The physics of light sails

PX2013 - Essay

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Abstract

Light sails are an alternative way to propel spacecraft. Their main advantage over other propulsion methods is that they do not require any fuel to operate and are therefore not limited by the same constraints as conventional propulsion systems. An overview of the base physics enabling this propulsion system is outlined. Specific focus lies on the the optical properties of such sails and how they impact the efficiency of the sail. Furthermore a specific light sail design is examined and its efficiency analyzed based on the previously outlined principles.

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1 Introduction

Conventional propulsion systems for space craft face a fundamental challange. In space there is nothing to push off to gain momentum. As such spacecraft have to carry Their own mass, that can be ejected as exhaust, which then propels the spacecraft forwards. For every kilo of fuel that is brought along, even more fuel has to be brought to propel that fuel. This results in diminishing returns and therefore poses a limit on the maximum delta-v[1] that can be delivered through such a system[1].

Light sails are one of the few propulsion methods, which don't face this problem. Light sails are essentially big mirrors that reflect light. As photons carry momentum, reflecting them leads to an opposite force applied to the spacecraft.

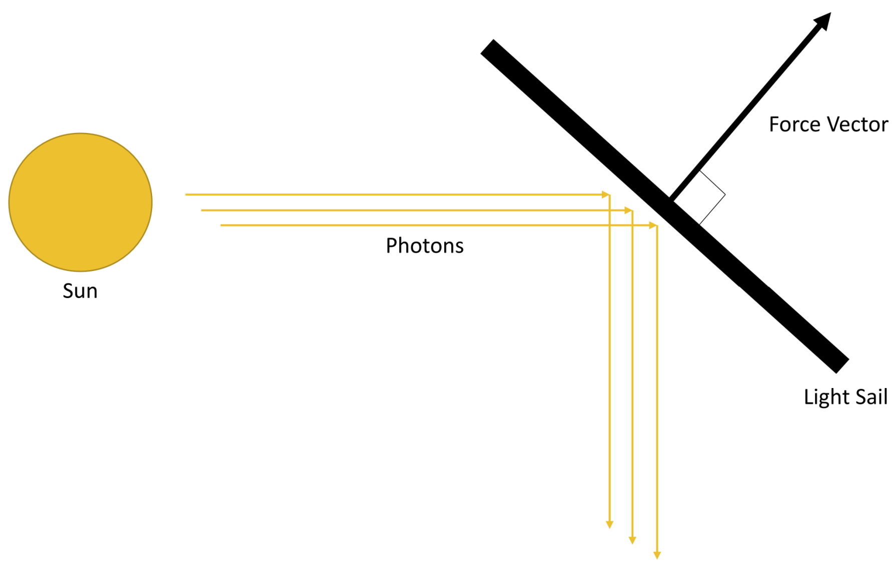

Figure 1: Basic principle behind a light sail

As this does not consume any fuel (or even any other type of power), the spacecraft can in theory gain any amount of delta-v given enough time.

2 Basic physical principles

2.1 Momentum of a photon

Even though photons don't have any mass, they have momentum. That photons have momentum is a consequence of general relativity[5], however as this essay is focused on optics, no further explanation will be given. The momentum of a single photon is given by:

$$p = \frac{h}{\lambda} \tag{1}$$

Important to the design of a light sail, is that the momentum (p) is inversely proportional to the wavelength. This means that for a light sail to be effective it has reflect a high percentage of these hight energy photons.

2.2 Momentum transfer

In order to later determine the actual efficiency of any given light sail it makes sense to look at the three base forces that can be applied by light on the sail:

[1]Delta-v denotes the change in velocity. It is the primary measure for the usefulness of any propulsion system in space. This is because to get to other places in space, a craft needs to change its orbit, which in turn means changing the velocity

1. The light is perfectly reflected off the sail's surface
2. The light is perfectly absorbed by the sail
3. The sail emmies light through blackbody radiation

Perfect reflection and perfect absorption can be modeled as completely elastic and inelastic collisions respectively[6]. In case of the photons being perfectly reflected, the inverse of a photons momentum change will be applied to the sail (perfect elastic collision).

