What is a monad?

A monad is a triple **(T, η, μ)** where T is an endofunctor **$T: X \rightarrow X$** and **$\eta: I \rightarrow T$** and **$\mu: T \times T \rightarrow T$** are 2 natural transformations satisfying these laws:

Identity law: $\mu(\eta(T)) = T = \mu(T(\eta))$

Associative law: $\mu(\mu(T \times T) \times T)) = \mu(T \times \mu(T \times T))$

In other words: "*a monad in X is just a monoid in the category of endofunctors of X, with product × replaced by composition of endofunctors and unit set by the identity endofunctor*"

What's the problem?

CS 252:

Advanced Programming Language Principles

Monads

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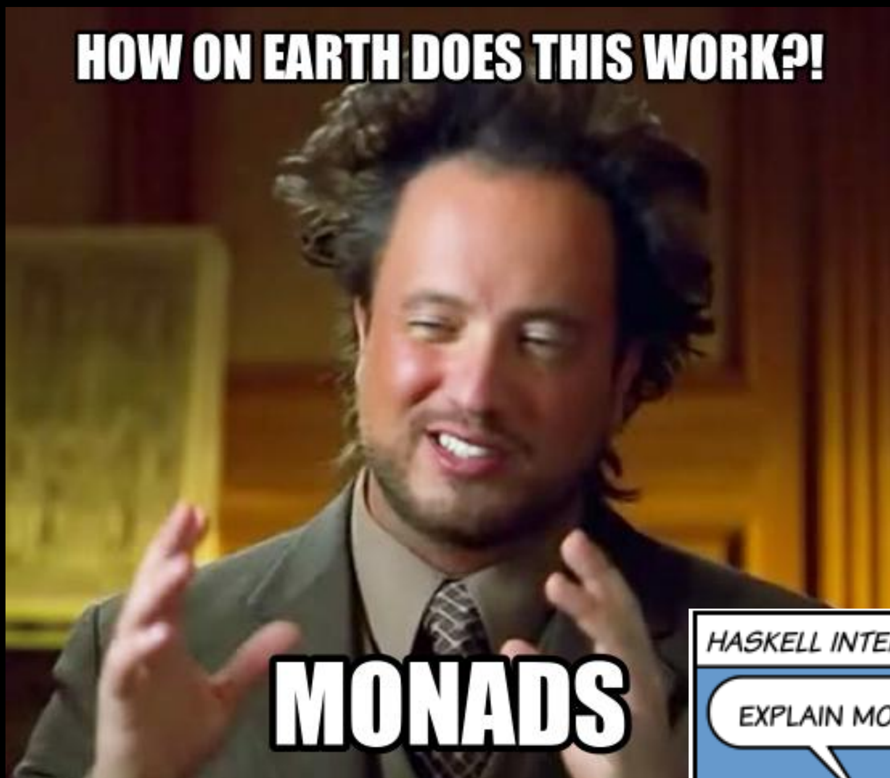
Review applicative functor lab

(in class)

Fear, uncertainty, & doubt



Review: what is a functor?

A functor is something that can be mapped over.

```
fmap :: Functor f =>
          (a -> b) -> f a -> f b
```

Problem with Functors

- This works great:
  `fmap (+1) (Just 3)`
- But this is an error:
  `fmap (+) (Just 3) (Just 4)`

```
Couldn't match expected type
`Maybe a1 -> t0'
with actual type
`Maybe (a0 -> a0)'
…
```

What is an *applicative functor*?

A functor that you can apply to other functors.

```
(<*>) ::  Applicative f =>
          f (a -> b) -> f a -> f b
```

Problem with Applicative Functors

- Now this works:

```
import Control.Applicative
fmap (+) (Just 3) <*> Just 4
```

- Which we could rewrite as:

```
Just (+3) <*> Just 4
```

- But this won't work:

```
Just (+3) <*> Just (+4) <*> Just 5
```

- `No instance for (Num (a0 -> b0)) arising from a use of `+'`

  …

Monads to the rescue

- Monads can chain through a series of functions:

```
Just 3 >>= (\x -> Just (x+4))
       >>= (\x -> Just (x+5))
```

- Or equivalently

```
return 3 >>= (\x -> return (x+4))
         >>= (\x -> return (x+5))
```

So what *is* a Monad?

The bind function

```
fmap :: Functor f =>
            (a -> b) -> f a -> f b

(<*>) :: Applicative f =>
          f (a -> b) -> f a -> f b

(>>=) :: Monad m =>
          m a -> (a -> m b) -> m b
```

called "bind"

```
> (\x -> x+1) <$> Just 1
Just 2

> Just (\x -> x+1) <*> Just 1
Just 2

> Just 1 >>= (\x -> Just $ x+1)
Just 2
```

Reimplementing >>=

`applyMaybe`

- Applies a function to a `Maybe` value
- Returns another `Maybe` value.

applyMaybe

```
applyMaybe :: Maybe a
           -> (a -> Maybe b)
           -> Maybe b
applyMaybe Nothing f = Nothing
applyMaybe (Just x) f = f x
```

```
> Just 3 `applyMaybe`
  (\x -> Just $ x * 2) `applyMaybe`
  (\x -> Just $ x - 1)
Just 5

> Just 3 `applyMaybe`
  (\_ -> Nothing) `applyMaybe`
  (\x -> Just $ x - 1)
Nothing
```

