Spacecraft Motion with Thrusting Normal to the Velocity Vector

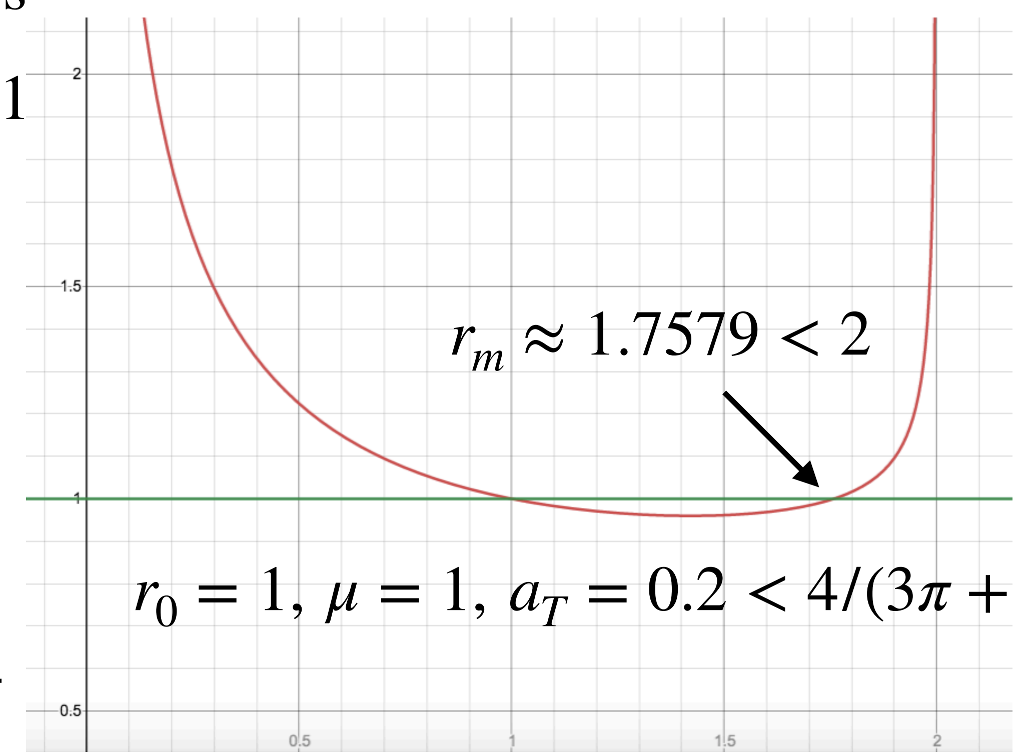
Consider $$w(r) = \frac{r_0}{\sqrt{r(2r_0 - r)}}\left[1 - \frac{a_T r_0^2}{\mu}\left(3\sin^{-1}\left(\sqrt{\frac{r}{2r_0}}\right) - \frac{3\pi}{4} + 2\right)\right] + \frac{a_T r_0}{2\mu}(r + 3r_0)$$

$w(r)$ is a function of the flight$-$direction angle,

the angle between the position and velocity vectors

$r \in [r_0, r_m]$, for some $r_m < 2r_0$ such that $w(r_m) = 1$

Sample plot of $w(r)$ v/s r



$$\mu > 0, \quad a_T \in \left[0, \frac{4\mu}{(3\pi + 8)r_0^2}\right]$$

Define $\beta = \sqrt{\dfrac{4\mu}{(3\pi + 8)a_T r_0^2}}$ such that $\beta \geq 1$

$r = 2r_0\sin^2(\phi + \pi/4)$, $\quad \phi \in [0,\phi_m]$, for some $\phi_m < \pi/4$

Your tasks:

(a) Express w in terms of ϕ and β

(b) Compute $y = \sqrt{\dfrac{\mu(2r_0 - r)}{r r_0}}\sqrt{1 - w^2}$ in terms of ϕ and β $\quad \leftrightarrow \quad$ radial velocity

(c) Find r when $w(r)$ is minimized and the minimum value of $w(r)$