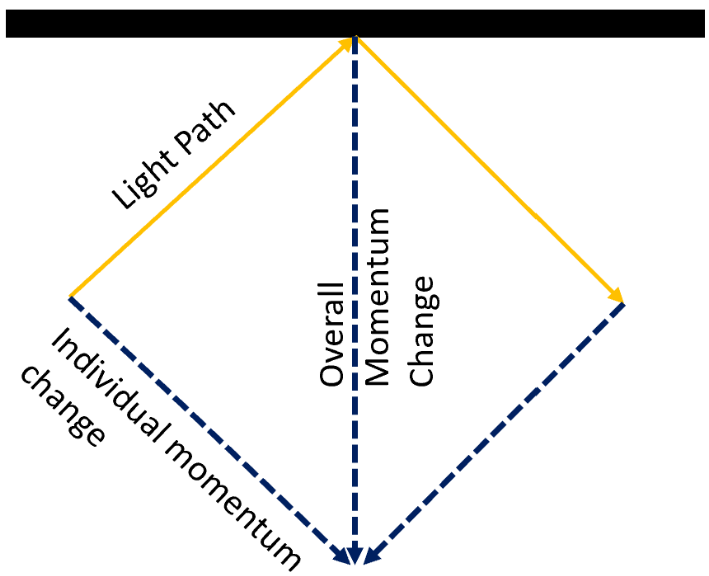

Figure 2: Momentum transfer assuming perfect reflection

The net momentum transferred will be twice the momentum normal to the sail. This is because the parallel component cancels out (figure 2)
If the photon is absorbed, all of the momentum that the photon previously carried by the photon is transferred to the sail (perfect inelastic collision). If the sail is not aligned with the light this can lead to a force component parallel to the sails surface.

The case of emission is the inverse of absorption. Instead of the sail gaining the momentum of the photon, it gains the inverse. Emission of blackbody radiation will happen in all directions. If all surfaces of the sail have the same geometry, temperature and emission characteristics, the blackbody radiation will lead to no net force.

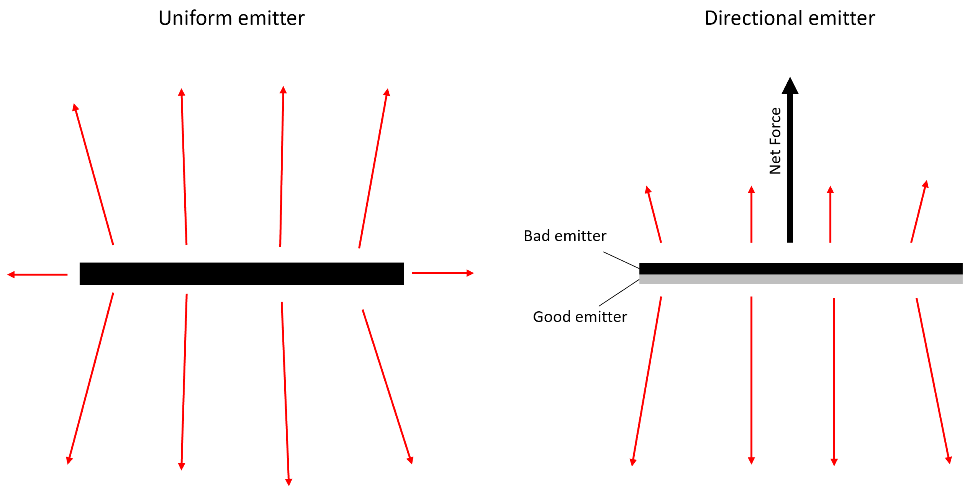

Figure 3: Uniform vs. none uniform emitter

To make a light sail even more efficient it is desireable to have more photons emitted in the direction of light source, as this would create an additional net force in the direction of the absorption and reflection forces (figure 3). To achieve this one of the surfaces needs to emit less blackbody radiation than the other. This can either be achieved by having a less emissive material on that side of the sail or by having a temperature gradient between the two sides (higher temperature leeds to more energy emitted).

