

# Introduction to **Information Retrieval**

CS276: Information Retrieval and Web Search

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Wildcard queries and Spelling Correction

# WILD-CARD QUERIES

# Wild-card queries: \*

- ***mon\****: find all docs containing any word beginning with “mon”.
- Easy with binary tree (or B-tree) dictionary: retrieve all words in range: ***mon ≤ w < moo***
- ***\*mon***: find words ending in “mon”: harder
  - Maintain an additional B-tree for terms *backwards*.  
Can retrieve all words in range: ***nom ≤ w < non***.

From this, how can we enumerate all terms meeting the wild-card query ***pro\*cent*** ?

# Query processing

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- At this point, we have an enumeration of all terms in the dictionary that match the wild-card query.
- We still have to look up the postings for each enumerated term.
- E.g., consider the query:

***se\*ate AND fil\*er***

This may result in the execution of many Boolean *AND* queries.

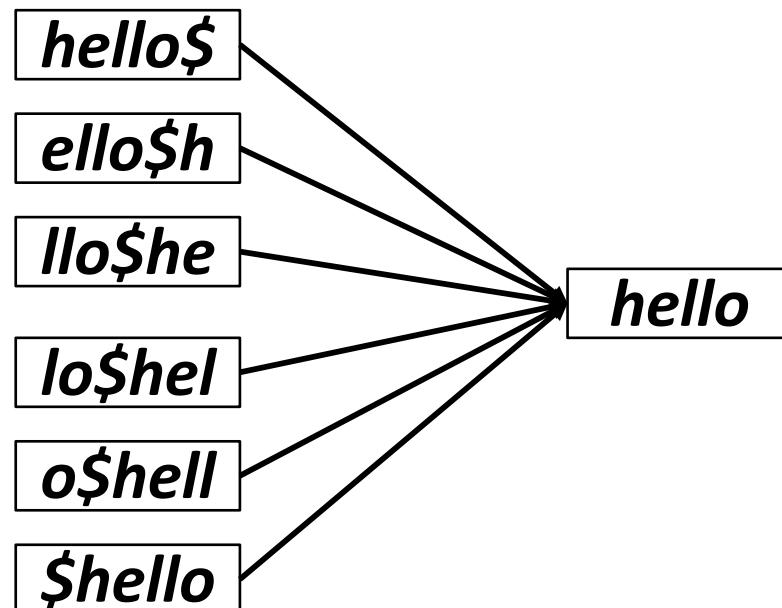
# B-trees handle \*'s at the end of a query term

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- How can we handle \*'s in the middle of query term?
  - *co\*tion*
- We could look up **co\*** AND **\*tion** in a B-tree and intersect the two term sets
  - Expensive
- The solution: transform wild-card queries so that the \*'s occur at the end
- This gives rise to the **Permuterm** Index.

# Permuterm index

- Add a \$ to the end of each term
- Rotate the resulting term and index them in a B-tree
- For term *hello*, index under:
  - *hello\$*, *ello\$h*, *llo\$he*, *lo\$hel*, *o\$hell*, *\$hello*where \$ is a special symbol.



Empirically, dictionary quadruples in size

# Permuterm query processing

- (Add \$), rotate \* to end, lookup in permuterm index
- Queries:
  - **X**      lookup on **X\$**      **hello\$** for **hello**
  - **X\***      lookup on **\$X\***      **\$hel\*** for **hel\***
  - **\*X**      lookup on **X\$\***      **llo\$\*** for **\*llo**
  - **\*X\***      lookup on **X\***      **ell\*** for **\*ell\***
  - **X\*Y**      lookup on **Y\$X\***      **lo\$h** for **h\*lo**
  - **X\*Y\*Z**      treat as a search for **X\*Z** and post-filter  
For **h\*a\*o**, search for **h\*o** by looking up **o\$h\***  
and post-filter **hello** and retain **halo**

# Bigram ( $k$ -gram) indexes

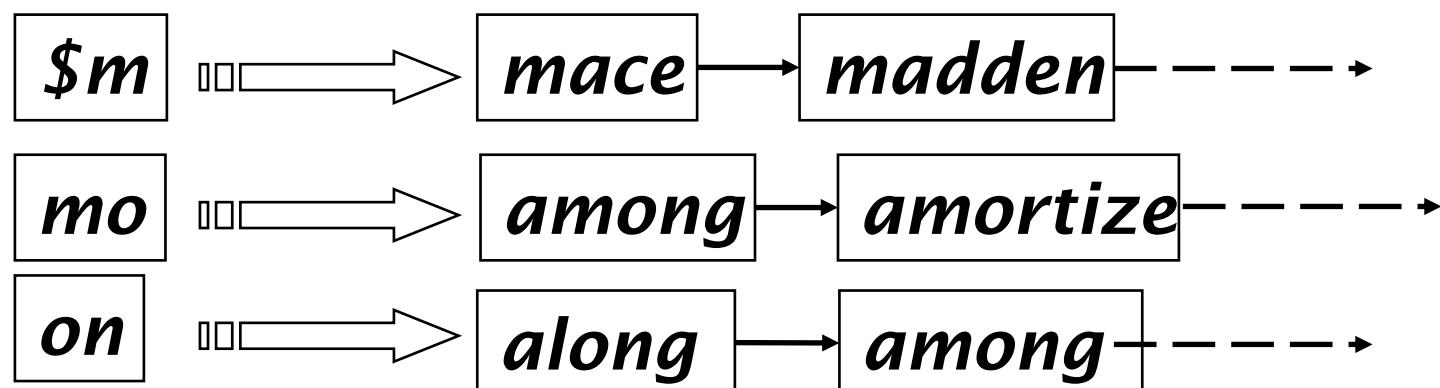
- Enumerate all  $k$ -grams (sequence of  $k$  chars) occurring in any term
- e.g., from text “***April is the cruelest month***” we get the 2-grams (*bigrams*)

```
$a,ap,pr,ri,il,I$, $i,is,s$, $t,th,he,e$, $c,cr,ru,  
ue,el,le,es,st,t$, $m,mo,on,nt,h$
```

- \$ is a special word boundary symbol
- Maintain a second inverted index from bigrams to dictionary terms that match each bigram.

