Laboratory: Circuit Analysis

Objectives

- ✓ Investigate what type of circuit to which Kirchhoff's rules must be applied.
- ✓ Quantify the effect of multiple resistors in series and in parallel.
- ✓ Explore the relationships between the flow of electrons, type of devices and type of circuit.
- ✓ Apply Kirchhoff's rules to several circuits, solve for the currents in the circuit, and compare the theoretical values predicted by Kirchhoff's rules to measured values.

Equipment List

- (3) Lightbulbs of various wattage
- (1) Multi-Lightbulb strip
- (2) Multimeters (or a voltmeter and ammeter)
- (1) AC Power Supply
- Electrical leads with banana jack plugs.

Background

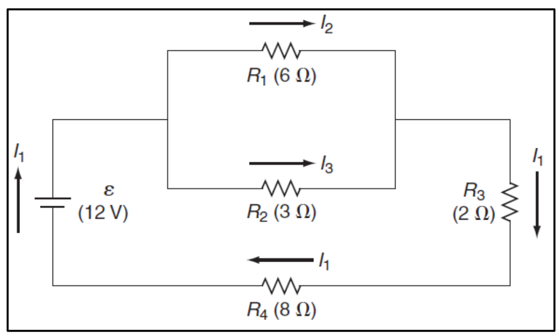

Figure 1 Single-loop circuit

Consider the circuit in Figure 1. The circuit is labeled with all of the currents. The 2 Ω resistor, 8 Ω resistor, and 12 V power supply have current I_1, the 6 Ω resistor has current I_2, and the 3 Ω resistor has current I_3. This circuit is called a single-loop circuit because it can be reduced to a single resistor in series with the power supply. The 6 Ω resistor and the 3 Ω resistor are in parallel with an equivalent resistance of 2 Ω. That equivalent 2 Ω resistance is in series with the 12 V power supply and the other two resistors, reducing the circuit to a single-loop circuit. The total resistance across the 12 V power supply is 12 Ω and its current is therefore $I_1 = 1$ A. Applying Ohm's law to the remaining part of the circuit gives $I_2 = 1/3$ A and $I_3 = 2/3$ A.

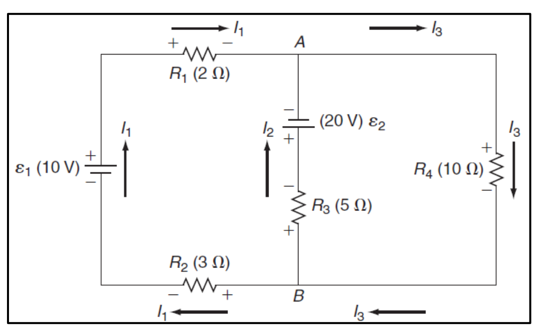

Figure 2 Multi-loop circuit

Consider now the circuit of Figure 2. This laboratory is concerned with the fundamental difference between circuits of the type depicted in Figure 1 and circuits of the type depicted in Figure 2. The circuit in Figure 2 cannot be reduced to a single-loop circuit, but instead is called a multi-loop circuit. Before analyzing this circuit, first we will define some terms. A point at which at least three possible current paths intersect is defined as a **junction**. For example, points A and B in Figure 2 are junctions. A closed loop is any path that starts at some point in a circuit and passes through elements of the circuit (in this case resistors and power supplies), and then arrives back at the same point without passing through any circuit element more than once. By this definition, there are three loops in the circuit of Figure 2: (1) starting at B, going through the 10-V power supply to A, and then down through the 20-V power supply back to B, (2) starting at B, up through the 20-V power supply, and then around the outside through the 10-Ω resistor and back to B, (3) completely around the outside part of the circuit. One can traverse a loop in either of two directions, but regardless of the direction, the resulting equations are equivalent.

The solution for the currents in a multi-loop circuit uses two rules developed by Gustav Robert Kirchhoff. The first of these rules is called Kirchhoff's current rule (KCR). It can be stated in the following way:

KCR—The sum of currents into a junction equals the sum of currents out of the junction.

This rule actually amounts to a statement of conservation of charge. In effect, it states that charge does not accumulate at any point in the circuit. The second rule is called Kirchhoff's voltage rule (KVR). It can be stated as:

KVR—The algebraic sum of the voltage changes around any closed loop is zero.

This rule is essentially a statement of the conservation of energy, which recognizes that the energy provided by the power supplies is absorbed by the resistors.

In a multi-loop circuit, the values of the resistors and the power supplies are known. It is necessary to determine how many independent currents are in the circuit, to label them, and then to assign a direction to each current. Application of Kirchhoff's rules to the circuits, treating the assigned currents as unknowns, will produce as many independent equations as there are unknown currents. Solving those equations will determine the values of the currents.