Assuming perfect reflection of all light and an angle of incidence $\theta_i = 0°$ a formula for a 100% efficient sail can be derived by putting together 1 with newtons 2nd law $F = \frac{dp}{dt}$ as well as the formula for the energy of a photon $E = h\frac{\lambda}{c}$:

$$F = \frac{dE}{dt} * \frac{1}{c} = \frac{P}{c} \tag{2}$$

If the light is reflected rather than absorbed this force needs to be multiplied as detailed.

3 Complications

In a real sail, these forces combine to produce the net force of the sail. Crucial to the analysis is that all the absorbed radiation eventually gets emitted again by the sail (as soon as it reaches a stable temperature). This means that that if the sail has bad reflectivity (reflects a low percentage of light), absorption and emission characteristics become more important.

Furthermore a sail should be as light as possible to get the most acceleration out of the generated force. A design out of a thin structural base layer made out of a high strength material such as kevlar, carbon fibre or similar coated with desirable optical layers is suited best. In turn that means that the interactions of the light with the sail get quite complex.

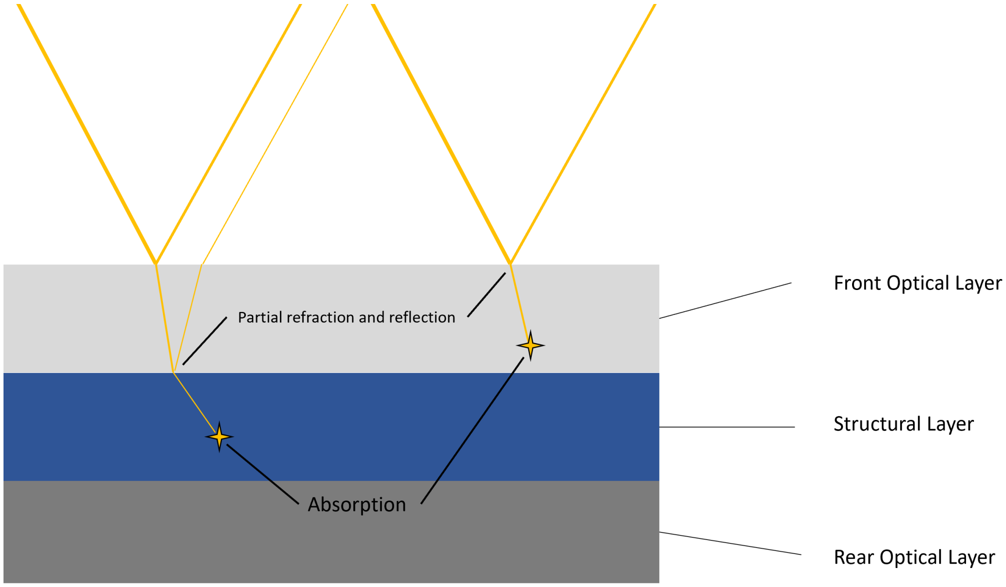

Figure 4: Example of paths the light hitting the sail could take

Figure 4 above shows two examples of how light might through the sail. The efficiency of a sail will dependent on the rate of reflection to absorption and the direction of the emitted light. Furthermore it is enough to look at only the energies of the photons, as any other effects (i.e. polarization) are not relevant to the momentum transfer. Therefore it needs to be examined how the light travels through the sail and how it is absorbed. Three optical effects are important to determine this path:

1. The refractive index at the different material interfaces, affecting the light path
2. The percentage of light reflected (reflectivity)
3. How likely it is for light to be absorbed given a distance traveled through the medium (absorption rate)

Note that scattering within the material might also contribute to how the sail behaves. For simplicity its effect is not looked at in this essay.

