Vel-Norm Problem Followup

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1 Simplifying expression for $w_p(\phi)$

1.1 Givens

The sine function is a phase shift of the cosine function, so

$$\sin(2k) = \sin(2\phi + \pi/2) = \cos(2\phi). \tag{1}$$

Further,

$$2\sin^2(2k) = 1 - \cos(2k). \tag{2}$$

Because $\cos(2k) = -\sin(2\phi)$ by a similar argument,

$$2\sin^2(2k) = 1 + \sin(2\phi). \tag{3}$$

1.2 Simplification

$$w_p(\phi) = \underbrace{\frac{1}{\sin(2\phi + \pi/2)}}_{\cos(2\phi)}[1 \; \underbrace{-\frac{a_T r_0^2}{\mu}}_{-\frac{4}{\beta^2(3\pi+8)}} (3\phi + 2)] + \frac{2}{\beta^2(3\pi + 8)}(2\sin^2(\phi + \pi/4) + 3) \tag{4}$$

$$= \frac{1}{\cos(2\phi)}\left[1 - \frac{4}{\beta^2(3\pi + 8)}(3\phi + 2)\right] + \frac{2}{\beta^2(3\pi + 8)}(2\sin^2(\phi + \pi/4) + 3) \tag{5}$$

2 Numerical evaluations of ϕ_m

2.1 Numerical evaluation of ϕ_m given β

2.2 Plot of ϕ_m vs. $\delta = \frac{1}{\beta^2}$

3 Numerical minimization of $w_p(\phi)$