Differential Equations

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3.1 Homogeneity

Definition 3.1 (Homogeneity of polynomials)**.** Polynomials where all terms are of the same degree are homogeneous.

Homoegeneity of functions is analogous to assigning physical dimensions (e.g. length) to all of the variables. If the function has the length dimension to the kth power, then it is homogeneous of degree k.

Example 3.2. If x, y are lengths, then the following is homogeneous of degree 3.

$$f(x, y) = 2y^3 \exp(\frac{y}{z}) - \frac{x^4}{x + 3y} \tag{1}$$

Alternate definition also suffices for generality.

Definition 3.3 (Homogeneous function)**.** $f(x, y)$ is homogeneous of degree k iff $f(\lambda x, \lambda y) = \lambda^k f(x, y)$.

Definition 3.4 (Alternate definition of homogeneity)**.** If $f(x, y)$ can be rewritten as $f(\frac{y}{x})$ or $f(\frac{x}{y})$ then it is homogeneous.

3.2 Homogeneous Differential Equations

Corollary 3.5 (Homogeneous DEs)**.** If $M(x, y)$ and $N(x, y)$ are homogeneous and of same degree, then $M(x, y)dx + N(x, y)dy = 0$ is a homogeneous DE.

Corollary 3.6 (Homogeneous DEs)**.** $M(x, y)/N(x, y)$ is homogeneous of degree 0.

Corollary 3.7 (Homogeneous DEs)**.** If $f(x, y)$ is homogeneous of degree 0 in x, y, then $f(x, y)$ is a function of y/x alone.

The ratio M/N is a function of y/x, so the above can be rewritten as

$$\frac{dy}{dx} + g(\frac{y}{x}) = 0 \tag{2}$$

$$\frac{d}{dx}(vx) + g(v) = \frac{dv}{dx} + v + g(v) = 0 \tag{3}$$

Can thus transform into SOV problem by substituting $y = vx$ or $x = vy$, where v is a function of y or x. Then, substitute back $v = \frac{y}{x}$ to obtain a general solution.