3.1 Refraction

To model refraction snell's law can be used with n_1 and n_2 being the refraction coefficients of the two materials and θ_i and θ_t as the angle of incidence and transmission[2]:

$$n_1 sin(\theta_i) = n_2 sin(\theta_t) \tag{3}$$

3.2 Reflectivity

Reflectivity describes how much of the light hitting a material gets reflected. This rate R of reflected light is dependent on the incident angle θ_i, the angle of transmission θ_t and the polarization of the incident light[2]. As for the material properties, the percentage is almost only dependent on the refraction coefficients n_1 and n_2 of the two materials if the light is in the optical range[3]. The equations describing this behavior are called the Fresnel equations, there is one for perpendicularly polarized light R_p and for parallel polarized light R_s (relative to the reflection surface, in this case the sail). The equations look like this:

$$R_s = \left| \frac{n_1 cos(\theta_i) - n_2 cos(\theta_t)}{n_1 cos(\theta_i) + n_2 cos(\theta_t)} \right|^2$$

$$R_p = \left| \frac{n_1 cos(\theta_t) - n_2 cos(\theta_i)}{n_1 cos(\theta_t) + n_2 cos(\theta_i)} \right|^2$$

If the light is unpolarized both cases combine to

$$R = \frac{1}{2}(R_s + R_p) \tag{4}$$

Note that on refraction and reflection the relative amount of polarization might change. This can lead to there being significantly more polarized perpendicular or parallel polarized light, which intern has further effects on how the light behaves in the material

3.3 Rate of absorption

When light travels through a material it might get absorbed. To model the light sail it is important to find out what percentage of light get's absorbed in a given distance l traveled through the material. This value can be determined experimentally and is given through a constant α which is called the absorption or extinction rate.
The coefficient can be used together with the Beer Lambert law to determine how much the intensity of light drops for a given distance l:

$$I = I_0 e^{-4\pi\alpha l/\lambda_0} \tag{5}$$

with λ_0 as the wavelength of the light to be absorbed.

3.4 Thermals

One additional challenge in a light sail are thermals. As the rate of absorption is somewhat constant and the rate of emission is dependent on the temperature the sail will reach an equilibrium temperature T. This temperature within the capabilities of the materials chosen, otherwise they will melt, deform or delaminate. At the same time it's not desireable to just increase the overall emissivity of the sail, as only light emitted in the right direction contributes.

4 Examining a sail design

4.1 External conditions

The characteristics of the light sail will be examined as if it where in earth's orbit as that is the place where any light sail might be used first. This means the light spectrum will be the sun's with diminished intensity given by the earth's distance to the sun. Furthermore it means that the light can be treated as being parallel as the distance to the light source is far enough. Finally no polarization effects need to be accounted for as the light from the sun is unpolarized.

The irradiance of the sun at the distance of earth is given by the solar constant:

$$G_{SC} = 1.362kW/m^2$$

It is roughly spread over the blackbody emission spectrum at the sun's temperature (In reality the irradiance differs slightly for specific wavelength due to absorption spectra).

Therefore plank's blackbody radiation law can be used:

$$B_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/(\lambda k_B T)} - 1}$$

with B_λ being the irradiance per square meter per wavelength per unit unit solid angle and T as the temperature of the given body.
To analyze the efficiency of the sail, it is enough to know how the irradiance distributes over the spectrum. This can be achieved by normalizing the blackbody radiation formula by the total energy emitted. The total energy emitted is calculated by integrating over the spectrum:

$$I = \int B_\lambda \, d\lambda$$

The normalized blackbody radiation formula then looks like this:

$$B_{\lambda n} = \frac{B_\lambda}{\int B_\lambda \, d\lambda}$$

4.2 The design of the sail

The design that will be examined is Aluminized Mylar. A sail of this construction is proposed in a NASA document as a suitable sail, given the suitability and availability of materials[4]. The sail proposed in the paper would consist of a structural layer made from $3\mu m$ of mylar and a couting of $50nm$ aluminum on each side. Mylar is a good choice as a structural material, as it has high tensile strength and is very light. Aluminum is one of the best options as a reflecting material, as its reflectivity is very high at almost all wavelength (further details can be found in the "Relevant Data" section below). Note that as the back of the sail is made from aluminum as well no additional thrust is to be expected from emission, unless a temperature gradient develops within the sail.