# Bigram index example

- The  $k$ -gram index finds *terms* based on a query consisting of  $k$ -grams (here  $k=2$ ).



# Processing wild-cards

- Query ***mon\**** can now be run as
  - **\$m AND mo AND on**
- Gets terms that match AND version of our wildcard query.
- But we'd enumerate ***moon***.
- Must post-filter these terms against query.
- Surviving enumerated terms are then looked up in the term-document inverted index.
- Fast, space efficient (compared to permuterm).

# Processing wild-card queries

- As before, we must execute a Boolean query for each enumerated, filtered term.
- Wild-cards can result in expensive query execution (very large disjunctions...)
  - pyth\* AND prog\*
- If you encourage “laziness” people will respond!

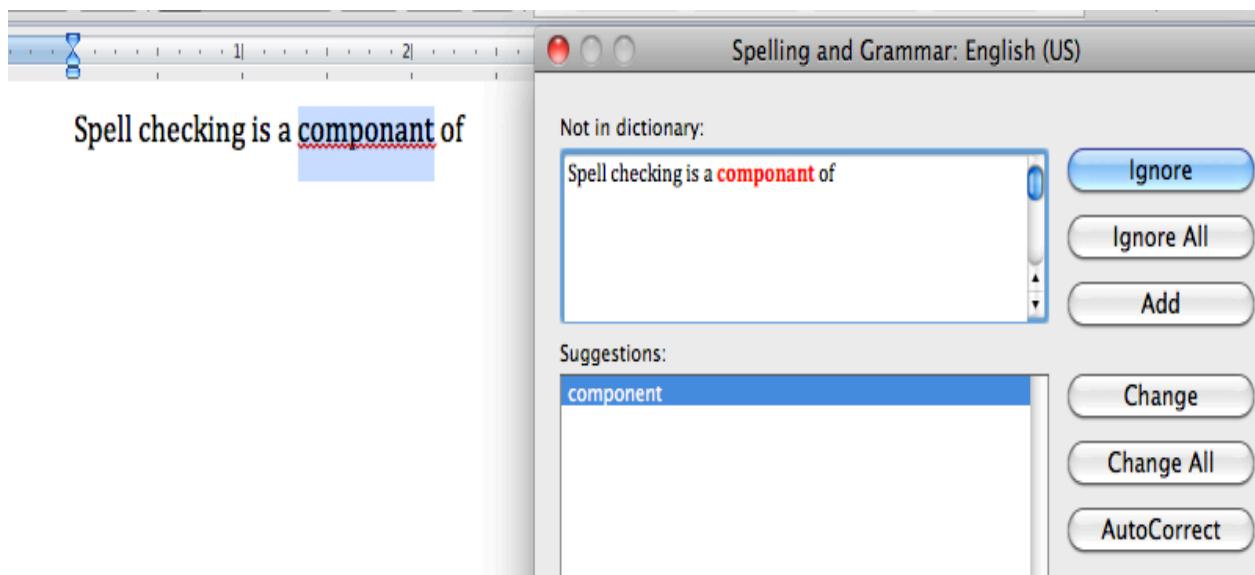
Search

Type your search terms, use '\*' if you need to.  
E.g., Alex\* will match Alexander.

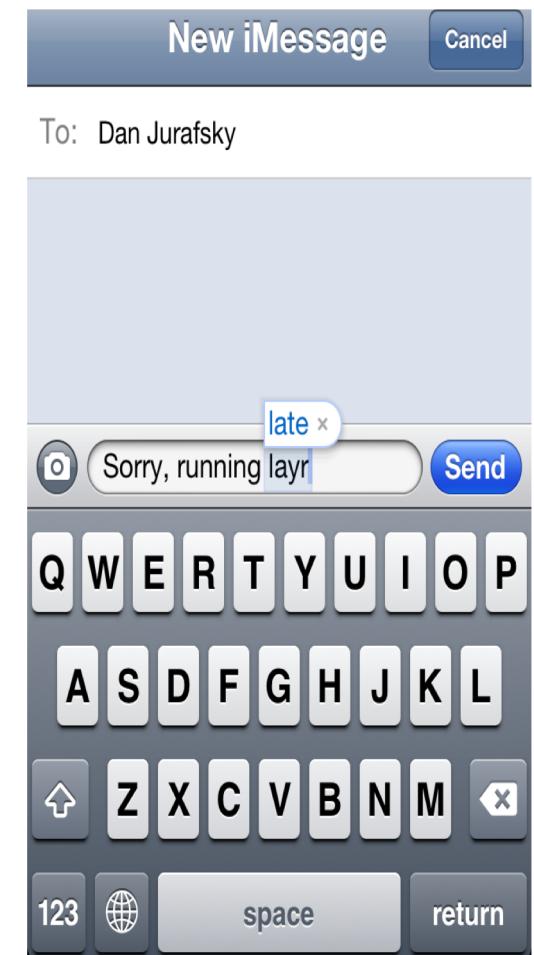
# SPELLING CORRECTION

# Applications for spelling correction

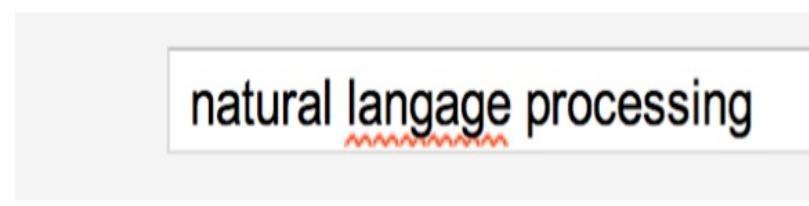
## Word processing



## Phones



## Web search



Showing results for natural *language* processing  
Search instead for natural langage processing

