Lab Report

PX1513 - Experiment 2 - The Strength of a Magnet

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Abstract

In the following experiment, the strength of the earth's magnetic field will be determined. It will be done by measuring the torque experienced by a magnetic dipole aligning itself with the magnetic field. As the magnetic dipole moment of this magnet is unknown, it will be determined using a magnetic field of known strength.

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1 Introduction

1.1 General idea

Measuring the magnetic field strength can be done by looking at how much force a magnetic dipole experiences when placed in the filled. To calculate the magnetic dipole moment of the magnet experiencing that force is needed. To determine the dipole moment the magnet can be placed in a magnetic field of known strength. Measuring what force it experiences in that field allows the calculating the magnetic dipole moment. Even though this simple procedure is conventionally easier it presents some practical challenges:

1. It is quite difficult to isolate the experiment from the earth's magnetic field while measuring the force experienced by the generated magnetic field.
2. As the earth magnetic field is comparatively weak, it is quite difficult to measure the small amount of force that the magnet experiences precisely
3. To figure out how the force relates to the filled the angle to the magnetic field is needed

To avoid to measure the force directly, the magnet will be allowed to oscillate in the magnetic field. By measuring the frequency of that oscillation and by applying the physical laws of harmonic oscillation the required quantities can be determined.

This also gets rid of the problem of removing earth magnetic fields from the measurements if it's aligned with the generated field. This is possible as one can look at the slope of the measurements thereby making it possible to ignore the absolute values.

The strength of a magnetic field can be determined by measuring the frequency of oscillation of a magnet allowed to freely turn within it. This can be done as the period of oscillation is proportional to the field strength.

1.2 Experimental setup

The experiment consists of three major parts:

1. A magnet suspended from a string, such that it can spin freely around the vertical axis
2. A Helmholz coil to produce a variable uniform magnetic field of known strength
3. A electronic counting device, connected to a laser-activated gate counting the oscillations of the magnet