In the application of KVR to a circuit, take care to assign the proper sign to a voltage change across a particular element. The value of the voltage change across an emf $\mathcal{E}$ can be either $+\mathcal{E}$ or $-\mathcal{E}$ depending upon which direction it is traversed in the loop. If the emf is traversed from the (−) terminal to the (+) terminal, the change in voltage is $+\mathcal{E}$. However, when going from the (+) terminal to the (−) terminal, the change in voltage is $-\mathcal{E}$. In the laboratory, we will measure the terminal voltage of the sources of emf. We will assume those values approximate the emf.

When a resistor R with an assumed current I is traversed in the loop in the same direction as the current, the voltage change is $-IR$. If the resistor is traversed in the direction opposite that of the current, the voltage change is $+IR$. The sign of the voltage change across an emf is not affected by the direction of the current in the emf. The sign of the voltage change across a resistor is determined by the current direction.

Consider the application of Kirchhoff's rules to the multi-loop circuit of Figure 2. At the junction *A* currents I_1 and I_2 go into the junction, current I_3 goes out of the junction, and KCR states

$$I_1 + I_2 = I_3 \quad \text{(Eq. 1)}$$

It might appear that applying KCR to the junction *B* would produce an additional useful equation, but in fact it would result in an equation that is identical to (Eq. 1).

Applying KVR to the loop that starts at B, goes through the 10-V power supply to A, and then down through the 20-V power supply back to B, gives the following equation with values of the resistances included.

$$-R_2I_1 + \mathcal{E}_1 - R_1I_1 + \mathcal{E}_2 + R_3I_2 = 0 \quad \text{or} \quad -3I_1 + 10 - 2I_1 + 20 + 5I_2 = 0 \qquad \text{(Eq. 2)}$$

The signs used in (Eq. 2) and the circuit diagrams are consistent with the description given above for determining the signs of voltage changes. Applying KVR to the loop that starts at B and goes clockwise around the right side of the circuit gives

$$-R_3I_2 - \mathcal{E}_2 - R_4I_3 = 0 \quad \text{or} \quad -5I_2 - 20 - 10I_3 = 0 \qquad \text{(Eq. 3)}$$

All three equations are the three needed equations for the three unknowns I_1, I_2, and I_3. The solution of these equations gives values for the currents of $I_1 = 2.800$ A, $I_2 = -3.200$ A, and $I_3 = -0.400$ A. The currents I_2 and I_3 are negative. This indicates that the original assumption of direction for these two currents was incorrect. The interpretation of the solution is that there is a current of 2.800 A in the direction indicated in the figure for I_1, a current of 3.200 A in a direction opposite to that indicated in Figure 2 for I_2, and a current of 0.400 A in a direction opposite to that indicated for I_3. This is a general feature of solutions using Kirchhoff's rules. Even if the original assumption of the direction of a current is wrong, the solution of the equations leads to the correct understanding of the proper direction by virtue of the sign of the current.

Procedure (50%, Group Exercise in Class)

Work in groups of three or four people. Proper units must be on ALL quantities for credit!

Part I - Series Circuits

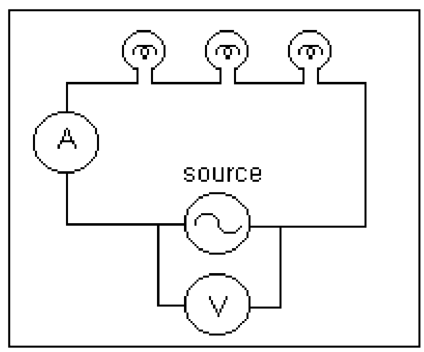

Figure 3 Setup of series circuit

1. Connect the circuit shown in Figure 3. Make sure that the lightbulbs do not have the same power rating. <u>Have your instructor check the circuit before you apply any power to the circuit.</u>
2. Slowly increase the voltage of the variable source power supply until you are at full scale, 120 volts. Measure the current that is flowing through the ammeter I_{bat} and the voltage ΔV_{bat} across the power supply. Record both the readings.
3. Calculate the total resistance $(R_{\text{eq}})_{\text{act}}$ of the circuit using Ohm's Law $\Delta V = IR$.
4. Calculate the total power supplied by the power source P_{bat} using $P = I\Delta V$.
5. Using the voltmeter, measure the potential difference ΔV_i across each lightbulb.
6. Add the voltages ΔV_1, ΔV_2, & ΔV_3. Calculate the %-difference between the voltage across the power supply and the sum of the voltages across the lightbulbs.
7. Turn off the power supply, move the ammeter between the first and second lightbulb in the circuit (wire from lightbulb #1 into ammeter and wire from ammeter to lightbulb #2). Turn the power supply back on; do not adjust the power supply's output. Measure the current I_i at this

point for lightbulb #1. Repeat the process for the other two lightbulbs, moving the ammeter in a similar manner.

8. Compare the reading of the currents. Should the readings all be approximately equal? Explain!

9. WARNING! Hot Lightbulbs! With the power supply on, unscrew one of the lightbulbs. What happened when you unscrewed the lightbulb? Explain!