# Rates of spelling errors

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Depending on the application, ~1–20% error rates

**26%**: Web queries [Wang et al. 2003](#)

**13%**: Retyping, no backspace: [Whitelaw et al. English&German](#)

**7%**: Words corrected retyping on phone-sized organizer

**2%**: Words uncorrected on organizer [Soukoreff &MacKenzie 2003](#)

**1-2%**: Retyping: [Kane and Wobbrock 2007, Gruden et al. 1983](#)

# Spelling Tasks

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- Spelling Error Detection
- Spelling Error Correction:
  - Autocorrect
    - hte → the
  - Suggest a correction
  - Suggestion lists

# Types of spelling errors

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- Non-word Errors
  - *graffe* → *giraffe*
- Real-word Errors
  - Typographical errors
    - *three* → *there*
  - Cognitive Errors (homophones)
    - *piece* → *peace*,
    - *too* → *two*
    - *your* → *you're*
- Non-word correction was historically mainly context insensitive
- Real-word correction almost needs to be context sensitive

# Non-word spelling errors

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- Non-word spelling error detection:
  - Any word not in a ***dictionary*** is an error
  - The larger the dictionary the better ... up to a point
  - (The Web is full of mis-spellings, so the Web isn't necessarily a great dictionary ...)
- Non-word spelling error correction:
  - Generate ***candidates***: real words that are similar to error
  - Choose the one which is best:
    - Shortest weighted edit distance
    - Highest noisy channel probability

# Real word & non-word spelling errors

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- For each word  $w$ , generate candidate set:
  - Find candidate words with similar *pronunciations*
  - Find candidate words with similar *spellings*
  - Include  $w$  in candidate set
- Choose best candidate
  - Noisy Channel view of spell errors
  - Context-sensitive – so have to consider whether the surrounding words “make sense”
  - *Flying form Heathrow to LAX → Flying from Heathrow to LAX*

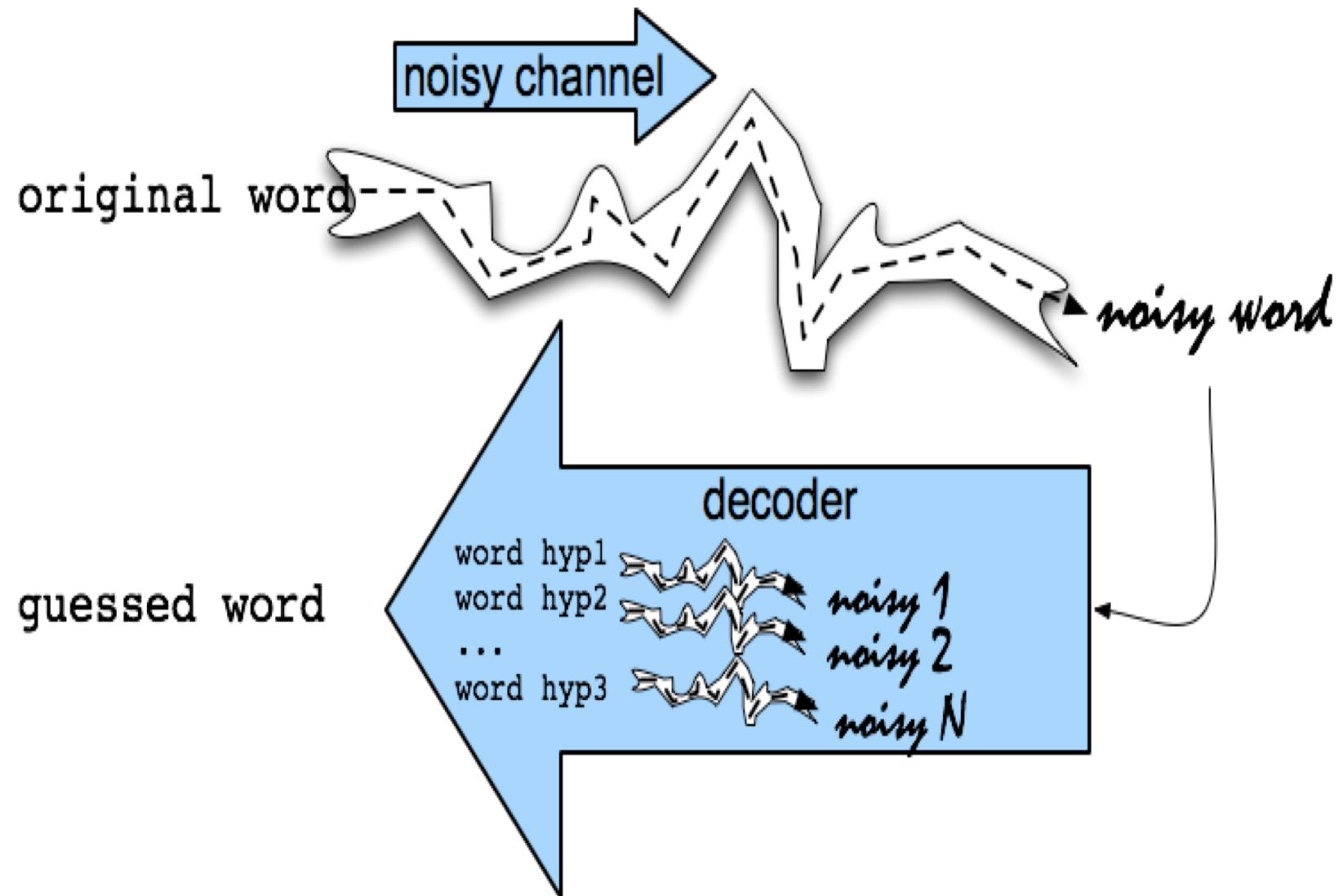
# Terminology

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- We just discussed character bigrams and k-grams:
  - *st, pr, an ...*
- We can also have word bigrams and n-grams:
  - *palo alto, flying from, road repairs*