4.3 Relevant data

Besides the natural constants, the only values needed for the analysis proposed above are the refractive index n_λ and the absorption index α_λ at the relevant wavelength.

Conveniently this data is openly available online from the website refractiveindex.info[7].

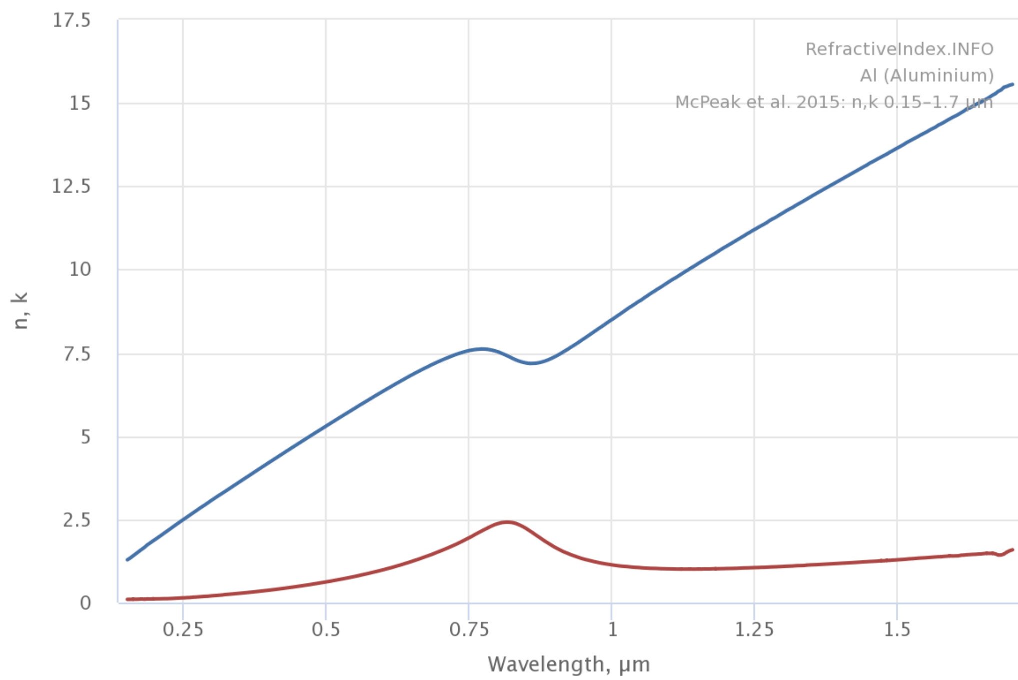

Figure 5: Refractive index (n) and absorption constant (k) of Aluminum

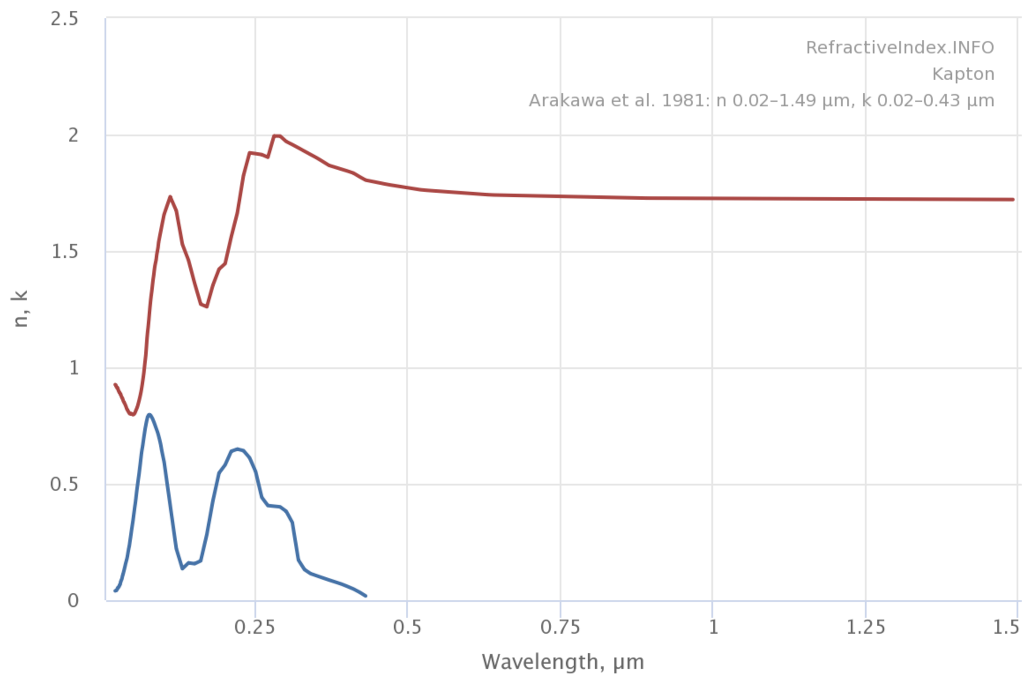

Figure 6: Refractive index (n) and absorption constant (k) of Kapton

The website provides multiple datasets on each of the elements. For the analysis a dataset roughly in the range of the sun's spectrum was chosen. Any values that where missing, where interpolated or assumed to be the same as at the closest available data point.

4.4 Modelling the sail

Analyzing what percentage of the light is reflected and absorbed by a given sail at a given angle and wavelength is not easily done analytically. Therefore a python program was created to do this. All python files can be either found online or attached to this document.

4.4.1 Calculation of the absorbed and absorbed intensities at a given wavelength

To determine how much light is reflected overall by the sail all possible paths suggested in figure 4 have to be taken into account. The program takes an intterative approach to this. The light is modeled as a uniform beam at a specific wavelength and angle. For every material the beam passes through it then gets analyzed how much of it is absorbed, how much is reflected at the next interface and how much of it gets transmitted into the next interface. The process is than repeated for the two resulting beams (reflected and transmitted beams) at the given reduced intensity. The initial intensity is normalized to one for simpler analysis later

The program first determines through trigonometry the path length taken through the material. This together with the absorption constant α_λ (interpolated from the above given dataset) can than be plugged into equation 5 to determine how much light is left after traveling through the material.

Next the percentage of reflected and absorbed light is calculated, this is done through formula 4. The relevant refractive indices n_λ are again interpolated from the data set obtained from refractiveindex.info[7].

The next two intensity reduced beams are then calculated by the same means. If there is no next material, this corresponds to the light exiting the material and it is added to the total reflected intensity.