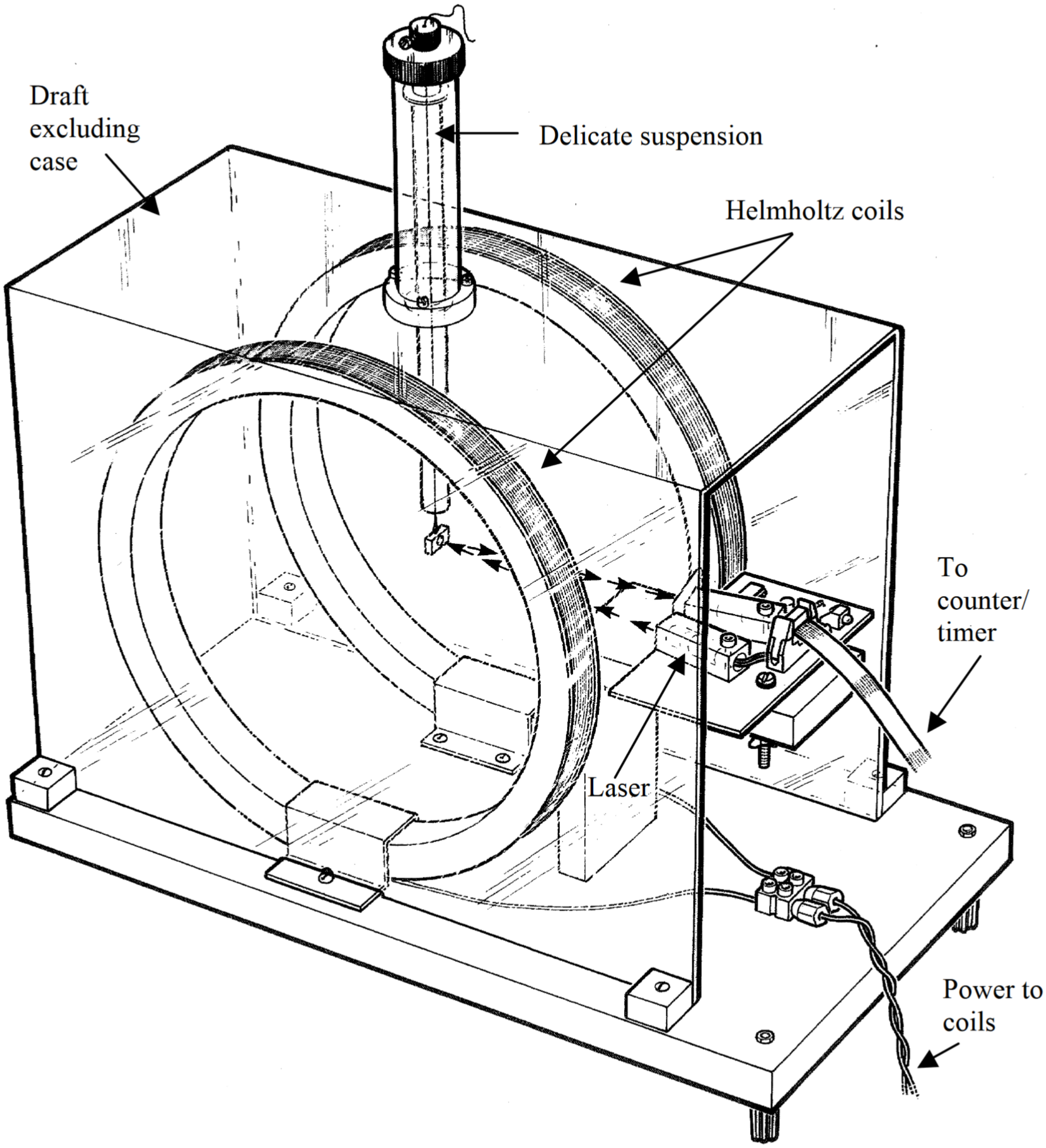


Figure 1: The experiment[1]

To allow the counting device to measure the oscillations it has a little mirror glued onto it. A laser shines onto this mirror. Depending on the angle of the magnet the laser light will be reflected at a different angle. The light activated gate is placed such that the light from the laser hits it once per oscillation.

The Helmholz coil is connected to a variable power supply. This makes it possible to adjust its field strength to a a desired known value.

2 Theory

We can start by looking at the magnetic force:

$$\tau = \mu \times B$$

Instead of using the cross product, the formula can be rearranged to be defined as the angle between the magnetic dipole moment μ and the magnetic field B:

$$\tau = -\mu B \sin(\theta)$$

The negative sign appears as it is a restoring force, such that the magnetic dipole experiences a force aligning it with the magnetic field.

This can be combined with the required torque needed to accelerate an object with inertia I:

$$\tau = I\omega$$

or

$$\tau = I\frac{d^2\theta}{dt^2}$$

thus resulting in (using the small angle approximation $\sin(\theta) = \theta$):

$$\frac{d^2\theta}{dt^2} + \frac{\mu B\theta}{I} = 0$$

Solving the differential equation results in:

$$\omega(t) = e^{\sqrt{-\frac{\mu B}{I}}t}$$

taking i outside gives us

$$\omega(t) = e^{it\sqrt{\frac{\mu B}{I}}}$$

From this we can get the period T:

$$T = 2\pi\sqrt{\frac{I}{\mu B}}$$

2.1 Moment of inertia

The moment of inertia for the magnet can be determined by using the definition for the inertia and then integrating over it:

$$I = d^2m$$

As the magnet is free to spin around the central vertical axis, we need to take the distance to the center:

$$dd = \sqrt{\frac{1}{2}dl^2 + \frac{1}{2}dw^2}$$

$$I = \int_0^l \int_0^w (\sqrt{\frac{1}{2}dl^2 + \frac{1}{2}dw^2})^2 dwdlm$$

reducing to

$$I = \frac{1}{12}(l^2 + w^2)m \tag{1}$$

2.2 Magnetic field

As the magnetic field of the earth (B_h) and Helmholz coils (B_c) line up the fields add up, thus:

$$B = B_h + B_c$$

with

$$B_c = ki \tag{2}$$

and

$$k = \frac{8}{5^{3/2}} \frac{\mu_0 n}{r} \tag{3}$$

results In

$$B = B_h + ki$$

2.3 Putting everything together

$$T^2 = 2\pi \frac{I}{\mu B_h + ki}$$

$$1/T^2 = 4\pi^2 \frac{\mu B_h + ki}{I}$$

$$1/T^2 = \frac{\mu k}{4\pi^2 I} i + \frac{\mu B_h}{4\pi^2 I} \tag{4}$$

this now being in the form $y = mx + b$ the slope (S) and the intercept (b) can be calculated to get the values for μ and B_h respectively:

$$\mu = \frac{S 4\pi^2 I}{k} \tag{5}$$

$$B_h = \frac{4\pi^2 I b}{\mu} \tag{6}$$

3 Experimental Procedure

3.1 Preperation

The first thing that needs to be done is align the Helmholtz magnetic field with the magnetic field of the earth. This can be done by utilizing a compass and aligning the Helmholtz coils in the direction it's pointing. In the case of this experiment the bar magnet already suspended in the middle of the Helmholz coil's frame can be used as the compass. One just has to wait until it stops oscillating. After that the coils can then be aligned in the direction the bar magnet is pointing.
The Helmholz coils are now aligned in such a way that they could either produce a magnetic field exactly in the direction of earth magnetic field or exactly opposite to it. To figure out which is the case the coil can be turned on a high setting (producing a magnetic field bigger than earth's). If the magnet flips violently, the magnetic fields are pointing in the opposite direction. If not the fields are aligned. Should the magnet flip around, there are three ways to solve this:

1. Flip the Helmholz coils around
2. Flip the polarity of the coils
3. Add a minus sign to the field strength in equation 4

In our case we simply flipped the polarity as this just required switching around the cables coming from the power supply.

Next the counting device needs to be prepared for measuring the frequency of the oscillation. The counting device used in the experiment has a setting allowing to measure the time until n number of events are counted. From this the frequency can be determined as such: $f = n/t$. We chose to set n to 50 to get a precise measurement.

As the last step before recording the measurements, the laser gate has to be set up to be triggered once per oscillation. To do so the power supply was set to a medium setting. The bar magnet should start oscillating as a result. If not it can be manually given a push. The laser can now be turned on. It now needs to be ensured that its beam hits the light gate for every oscillation of the magnet. There are two ways to do this:

1. Adjust the rotation of the laser, such that it hits the mirror on the magnet at a different angle
2. Adjust the feet of the frame, such that the magnet (which is suspended from a thread) is being translated relative to the laser light gate setup.

The LED on the light gate lights should now light up reliably.

3.2 Taking the measurements

With the setup done the measurements can now be taken. To do this the power supply is first set to the desired current. It is crucial to wait for the current to stabilize before proceeding. This might take a short while as adjusting current results in the magnetic field changing which in reverse again induces a current.
With a stable magnetic field, the counter can now be stared. After the counter is done counting the time should be recorded and the procedure can be repeated with a new current setting.
If the counting becomes unreliable during the experiment (which it did in our case), it might help to give the magnet a bigger push at the beginning (increasing he oscillation magnitude[1]) or by repeating the adjustment step. If this happens all measurements should get retaken to reduce the error.

After taking all the measurements the dimensions and the mass of the magnet needs to be measured and recorded. This is best done with calipers and a high-precision scale.