# The Noisy Channel Model of Spelling **INDEPENDENT WORD SPELLING CORRECTION**

# Noisy Channel Intuition



# Noisy Channel = Bayes' Rule

- We see an observation  $x$  of a misspelled word
- Find the correct word  $\hat{w}$

$$\hat{w} = \operatorname{argmax}_{w \in V} P(w | x)$$

$$= \operatorname{argmax}_{w \in V} \frac{P(x | w)P(w)}{P(x)}$$

$$= \operatorname{argmax}_{w \in V} P(x | w)P(w)$$

↑  
Noisy channel model

Prior



Bayes

# History: Noisy channel for spelling proposed around 1990

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- **IBM**
  - Mays, Eric, Fred J. Damerau and Robert L. Mercer. 1991. Context based spelling correction. *Information Processing and Management*, 23(5), 517–522
- **AT&T Bell Labs**
  - Kernighan, Mark D., Kenneth W. Church, and William A. Gale. 1990. [A spelling correction program based on a noisy channel model](#). Proceedings of COLING 1990, 205-210

# Non-word spelling error example

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acress

# Candidate generation

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- Words with similar spelling
  - Small *edit distance* to error
- Words with similar pronunciation
  - Small distance of pronunciation to error

# Candidate Testing: Damerau-Levenshtein edit distance

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- Minimal edit distance between two strings, where edits are:
  - Insertion
  - Deletion
  - Substitution
  - Transposition of two adjacent letters
- See *IIR* sec 3.3.3 for edit distance

# Words within 1 of acress

Error	Candidate Correction	Correct Letter	Error Letter	Type
acress	actress	t	–	deletion
acress	cress	–	a	insertion
acress	caress	ca	ac	transposition
acress	access	c	r	substitution
acress	across	o	e	substitution
acress	acres	–	s	insertion

# Candidate generation

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- 80% of errors are within edit distance 1
- Almost all errors within edit distance 2
- Also allow insertion of **space** or **hyphen**
  - `thisidea` → `this idea`
  - `inlaw` → `in-law`
- Can also allow merging words
  - `data base` → `database`
  - For short texts like a query, can just regard whole string as one item from which to produce edits

# How do you generate the candidates?

1. Run through dictionary, check edit distance with each word
2. Generate all words within edit distance  $\leq k$  (e.g.,  $k = 1$  or 2) and then intersect them with dictionary
3. Use a character  $k$ -gram index and find dictionary words that share “most”  $k$ -grams with word (e.g., by Jaccard coefficient)
  - see *IIR* sec 3.3.4
4. Compute them fast with a Levenshtein finite state transducer
5. Have a precomputed map of words to possible corrections

# A paradigm ...

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- We want the best spell corrections
- Instead of finding the very best, we
  - Find a subset of pretty good corrections
    - (say, edit distance at most 2)
  - Find the best amongst them
- *These may not be the actual best*
- This is a recurring paradigm in IR including finding the best docs for a query, best answers, best ads ...
  - Find a good candidate set
  - Find the top  $K$  amongst them and return them as the best

# Let's say we've generated candidates: Now back to Bayes' Rule

- We see an observation  $x$  of a misspelled word
- Find the correct word  $\hat{w}$

$$\hat{w} = \operatorname{argmax}_{w \in V} P(w | x)$$

$$= \operatorname{argmax}_{w \in V} \frac{P(x | w)P(w)}{P(x)}$$

$$= \operatorname{argmax}_{w \in V} P(x | w)P(w)$$

What's  $P(w)$ ?

# Language Model

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- Take a big supply of words (your document collection with  $T$  tokens); let  $C(w)$  = # occurrences of  $w$

$$P(w) = \frac{C(w)}{T}$$

- In other applications – you can take the supply to be typed queries (suitably filtered) – when a static dictionary is inadequate

# Unigram Prior probability

Counts from 404,253,213 words in Corpus of Contemporary English (COCA)

word	Frequency of word	$P(w)$
actress	9,321	.0000230573
cress	220	.0000005442
caress	686	.0000016969
access	37,038	.0000916207
across	120,844	.0002989314
acres	12,874	.0000318463

# Channel model probability

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- **Error model probability, Edit probability**
- *Kernighan, Church, Gale 1990*
  
- *Misspelled word  $x = x_1, x_2, x_3 \dots x_m$*
- *Correct word  $w = w_1, w_2, w_3, \dots, w_n$*
  
- $P(x/w)$  = probability of the edit
  - (deletion/insertion/substitution/transposition)

# Computing error probability: confusion “matrix”

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del[x,y]:	count(xy typed as x)
ins[x,y]:	count(x typed as xy)
sub[x,y]:	count(y typed as x)
trans[x,y]:	count(xy typed as yx)