10. Calculate the resistance R_i in each lightbulb using Ohm's Law $\Delta V = IR$. Calculate the equivalent resistance $(R_{\text{eq}})_{\text{theo}}$ for the entire circuit, using the resistances R_i calculated for each bulb.
11. Calculate the %-error between the total resistance $(R_{\text{eq}})_{\text{act}}$ and the theoretical equivalent resistance $(R_{\text{eq}})_{\text{theo}}$.
12. Calculate the power P_i dissipated in each lightbulb using $P = I\Delta V$.
13. Add the powers P_1, P_2, & P_3. Calculate the %-difference between the power from the power supply P_{bat} and the total power dissipated by the lightbulbs ΣP_i.

Part II - Parallel Circuits

14. Connect the circuit shown in Figure 4. Make sure that the lightbulbs do not have the same power rating. <u>Have your instructor check the circuit before you apply any power to the circuit.</u>
15. Slowly increase the voltage of the variable source power supply until you are at full scale, 120 volts. Measure the current that is flowing through the ammeter I_{bat} and the voltage ΔV_{bat} across the power supply. Record both the readings.
16. Calculate the total resistance $(R_{\text{eq}})_{\text{act}}$ of the circuit using Ohm's Law $\Delta V = IR$.
17. Calculate the total power supplied by the power source P_{bat} using $P = I\Delta V$.

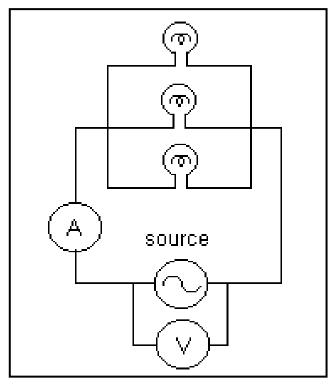

Figure 4 Setup of parallel circuit

18. Using the voltmeter, measure the potential difference ΔV_i across each lightbulb.
19. Turn off the power supply, move the ammeter between the first junction point and first lightbulb in the circuit. Turn the power supply on; do not adjust the power supply's output. Measure the current I_i at this point for lightbulb #1. Repeat the process for the other two lightbulbs, moving only the output lead from the ammeter in a similar manner.

20. Compare the voltage readings. Should the readings all be equal? Explain!

21. Add the current measurements I_1, I_2, & I_3. Calculate the %-difference between the current delivered from the power supply and the sum of the currents through each of the lightbulbs.
22. WARNING! Hot Lightbulbs! With the power supply on, unscrew one of the lightbulbs. What happened when you unscrewed the lightbulb? Explain!

23. Calculate the resistance R_i in each lightbulb using Ohm's Law $\Delta V = IR$. Calculate the equivalent resistance $(R_{\text{eq}})_{\text{theo}}$ for the entire circuit, using the resistances R_i calculated for each bulb.
24. Calculate the %-error between the total resistance $(R_{\text{eq}})_{\text{act}}$ and the theoretical equivalent resistance $(R_{\text{eq}})_{\text{theo}}$.
25. Calculate the power P_i dissipated in each lightbulb using $P = I\Delta V$.
26. Add the powers P_1, P_2, & P_3. Calculate the %-difference between the power from the power supply P_{bat} and the total power dissipated by the lightbulbs ΣP_i.

Part III - Parallel - Series Circuits

27. Connect the circuit shown in Figure 5. Make sure that the lightbulbs do not have the same power rating. <u>Have your instructor check the circuit before you apply any power to the circuit</u>.
28. Slowly increase the voltage of the variable source power supply until you are at full scale, 120 volts. Measure the current that is flowing through the ammeter I_{bat} and the voltage ΔV_{bat} across the power supply. Record both the readings.
29. Calculate the total resistance $(R_{\text{eq}})_{\text{act}}$ of the circuit using Ohm's Law $\Delta V = IR$
30. Turn off the power supply, and move the ammeter to a position that will read the current through each of the three lightbulbs.

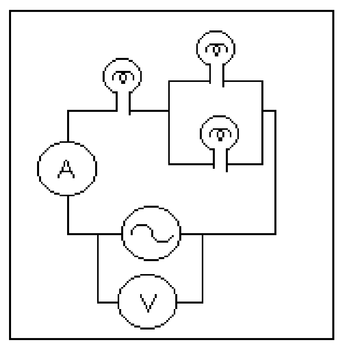

Figure 5 Setup of parallel-series circuit

31. Calculate the resistance R_i in each lightbulb using Ohm's Law $\Delta V = IR$. Calculate the equivalent resistance $(R_{\text{eq}})_{\text{theo}}$ for the entire circuit, using the resistances R_i calculated for each bulb.
32. Calculate the %-error between the total resistance $(R_{\text{eq}})_{\text{act}}$ and the theoretical equivalent resistance $(R_{\text{eq}})_{\text{theo}}$.
33. Calculate the power P_i dissipated in each lightbulb using $P = I\Delta V$.
    Add the powers P_1, P_2, & P_3. Calculate the %-difference between the power from the power supply P_{bat} and the total power dissipated by the lightbulbs ΣP_i.