be project and state based, it would be extremely difficult to implement.

4.4.3 Smart Contract

Is there a more efficient way to achieve the social optimum? This optimal subsidy may pay a given agent in one state even though that agent may get rent in another state. Netting this benefit over states would make the total size of the intervention smaller.

To take advantage of this netting, before the state is realized, agents could agree to enter into a contract that would trade a certain amount of goods in different states. To incentivize such a contract, the policy maker could subsidize the agents to enter into the contract by paying the agents an amount equal to $V^* - \Pi$. If this incentive was allocated correctly between agents, then all agents would be indifferent to entering into such a contract.

I show that subsidizing this contract would be cheaper than the optimal subsidy. In particular:

Proposition 5. $|V^* - \Pi| \leq |S|$

Even this policy which requires a smaller total cost could still not be worth the cost to achieve the social optimum. The gains from achieving the social optimum are $V^* - V^m$ where $V^m = \sum_{i \in I} V_i(\vec{\theta}^m)$. Therefore policy intervention is only cost effective if $\Pi > V^m$.

4.4.4 Implementation Limitations

A drawback of this analysis is that to implement these policies, the social planner must know the functional forms of the firms' value functions and the firms' private incentives. Ultimately, the social planner would want to create a mechanism that is incentive compatible so that firms' accurately report their own value functions and private incentives. In ongoing work I am looking for such a mechanism.

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Appendix

A Model of Excess Capacity

A.1 Model Setup

A.2 Social Planner's Problem

The social planner can be thought of as an agent that gets value from all projects. The only subtlety is how to aggregate costs and quantities across agents.

If the social planner targets the same quantity of the critical good as all the agents combined, the social planner should receive at least the same quantity of the critical good as all the agents combined in each state. Denote θ as the social planner's target, $q(\theta, \omega)$ as the quantity of the critical good the social planner receives, and $c(\theta)$ as the cost of investing in the social planner's critical good target. Then:

$$\begin{aligned} c(\theta) = \min_{\vec{\theta}} &\sum_{i \in I} c_i(\theta_i) \\ \text{s.t. } &\sum_{i \in I} \theta_i \leq \theta \end{aligned} \tag{4}$$

and for all $\omega \in \Omega$:

$$\begin{aligned} q(\theta, \omega) = \max_{\vec{\theta}} &\sum_{i \in I} q_i(\theta_i, \omega) \\ \text{s.t. } &\sum_{i \in I} \theta_i \leq \theta \end{aligned} \tag{5}$$

The social planner does not necessarily do strictly better than the agents' aggregate costs and quantities. Now I give an example where the social planner's costs and quantities align with the agents' aggregate costs and quantities for any vector of targets, $\vec{\theta}$. Let $c_i(\theta) = c_0\theta$. Let the state space be defined by $\omega = (\omega_I, \omega_S) \in \Omega = \Omega_I \times \Omega_S$. Finally, let $q(\vec{\theta}, \omega) = \theta_i \gamma_i(\omega_I)\delta(\omega_S)$ for some $\gamma_i(\cdot)$ and $\delta(\cdot)$ where $\sum_{i \in I} \gamma_i(\omega_I) = 1$ for all ω_I, and $\int_{\Omega_I} \gamma_i(\omega_I) = 1$. The function γ provides an idiosyncratic risk and the function δ provides systemic risk, although either is sufficient. Even in this case the market solution does not align with the social optimum. In this case, there is no aggregate uncertainty. I use this structure of costs and targets in the symmetric agents example in Section 3.2.1.

These definitions of the social planner's cost functions give the full social planner's value function:

$$V = \max_{\theta} V(\theta) = \max_{\theta} \mathbb{E}[v(q(\theta))] - c(\theta) = \max_{\theta} \int_{\Omega} v(q(\theta, \omega), \omega) f(\omega)\, d\omega \; - c(\theta)$$

A.3 Comparing the Market Solution with the Social Optimum

A.4 Using Scarce Resources

To better understand an agent's decision problem, I derive an expression for agent i's shadow price given q_i units of the critical good, $\lambda_i(q_i)$. To understand the shadow price, first consider the following dual problem. Since v_{ij} is increasing, its inverse exists. Denote $q^*_{ij}(\cdot)$ as the inverse of v'_{ij}. More specifically:

$$q^*_{ij}(\lambda) = \{q : \lambda = v'_{ij}(q)\}$$

$q^*_{ij}(\lambda)$ represents the quantity of goods that the agent can allocate to project j and still have the last unit be worth at least λ. Let:

$$q^*_i(\lambda) = \sum_{j \in J_i} q^*_{ij}(\lambda)$$

Similarly $q^*_{ij}(\lambda)$, asks how many goods can an agent allocate to her projects and still get a marginal value of λ. This serves as a ranking of the marginal value of an agent's allocation decisions. The agent will follow this ranking to maximize profits. I call this ranking, the pecking order.