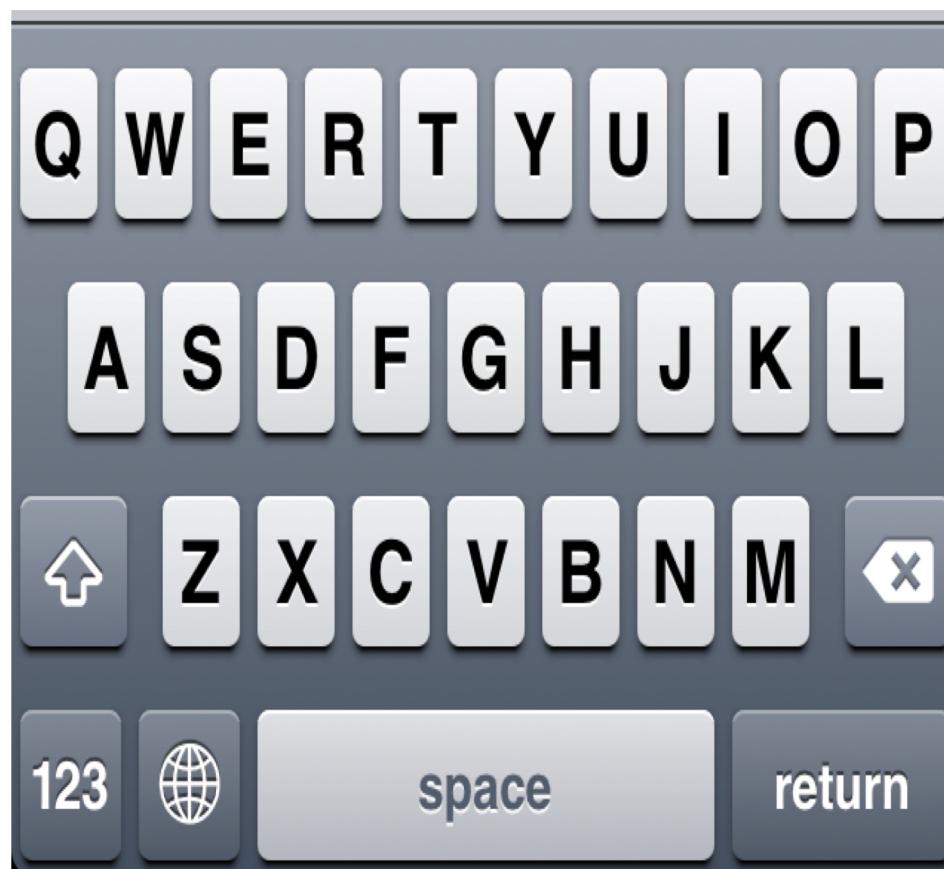
Insertion and deletion conditioned on previous character

# Confusion matrix for substitution

X	sub[X, Y] = Substitution of X (incorrect) for Y (correct)																									
	Y (correct)																									
a	0	0	7	1	342	0	0	2	118	0	1	0	0	3	76	0	0	1	35	9	9	0	1	0	5	0
b	0	0	9	9	2	2	3	1	0	0	0	5	11	5	0	10	0	0	2	1	0	0	8	0	0	0
c	6	5	0	16	0	9	5	0	0	0	1	0	7	9	1	10	2	5	39	40	1	3	7	1	1	0
d	1	10	13	0	12	0	5	5	0	0	2	3	7	3	0	1	0	43	30	22	0	0	4	0	2	0
e	388	0	3	11	0	2	2	0	89	0	0	3	0	5	93	0	0	14	12	6	15	0	1	0	18	0
f	0	15	0	3	1	0	5	2	0	0	0	3	4	1	0	0	0	6	4	12	0	0	2	0	0	0
g	4	1	11	11	9	2	0	0	0	1	1	3	0	0	2	1	3	5	13	21	0	0	1	0	3	0
h	1	8	0	3	0	0	0	0	0	0	2	0	12	14	2	3	0	3	1	11	0	0	2	0	0	0
i	103	0	0	0	146	0	1	0	0	0	0	6	0	0	49	0	0	0	2	1	47	0	2	1	15	0
j	0	1	1	9	0	0	1	0	0	0	0	2	1	0	0	0	0	0	5	0	0	0	0	0	0	0
k	1	2	8	4	1	1	2	5	0	0	0	0	5	0	2	0	0	0	6	0	0	0	4	0	0	3
l	2	10	1	4	0	4	5	6	13	0	1	0	0	14	2	5	0	11	10	2	0	0	0	0	0	0
m	1	3	7	8	0	2	0	6	0	0	4	4	0	180	0	6	0	0	9	15	13	3	2	2	3	0
n	2	7	6	5	3	0	1	19	1	0	4	35	78	0	0	7	0	28	5	7	0	0	1	2	0	2
o	91	1	1	3	116	0	0	0	25	0	2	0	0	0	0	14	0	2	4	14	39	0	0	0	18	0
p	0	11	1	2	0	6	5	0	2	9	0	2	7	6	15	0	0	1	3	6	0	4	1	0	0	0
q	0	0	1	0	0	0	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
r	0	14	0	30	12	2	2	8	2	0	5	8	4	20	1	14	0	0	12	22	4	0	0	1	0	0
s	11	8	27	33	35	4	0	1	0	1	0	27	0	6	1	7	0	14	0	15	0	0	5	3	20	1
t	3	4	9	42	7	5	19	5	0	1	0	14	9	5	5	6	0	11	37	0	0	2	19	0	7	6
u	20	0	0	0	44	0	0	0	64	0	0	0	0	2	43	0	0	4	0	0	0	0	2	0	8	0
v	0	0	7	0	0	3	0	0	0	0	0	1	0	0	1	0	0	0	8	3	0	0	0	0	0	0
w	2	2	1	0	1	0	0	2	0	0	1	0	0	0	0	7	0	6	3	3	1	0	0	0	0	0
x	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0	0	0	0
y	0	0	2	0	15	0	1	7	15	0	0	0	2	0	6	1	0	7	36	8	5	0	0	1	0	0
z	0	0	0	7	0	0	0	0	0	0	0	7	5	0	0	0	2	21	3	0	0	0	0	3	0	

# Nearby keys

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# Generating the confusion matrix