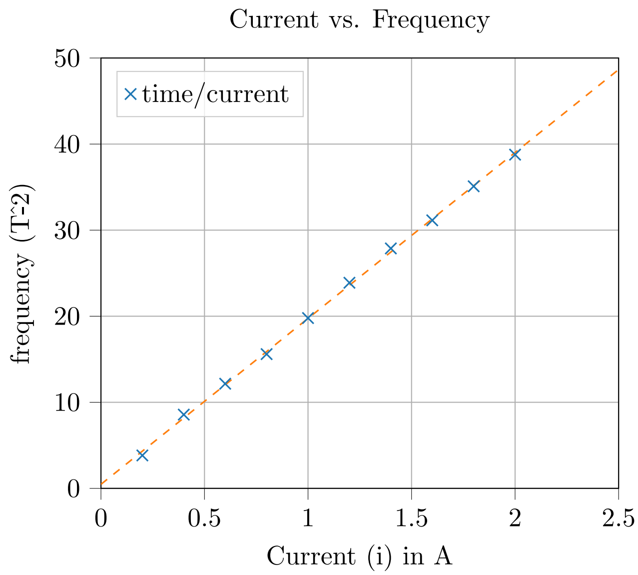
Furthermore, the constant k for the Helmholtz coil (introduced in equation3) needs to be taken or calculated from the manufacturer's datasheet.

4 Experimental Results

We decided to take 10 measurements between $2A$ and 0.2:

current	time	Frequency	Frequencyˆ2
2.0	8.03	0.1606	38.771171
1.8	8.44	0.1688	35.095797
1.6	8.96	0.1792	31.140386
1.4	9.47	0.1894	27.876616
1.2	10.23	0.2046	23.888492
1.0	11.24	0.2248	19.788250
0.8	12.66	0.2532	15.598132
0.6	14.34	0.2868	12.157428
0.4	17.08	0.3416	8.569674
0.2	25.56	0.5112	3.826646

[1] Be careful to not exceed $> 5\,\deg$ to not exceed the limit of the small-angle approximation used in the formulas



We also measured the width, length and mass of the magnet as well as recorded the k value for the Helmholtz coils used:

Helmholz coil constant	Magnet length (l)	Magnet width (w)	Magnet mass (m)
0.000745	0.019897	0.005257	0.007413

5 Analysis

All analysis was done in python. If you need the source code please have a look at the github repository.

Analyzing the data is now fairly straightforward. The slope and intercept can be either read of the graph or as done here be calculated with a regression algorithm:

Slope	Intercept	Standard deviation	Standard deviation(%)
19.27376	0.470123	0.293034	1.520378

To now calculate the magnetic dipole moment μ and the strength of earth's magnetic field B_h we now need the values using formulas 5 and 6 respectively, we need the inertia of the magnet I and the constant k relating the Helmholtz coil's field strength B_c to current.
k can be found in table4. The inertia was calculated from the values in the same table using formula1. Plugging everything into formulas 5 and 6 gives the following final results:

Intertia (I)	Dipole Moment	Earth magnetic field (expected)	Earth magnetic field (measured)	Error
2.616384e-07	0.33928	0.000017	0.000018	1.101331

6 Discussion

The final calculated result is just 10% off from the expected value. While the result agrees with the literature value it is not really precise. From the linear regression 5 we can see that the standard division of the measurements is only around 1.5% which does not explain the final error of ¿10%. This means the error either comes from one of the other measurements taken (Magnetic field strength or dimensions/weight of the magnet) or it's a systematic error.

While the error could very well come from the other measurements, it's more likely that it is a systematic error.

There are four possible identifiable causes for such a systematic error:

1. Small-angle approximation: the small-angle approximation is just that: an approximation so this might contribute to the error
2. The two magnetic fields might not be aligned perfectly, therefore their forces would not exactly add up
3. The thread on which the magnet is suspended could introduce a torque, thus, the magnetic torque would not be the only relevant force
4. Foreign objects could interfere with the magnetic field around the experiment. In the setting of the experiment this is quite likely as there were multiple experiments utilizing magnetic fields running in the same lab at the same time

References

[1] *PX1513 - THE PHYSICAL UNIVERSE B - LABORATORY MANUAL*. 2018 - 2019.