Representing Neural Networks

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Non-linear hypotheses

- Computer vision - section matrix of pixel intensities correspond to an image
  - Denote + and – for affirming if an image fits a classification
  - Would need nonlinear hypothesis
  - Feature vector x - pixel intensities in a column vector
- Example
  - Assume 50×50 pixel images - 2500 pixels
    * $n = 2500$ features
  - Quadratic features would mean ~ 3 mil features
    * number of features $= 50^2 + C(50^2, 2)$ as need all possible ways of 2 terms from features in addition to number of features present

Neural Network Model

- Neuron structure
  - Dendrite - input wires
  - Computation in nucleus
  - Axon - output wires
- Neuron model - logistic unit
  - Input wires from features x through computation to output $h_\theta(x)$
  - Sigmoid activation function $g(z) = \frac{1}{1+e^{-z}}$
  - Parameters θ are same as weights
- Layers of neural networks
  - Layer 1 of features/inputs
  - Layer 2 of bias units - is *hidden* as is not an output
  - Layer 3 is the output

$$[x_0 x_1 x_2 x_3] \rightarrow \left[a_1^{(2)} a_2^{(2)} a_3^{(2)}\right] \rightarrow h_\theta(x)$$

Node values are

$$\begin{aligned}
a_1^{(2)} &= g\left(\Theta_{10}^{(1)} x_0 + \Theta_{11}^{(1)} x_1 + \Theta_{12}^{(1)} x_2 + \Theta_{13}^{(1)} x_3\right) \\
a_2^{(2)} &= g\left(\Theta_{20}^{(1)} x_0 + \Theta_{21}^{(1)} x_1 + \Theta_{22}^{(1)} x_2 + \Theta_{23}^{(1)} x_3\right) \\
a_3^{(2)} &= g\left(\Theta_{30}^{(1)} x_0 + \Theta_{31}^{(1)} x_1 + \Theta_{32}^{(1)} x_2 + \Theta_{33}^{(1)} x_3\right) \\
h_\Theta(x) = a_1^{(3)} &= g\left(\Theta_{10}^{(2)} a_0^{(2)} + \Theta_{11}^{(2)} a_1^{(2)} + \Theta_{12}^{(2)} a_2^{(2)} + \Theta_{13}^{(2)} a_3^{(2)}\right)
\end{aligned}$$

Arguments are $z_c^{(k)}$ where c is which element of the layer and k is the layer.

- Notation

- $a_i^{(j)}$ is activation of unit i in layer j
- $\Theta^{(j)}$ is matrix of weights controlling function mapping from layer j to layer $j+1$
- If network has s_j units in layer j, s_{j+1} units in layer $j+1$, then $\Theta^{(j)}$ is of dimension $\boxed{s_{j+1} \times (s_j + 1)}$
  * x_0 and $\Theta_0^{(j)}$ bias nodes are not shown in a NN diagram

- Vectorized
  - Arguments of g, $z_c^{(k)} = \theta^{(k)} x$
  - Can let $a^{(k)} = g(z^{(k)}) = \Theta^{(k)} a^{(k)}$
  - $x = \begin{bmatrix} x_0 \\ x_1 \\ \cdots \\ x_n \end{bmatrix}$, $z^{(j)} = \begin{bmatrix} z_1^{(j)} \\ z_2^{(j)} \\ \cdots \\ z_n^{(j)} \end{bmatrix}$
  - Thus $z^{(j)} = \Theta^{(j-1)} a^{(j-1)}$

Multiclass Classification

- One-vs-all method extension
- Multiple output units for multiple classifications
- $h_\Theta(x)$ is a vector