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- Peter Norvig's list of errors
- Peter Norvig's list of counts of single-edit errors
  - All Peter Norvig's ngrams data links: <http://norvig.com/ngrams/>

# Channel model

Kernighan, Church, Gale 1990

$$P(x|w) = \begin{cases} \frac{\text{del}[w_{i-1}, w_i]}{\text{count}[w_{i-1} w_i]}, & \text{if deletion} \\ \frac{\text{ins}[w_{i-1}, x_i]}{\text{count}[w_{i-1}]}, & \text{if insertion} \\ \frac{\text{sub}[x_i, w_i]}{\text{count}[w_i]}, & \text{if substitution} \\ \frac{\text{trans}[w_i, w_{i+1}]}{\text{count}[w_i w_{i+1}]}, & \text{if transposition} \end{cases}$$

# Smoothing probabilities: Add-1 smoothing

- But if we use the confusion matrix example, unseen errors are impossible!
- They'll make the overall probability 0. That seems too harsh
  - e.g., in Kernighan's chart  $q \rightarrow a$  and  $a \rightarrow q$  are both 0, even though they're adjacent on the keyboard!
- A simple solution is to add 1 to all counts and then if there is a  $|A|$  character alphabet, to normalize appropriately:

$$\text{If substitution, } P(x|w) = \frac{\text{sub}[x,w]+1}{\text{count}[w]+A}$$

# Channel model for acress

Candidate Correction	Correct Letter	Error Letter	$x w$	$P(x w)$
actress	t	-	c ct	.000117
cress	-	a	a #	.00000144
caress	ca	ac	ac ca	.00000164
access	c	r	r c	.000000209
across	o	e	e o	.0000093
acres	-	s	es e	.0000321
acres	-	s	ss s	.0000342

Candidate Correction	Correct Letter	Error Letter	$x/w$	$P(x/w)$	$P(w)$	$10^9 * P(x/w) * P(w)$
actress	t	-	c   ct	.000117	.0000231	2.7
cress	-	a	a   #	.00000144	.000000544	.00078
caress	ca	ac	ac   c a	.00000164	.00000170	.0028
access	c	r	r   c	.000000209	.0000916	.019
across	o	e	e   o	.0000093	.000299	2.8
acres	-	s	es   e	.0000321	.0000318	1.0
acres	-	s	ss   s	.0000342	.0000318	1.0 <sup>42</sup>

Candidate Correction	Correct Letter	Error Letter	$x/w$	$P(x/w)$	$P(w)$	$10^9$ $*P(x/w)P(w)$
actress	t	-	c   c t	.000117	.0000231	2.7
cress	-	a	a   #	.00000144	.000000544	.00078
caress	ca	ac	ac   ca	.00000164	.00000170	.0028
access	c	r	r   c	.000000209	.0000916	.019
across	o	e	e   o	.0000093	.000299	2.8
acres	-	s	es   e	.0000321	.0000318	1.0
acres	-	s	ss	.0000342	.0000318	1.0 <sup>43</sup>

# Evaluation

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- Some spelling error test sets
  - [Wikipedia's list of common English misspelling](#)
  - [Aspell filtered version of that list](#)
  - [Birkbeck spelling error corpus](#)
  - [Peter Norvig's list of errors \(includes Wikipedia and Birkbeck, for training or testing\)](#)

# Context-Sensitive Spelling Correction

## **SPELLING CORRECTION WITH THE NOISY CHANNEL**

# Real-word spelling errors

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- ...leaving in about fifteen **minuets** to go to her house.
- The design **an** construction of the system...
- Can they **lave** him my messages?
- The study was conducted mainly **be** John Black.
- 25-40% of spelling errors are real words      **Kukich 1992**

# Context-sensitive spelling error fixing

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- For each word in sentence (phrase, query ...)
  - Generate *candidate set*
    - the word itself
    - all single-letter edits that are English words
    - words that are homophones
    - (all of this can be pre-computed!)
  - Choose best candidates
    - Noisy channel model

# Noisy channel for real-word spell correction

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- Given a sentence  $x_1, x_2, x_3, \dots, x_n$
- Generate a set of candidates for each word  $x_i$ 
  - Candidate( $x_1$ ) =  $\{x_1, w_1, w'_1, w''_1, \dots\}$
  - Candidate( $x_2$ ) =  $\{x_2, w_2, w'_2, w''_2, \dots\}$
  - Candidate( $x_n$ ) =  $\{x_n, w_n, w'_n, w''_n, \dots\}$
- Choose the sequence  $W$  that maximizes  $P(W | x_1, \dots, x_n)$

$$\begin{aligned}\hat{w} &= \operatorname{argmax}_{w \in V} P(w | x) \\ &= \operatorname{argmax}_{w \in V} P(x | w)P(w)\end{aligned}$$

# Incorporating context words: Context-sensitive spelling correction

- Determining whether **actress** or **across** is appropriate will require looking at the context of use
- We can do this with a better **language model**
  - You learned/can learn a lot about language models in CS124 or CS224N
  - Here we present just enough to be dangerous/do the assignment
- A **bigram language model** conditions the probability of a word on (just) the previous word

$$P(w_1 \dots w_n) = P(w_1)P(w_2 | w_1) \dots P(w_n | w_{n-1})$$

# Incorporating context words

- For unigram counts,  $P(w)$  is always non-zero
  - if our dictionary is derived from the document collection
- This won't be true of  $P(w_k | w_{k-1})$ . We need to **smooth**
- We could use add-1 smoothing on this conditional distribution
- But here's a better way – interpolate a unigram and a bigram:

$$P_{\text{li}}(w_k | w_{k-1}) = \lambda P_{\text{uni}}(w_k) + (1-\lambda)P_{\text{bi}}(w_k | w_{k-1})$$