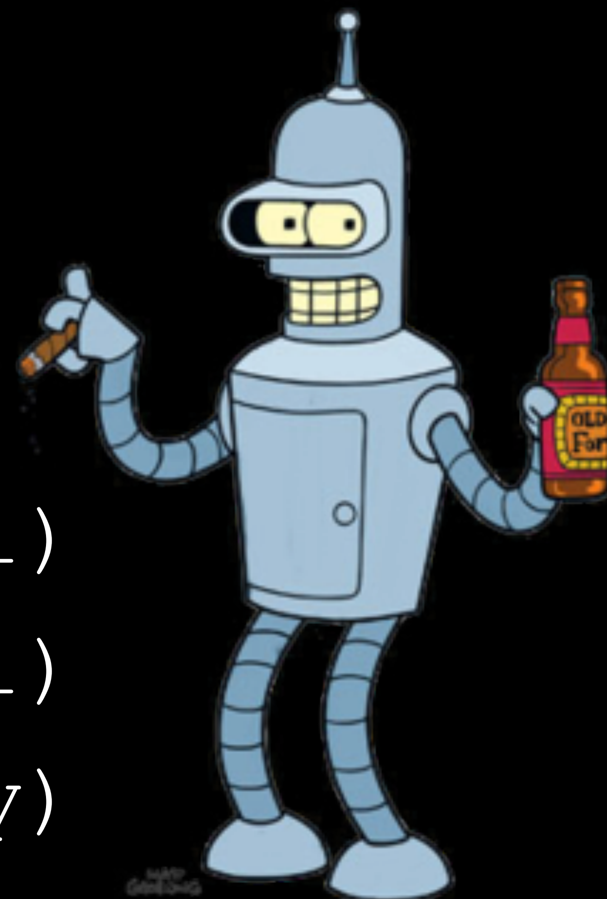
The Monad Typeclass

```
class Monad m where
    return :: a -> m a

    (>>=) :: m a -> (a -> m b) -> m b

    (>>) :: m a -> m b -> m b
    x >> y = x >>= \_ -> y

    fail :: String -> m a
    fail msg = error msg
```

Robot example (sans monads)

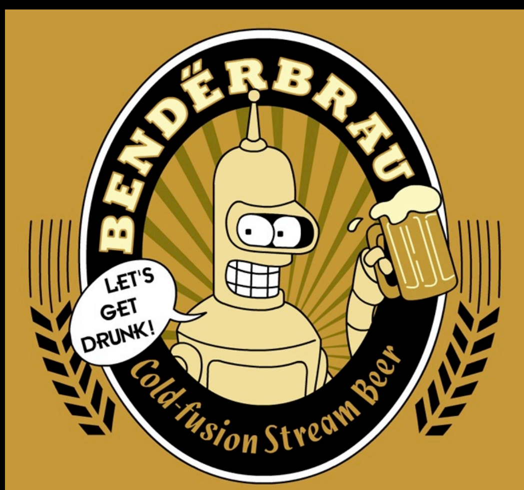
Model a robot moving on a grid:

```
type Pos = (Int, Int)


up (x, y)    = (x, y+1)
down (x, y)  = (x, y-1)
left (x, y)  = (x-1, y)
right (x, y) = (x+1, y)
```



```
x -: f = f x
start = (0,0)

> start -: up -: right
(1,1)

> start -: up -: left -: left -:
  right -: down
(-1,0)
```

Now let's modify our program to account for failure.



If Bender finds beer, he ignores all future commands.

The `Maybe` Monad

```
instance Monad Maybe where
   return x         = Just x
   Nothing >>= f = Nothing
   Just x >>= f  = f x
   fail _           = Nothing
```

Defining where Bender ignores commands

```
beerPos = Map.empty
   -: Map.insert (0,2) True
   -: Map.insert (-1,3) True
   -: Map.insert (-3,-8) True
```

```
moveTo :: Pos -> Maybe Pos
moveTo p =
  if Map.member p beerPos
    then Nothing
    else Just p

up (x, y)    = moveTo (x, y+1)
down (x, y)  = moveTo (x, y-1)
left (x, y)  = moveTo (x-1, y)
right (x, y) = moveTo (x+1, y)
```

```
> return start >>= up >>= left
    >>= left >>= right >>= down
Just (-1,0)

> return start >>= left >>= left
    >>= up >>= up >>= right >>= up
    >>= right >>= right >>= down
Nothing
```

What if we have many `Maybe` values that we need to compute?

Theirs not to reason why,
Theirs but to do and die.

--**Alfred Tennyson**

Division example, sans `do`

```
mydiv x y =
  x >>= (\numer ->
  y >>= (\denom ->
  if denom > 0 then
    Just $ numer `div` denom
    else fail "div by 0"))
```

Division example, with `do`

```
mydiv' x y = do
  numer <- x
  denom <- y
  if denom > 0 then
    Just $ numer `div` denom
    else fail "div by 0"
```

Division example, with `do` & `return`

```
mydiv' x y = do
  numer <- x
  denom <- y
  if denom > 0 then
    return $ numer `div` denom
    else fail "div by 0"
```

List Monad

```
instance Monad [] where
    return x = [x]
    xs >>= f = concat (map f xs)
    fail _ = []
```

```
listOfTuples :: [(Int,Char)]
listOfTuples = do
    n <- [1,2]
    ch <- ['a','b']
    return (n,ch)

 [(n,ch) | n <- [1,2],
           ch <- ['a','b']]
```

list comprehensions: syntactic sugar for using lists as monads.

Lab: Monads

This lab is available in Canvas.

Starter code is available on the course website.