Agent i's dual optimization problem is:

$$\begin{aligned} &\min_{\lambda_i} \lambda_i \\ \text{s.t. } &q^*_i(\lambda_i) \leq q_i \end{aligned} \tag{6}$$

The interpretation is that the agent allocates the critical good unit-by-unit to the most valuable available project until the agent runs out of the critical good. The optimization problem asks: following this pecking order, what is the least beneficial use of the critical good an agent can make before running out of the critical good?

The dual problem provides a relationship between an agent's quantity and her marginal value. We can invert this relationship to uncover her marginal value as a function of her quantity of the critical good. Since q^*_i is strictly increasing in λ_i, then we can define:

$$\lambda_i(q_i) = \{\lambda : q_i = q^*_i(\lambda_i)\}$$

which is an agent's marginal value of the critical good if she gets q_i units of the critical good. This function comes from simply manipulating the functions $\{v_{ij}\}_{j \in J_i}$.

The same steps also yield the social planner's marginal value of the critical good given a fixed quantity of the good q. I denote the marginal value $\lambda(q)$. For the social planner, the summation is over all the value functions, so $q^*(\lambda) = \sum_{j \in J} q^*_{ij}(\lambda)$.

With this computation for each agent's marginal value of the critical good, I return to Proposition 1. From the dual problem, we can restate the proposition as follows:

Proposition A1. *The market solution is socially optimal if and only if $q^*_i(\lambda) = q_i$ for all $i \in I$.*

Proof. Follows from Lagrange multipliers. □

In this formulation, the proposition says that to achieve the social optimum, all agents must have the correct quantity to make their shadow prices align with that of the social optimum.

A.4.1 Preparing for Shortages

Behavior during shortages implies that investment capacity is too low. I prove this result here.

Proposition A2. *If $v_0'(q,\omega)$ is strictly convex (concave) in q for each $\omega \in \Omega$ then the target in the market solution is lower (higher) than the target in the social optimum.*

Proof. To prove this, I evaluate the social planner's first order conditions evaluated at the optimal target of the market solution. I show that if the marginal value a project is convex (concave), the benefit to the social planner of increasing the target falls short of (exceeds) the cost. Since the value function is increasing and concave, this means the social planner's total target should be lower (higher) than the market solution.

Agent i's first order conditions for θ_i^m is:

$$\int_\Omega \lambda_i(q_i(\vec{\theta}^{\,m},\omega),\omega)q_i'(\vec{\theta}^{\,m},\omega)f(\omega)\,d\omega \;= c_i'(\theta_i^m)$$

where $q_i'(\vec{\theta},\omega) = \frac{\partial}{\partial\theta_i}q_i(\vec{\theta},\omega)$. Under Assumption 1, this simplifies to:

$$\int_{\Omega_I}\int_{\Omega_S} \lambda_0(q_i(\vec{\theta}^{m},\omega),\omega)\gamma_i(\omega_S)\delta(\omega_I)f(\omega)\,d\omega_S\,d\omega_I \;= c_0$$

Where λ_0 is an agent's marginal value function. The expected marginal value of an extra target if the social planner total aims for the market solution's target, θ^m, follows:

$$\begin{aligned}
V'(\theta) = \int_\Omega \lambda(q(\theta^m,\omega),\omega)q'(\theta^m,\omega)f(\omega)\,d\omega \; - c'(\vec{\theta}^m) = \\
\int_{\Omega_I} \lambda(q(\theta^m,\omega),\omega)\delta(\omega_I)f(\omega)\,d\omega_I \; - c_0 = \\
\int_{\Omega_I}\int_{\Omega_S} \lambda(q(\theta^m,\omega),\omega)\sum_{i\in I}\gamma_i(\omega_S)\delta(\omega_I)f(\omega)\,d\omega_S\,d\omega_I \; - c_0 > (<) \\
\int_{\Omega_I}\int_{\Omega_S}\sum_{i\in I}\lambda_0(q_i(\vec{\theta}^m,\omega),\omega)\gamma_i(\omega_S)\delta(\omega_I)f(\omega)\,d\omega_S\,d\omega_I \; - c_0 = \\
\sum_{i\in I}\int_\Omega \lambda_i(q_i(\vec{\theta}^m,\omega),\omega)q_i'(\vec{\theta}^m,\omega)f(\omega)\,d\omega \; - c_i'(\vec{\theta}^m) = 0
\end{aligned} \tag{7}$$

Where the inequality follows from the convexity (concavity) of v_0'. It is a strict inequality because v_0' is strictly concave (convex) and because there are idiosyncratic shocks since $\gamma_i(\omega)$ is not constant. □

Note that this proof does not use any properties of the market solution except that it is not socially optimal. As such, I use this proof to consider distorted secondary markets.