- $P_{\text{bi}}(w_k | w_{k-1}) = C(w_{k-1}, w_k) / C(w_{k-1})$

# All the important fine points

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- Note that we have several probability distributions for words
  - Keep them straight!
- You might want/need to work with log probabilities:
  - $\log P(w_1 \dots w_n) = \log P(w_1) + \log P(w_2 | w_1) + \dots + \log P(w_n | w_{n-1})$
  - Otherwise, be very careful about floating point underflow
- Our query may be words anywhere in a document
  - We'll start the bigram estimate of a sequence with a unigram estimate
  - Often, people instead condition on a start-of-sequence symbol, but not good here
  - Because of this, the unigram and bigram counts have different totals – not a problem

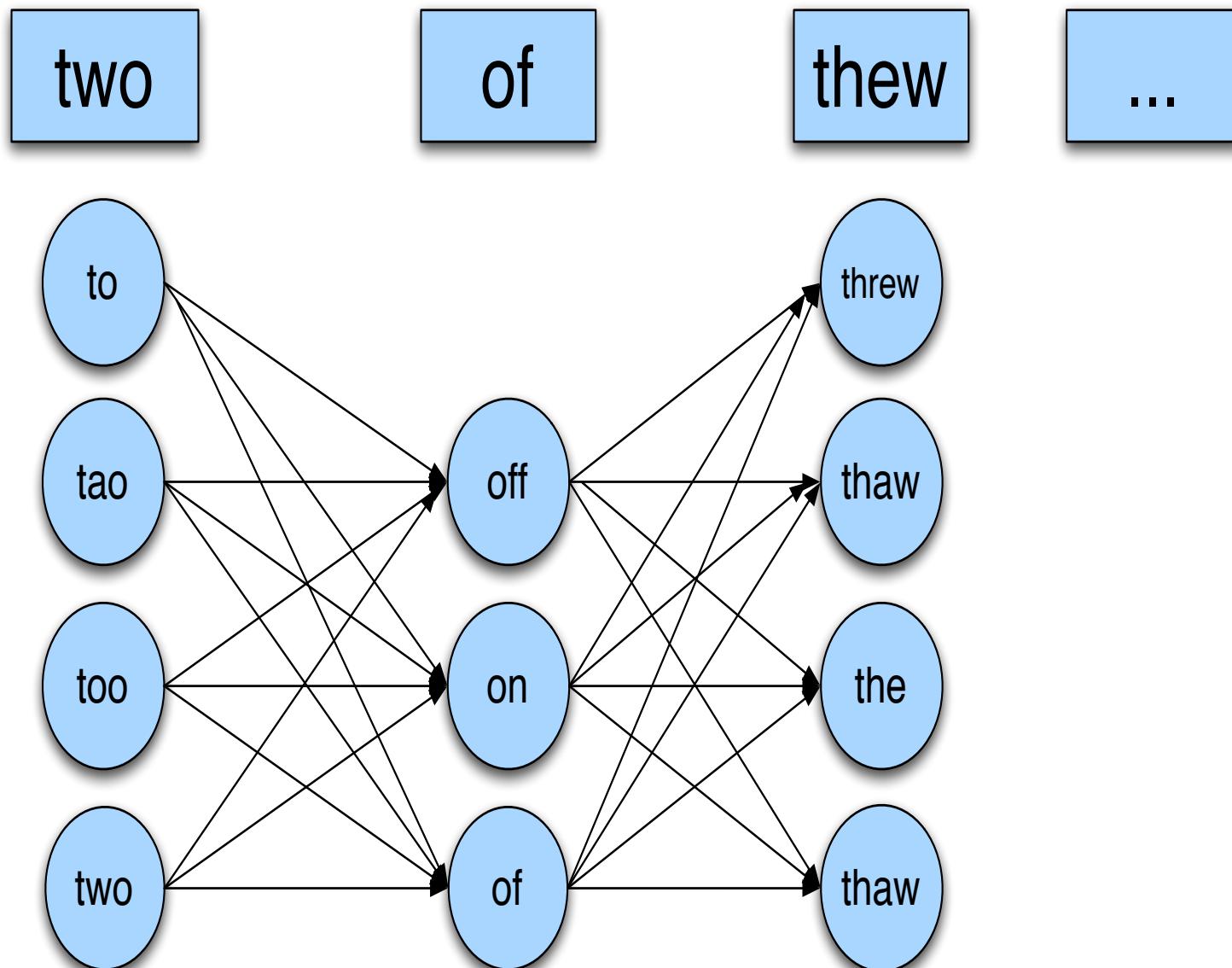
# Using a bigram language model

- “**a stellar and versatile across whose combination of sass and glamour...**”
- Counts from the Corpus of Contemporary American English with add-1 smoothing
- $P(\text{actress} \mid \text{versatile}) = .000021$
- $P(\text{across} \mid \text{versatile}) = .000021$
- $P(\text{whose} \mid \text{actress}) = .0010$
- $P(\text{whose} \mid \text{across}) = .000006$
- $P(\text{"versatile actress whose"}) = .000021 * .0010 = 210 \times 10^{-10}$
- $P(\text{"versatile across whose"}) = .000021 * .000006 = 1 \times 10^{-10}$