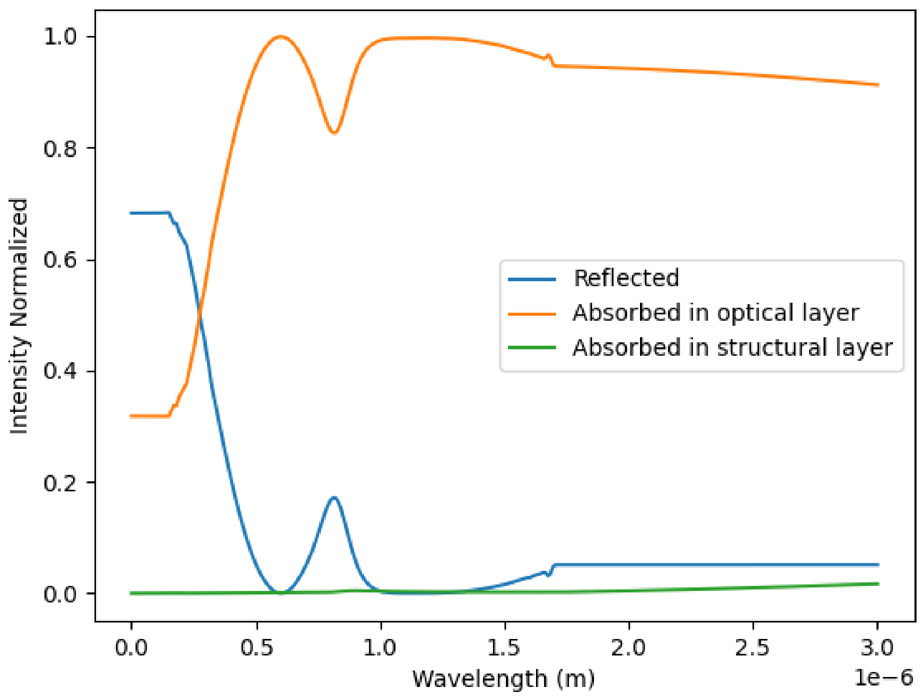

Figure 7: Relative intensity of the absorbed and reflected light hitting the proposed sail

The process can then be repeated for different wavelength which leads to the output above (figure 7)

4.4.2 Analysing the resulting efficiency

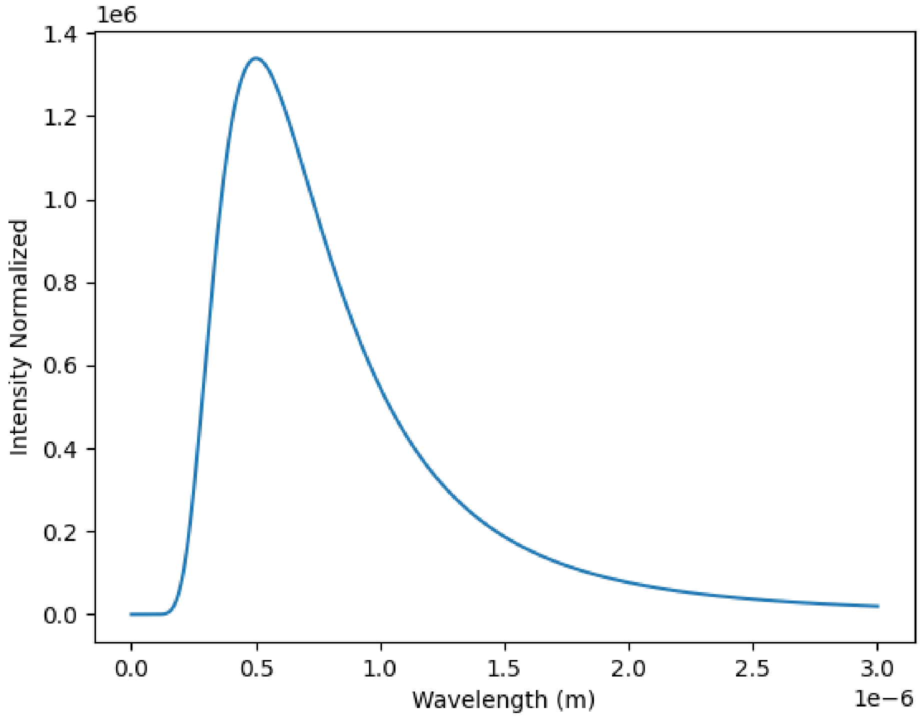

Figure 8: Normalized and idealized spectrum of the sun

These intensities now have to be related to the wavelength given of by the sun. Figure 8 above shows the idealized (no absorption and emission lines) and normalized (combined intensity, i.e. area under the graph of one). As both the absorption/reflection intensities of the sail and the sun's light spectrum are given in a normalized form they can simply be multiplied to give the weighted intensity of reflected and absorbed light energy:

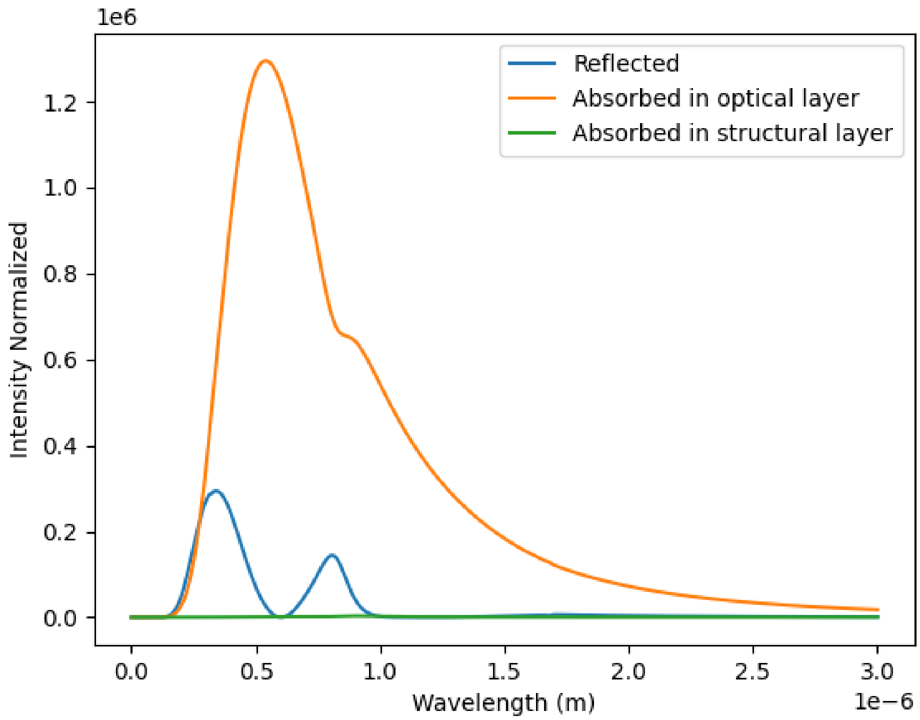

Figure 9: Relative intensity of the absorbed and reflected light weighted by the amount of light at the given wavelength

Integrating over each of the curves will result in the normalized overall energy per unit area absorbed or reflected. These two quantities will be called I_a and I_r respectively
Recalling formula 2 and the fact that the force doubles if light is reflected we can derive the following formula for the overall force per area (A) can be derived. Note that this is the force perpendicular to the sail $F_\perp$.

$$\frac{F_\perp}{A} = \frac{2cos(\theta)I_r + cos(\theta)I_a}{c} = cos(\theta)\frac{2I_r + I_a}{c}$$

Any light absorbed will also contribute to a force at a direction parallel to the sail's surface $F_\parallel$:

$$\frac{F_\parallel}{A} = \frac{sin(\theta)I_a}{c}$$

The combined force will therefore be given by

$$\frac{F}{A} = \sqrt{F_\parallel^2 + F_\perp^2}$$

From this the efficiency of the sail can be calculate. A 100% efficient sail would be one that reflects 100% of the light that hits it. Note that as we are dealing with the force per unit area it has to be considered that the area of the sail actually facing the sun reduces as the sail is rotated. This factor will not be factored into the following efficiency. This is done as the analysis is focused on how the sail behaves optically. This optic efficiency will be given as ϵ_o. The actual efficiency can be easily obtained by multiplying this efficiency with $cos(\theta)$.

$$\epsilon_o = \frac{\sqrt{(cos(\theta)\frac{2I_r+I_a}{c})^2 + (\frac{sin(\theta)I_a}{c})^2}}{\frac{2cos(\theta)I_s}{c}}$$

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