Since the value function is the same across projects, the idiosyncrasy is necessary to ensure all agents are not identical. If the projects had different value functions then this idiosyncrasy would not be necessary to create a distortion to targets in the market solution; however, I rely on the symmetry of projects to pin down the direction of the distortion.

B Hoarder Example

In this section I prove the conclusions from the hoarder example.

B.1 Private Incentives in a Secondary Market

To prove that the secondary market is inefficient under the hoarders example, I show there is excess demand at the socially optimal price.

Proposition A3. $p(\omega) > \lambda(\omega)$

Proof. Since I assumed a non-degenerate problem, there must be trading on the secondary market to achieve the social optimum. As such, some agent i must be a seller of goods in the socially optimal. I consider the market price required to encourage agent i to follow the socially optimal allocation. Such a price must follow:

$$v'_{ij}(q^*_{ij}) + \phi'_{ij}(q_i^{\tau\,*}) > v'_{ij}(q^*_{ij}) = \lambda(\omega)$$

Where the strict inequality follows since the agent sells the good. As a result agent i, will sell fewer goods on the secondary market than is socially optimal. Therefore there will be excess demand at the socially optimal price so the market price will be higher than the social optimum.

□

Corollary 3. $\sum_{i\in I} |q_i^{\tau\,m}| < \sum_{i\in I} |q_i^{\tau\,*}|$

Proof. Consider a buyer in the social optimal. Since the price is higher than in the social optimum, then the buyer will buy less than in the social optimum since the buyer's value function for a given project is the same as the social planner's value function for that project.

In particular if $q_i^{\tau\,m} > 0$, then the first order conditions for agent i are:

$$v'_{ij}(q^m_{ij}) = p(\omega) > \lambda(\omega) = v'_{ij}(q^*_{ij})$$

By decreasing marginal value of a project, we get that $q^m_{ij} < q^*_{ij}$, which implies $q_i^{\tau\,m} < q_i^{\tau\,*}$. By market clearing. If this is true for all buyers then the absolute quantity traded must be smaller in the market solution than in the social optimum. □

Corollary 4. $\sum_{i\in I} \pi_i(\vec{q}^{\,m}, p, \omega) < \sum_{i\in I} v_i(\vec{q}^{\,m}, \omega) < \sum_{i\in I} v_i(\vec{q}^{\,*}, \omega)$.

Proof. The first inequality follows from the Assumptions 2 and 3. In the hoarder example, agent's profits from a project are lower than its social value. The second inequality follows from the inefficiency of the secondary market. □

B.2 Preparing for Shortages

Now in addition to the the hoarder assumption, additionally assume the agents are symmetric following the symmetric example described in Assumption 1.

Proposition A4. *In the case of hoarding agents, if the marginal value function is strictly convex (concave) in all states then the target in the market solution is lower (higher) than the target in the social optimum.*

Proof. Since, the market solution is different from the social optimum then the proof to Proposition A2 applies. □

C Policy Solutions

For the policy analysis, I assume that the social planner does not do strictly better than the agents in aggregate costs and quantities. This allows the policy maker to achieve the social optimum without combining agents. In particular this means that the equations 4 and 5 both hold no matter the vector $\vec{\theta}$.

C.1 Smart Contract

I prove the result that a smart contract will require a smaller expected cost than the optimal subsidy. Before the proposition, I define the set of states where an agent is a net seller in the social optimum. In particular define:

$$\begin{aligned} \Omega_i^+ &= \{\omega : q_i(\vec{\theta}^*, \omega) > q_i^*\} \\ \Omega_i^- &= \{\omega : q_i(\vec{\theta}^*, \omega) < q_i^*\} \end{aligned} \tag{8}$$

Where $\vec{\theta}^*$ the vector of agents' targets if goods are allocated efficiently in every state. Under the assumption that the social planner cannot do strictly better in orders and cost, then $\theta^* = \sum_{i \in I} \theta_i^*$. Therefore Ω_i^+ (Ω_i^-) is the set of states in which agent i sells (buys) the critical good.

Proposition A5. $|V^* - \Pi| \leq |S|$ *with equality if and only if* $\Omega_i^+ = \varnothing$ *or* $\Omega_i^- = \varnothing$ *for all* $i \in I$.

Proof. □