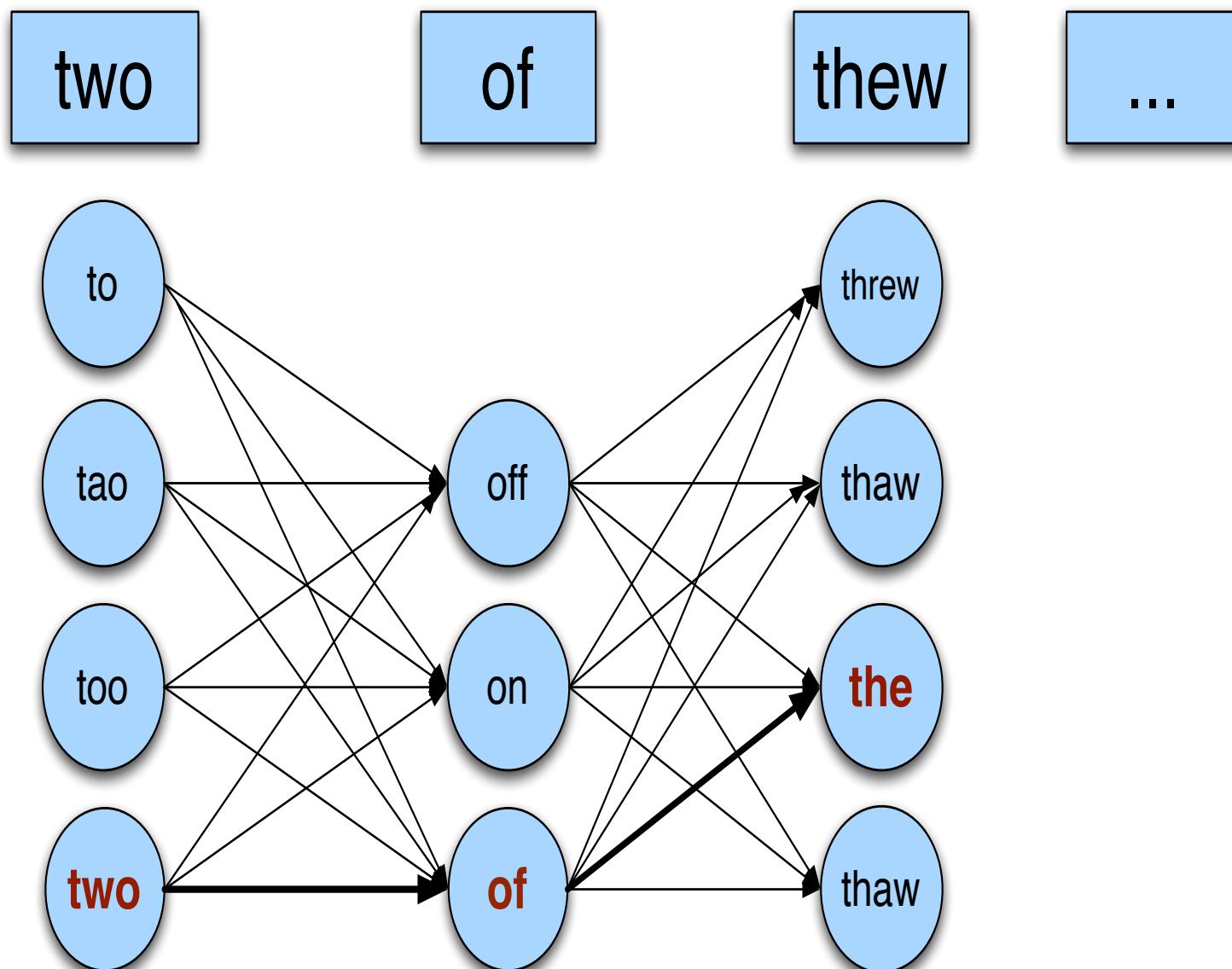
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- $P(\text{actress} \mid \text{versatile}) = .000021$   $P(\text{whose} \mid \text{actress}) = .0010$
- $P(\text{across} \mid \text{versatile}) = .000021$   $P(\text{whose} \mid \text{across}) = .000006$
  
- $P(\text{"versatile actress whose"}) = .000021 * .0010 = 210 \times 10^{-10}$
- $P(\text{"versatile across whose"}) = .000021 * .000006 = 1 \times 10^{-10}$

# Noisy channel for real-word spell correction



# Noisy channel for real-word spell correction



# Simplification: One error per sentence

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- Out of all possible sentences with one word replaced
  - $w_1, w''_2, w_3, w_4$       two off thew
  - $w_1, w_2, w'_3, w_4$       two of the
  - $w'''_1, w_2, w_3, w_4$       too of thew
  - ...
- Choose the sequence W that maximizes  $P(W)$

# Where to get the probabilities

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- Language model
  - Unigram
  - Bigram
  - etc.
- Channel model
  - Same as for non-word spelling correction
  - Plus need probability for no error,  $P(w/w)$

# Probability of no error

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- What is the channel probability for a correctly typed word?
- $P(\text{"the"} | \text{"the"})$ 
  - If you have a big corpus, you can estimate this percent correct
- But this value depends strongly on the application
  - .90 (1 error in 10 words)
  - .95 (1 error in 20 words)
  - .99 (1 error in 100 words)

# Peter Norvig's “thew” example

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x	w	x w	$P(x w)$	$P(w)$	$10^9 P(x w)P(w)$
thew	the	ew e	0.000007	0.02	144
thew	thew		0.95	0.0000009	90
thew	thaw	e a	0.001	0.0000007	0.7
thew	threw	h hr	0.000008	0.000004	0.03
thew	thwe	ew w	0.000003	0.00000004	0.0001

# State of the art noisy channel

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- We never just multiply the prior and the error model
- Independence assumptions → probabilities not commensurate
- Instead: Weight them

$$\hat{w} = \operatorname{argmax}_{w \in V} P(x | w) P(w)^\lambda$$

- Learn  $\lambda$  from a development test set

# Improvements to channel model

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- Allow richer edits [\(Brill and Moore 2000\)](#)
  - ent → ant
  - ph → f
  - le → al
- Incorporate pronunciation into channel [\(Toutanova and Moore 2002\)](#)
- Incorporate device into channel
  - Not all Android phones need have the same error model
  - But spell correction may be done at the system level