Limits to Firm Growth: All in the Family?

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This Version: April 2, 2021
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Abstract

We model a firm as a collection of managers who coordinate on joint production. The firm-level production technology features increasing returns to the number of managers and complementarity across managers of heterogeneous skills. Individuals are born into families that differ in size and managerial skill endowment. Each member of a family has the option to (i) work as a manager in the family firm; (ii) work as a manager in a non-family firm; or (iii) supply non-managerial labor for a wage. Non-family firms are constrained by a basic moral-hazard constraint: individual managers can steal a fraction of the joint output and forgo their managerial remunerations. The fraction that they may steal can be reduced by costly monitoring, which determines the optimal size of the firm. The limitation of family firms is, naturally, that the size and the managerial skill endowment of a family are exogenously given and immutable.

Keywords: Family Firms, Contracting Frictions, Firm Growth
JEL Classification: O4, J24, L1

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1. Introduction

There are substantial differences in per-capita income across countries, driven primarily by total factor productivity (TFP). For instance, GDP per capita of India in 2018 was a little over \$2,000 compared to about \$60,000 for the US. At the same time, literature has documented important differences in organization of production across countries.

In less developed countries such as India, there are smaller firms and establishments with substantially lower growth compared to firms in the US (see for example, Tybout (2000), Hsieh and Klenow (2014), Hsieh and Olken (2014)) resulting in aggregate productivity loss. There is also evidence of centralization of decisions within firms in developing countries, as a result of lack of trust, leading to lower aggregate productivity, as documented by Bloom, Sadun, and Van Reenen (2012). Another feature of developing countries is the prominence of family firms. Literature has documented that generally family firms are less productive than their non-family counterparts (see for example, Bertrand and Schoar (2006) and Bertrand, Johnson, Samphantharak, and Schoar (2008) for a review of the literature documenting this). Another strand of literature has documented cross-country differences in the rule of law and contract enforcement and its implications on shareholders and ownership (Porta, Lopez-de Silanes, Shleifer, and Vishny (1998)).

In this paper, we ask the question as to what explains the substantial differences in firm-size distribution across India and the US as seen clearly in figure 1? In particular, we ask how does weak rule of law and enforcement affects the organization of production in the less developed countries whereby we see smaller establishments, more centralization of decisions and existence of family firms? Lastly, and most importantly, what are the implications of weak enforcement for aggregate productivity and output. To this end, we develop a model of firms as a collection of managers and workers where managers have heterogeneous productivity and there exists complementarity in production function. The model features increasing returns to the number of managers or gains from specialization à la Becker and Murphy (1992).

Our model gives rise to the existence of a fairly rich set of firms: single person firms, professional firms, family firms without any outside managers and family firms with outside managers. In our model, the size of any non-family firm is limited by the ability of managers to divert a fraction of output, i.e., imperfect enforcement of contracts. On the other hand, size of a family firm is limited by the number of family members and their productivity endowment. This gives rise to a large number of small and unproductive firms and a few very large and productive professional firms.

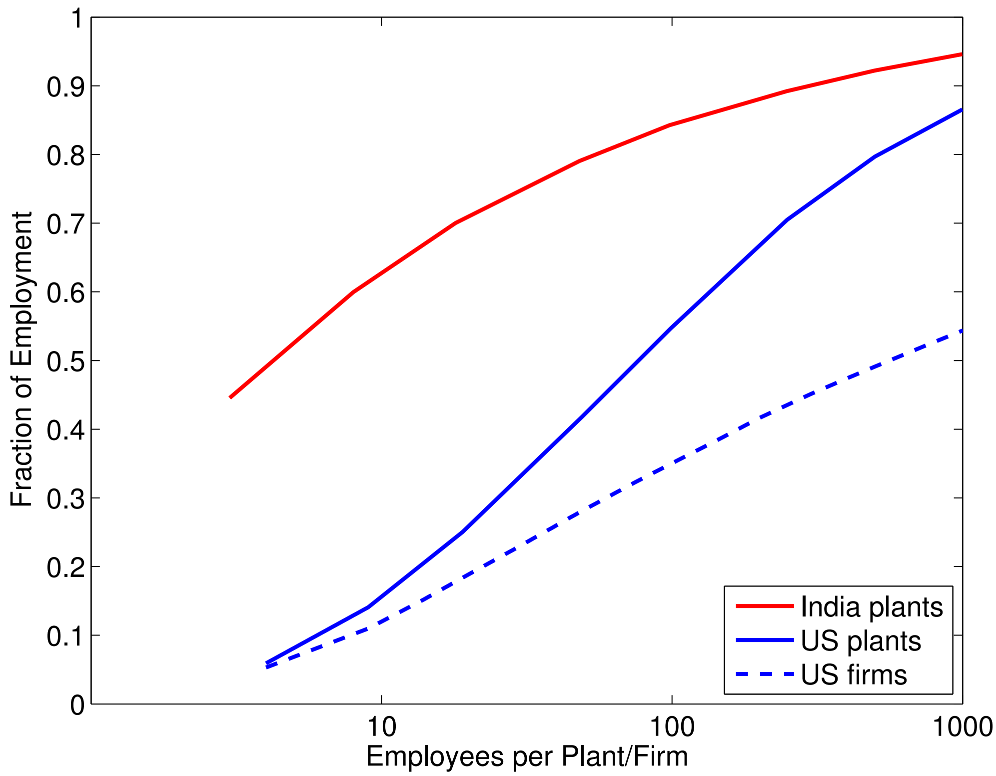


Figure 1: Size Distribution of Plants (Firms)

2. Model

Consider the following production function mapping the productivity vector of n managers $\mathbf{z} = (z_1, ..., z_n)$ and workers $\mathbf{l} = (l_1, ..., l_n)$ in an organization into final output,

$$y = f(\mathbf{z}, \mathbf{l}) = n^{\alpha} \left[\frac{1}{n} \sum \left(z_i l_i^{\theta} \right)^{\rho} \right]^{1/\rho}$$

where $\rho < 1$. The complementarity between managers implies that organizations that only employ non-family managers will be perfectly sorted. $\theta \in [0, 1)$ is the span of control of an individual manager and $\alpha \geq 1$ governs the gains from specialization. We assume $\alpha + \theta < 2$.

Its worthwhile to emphasize some special cases of the production technology:

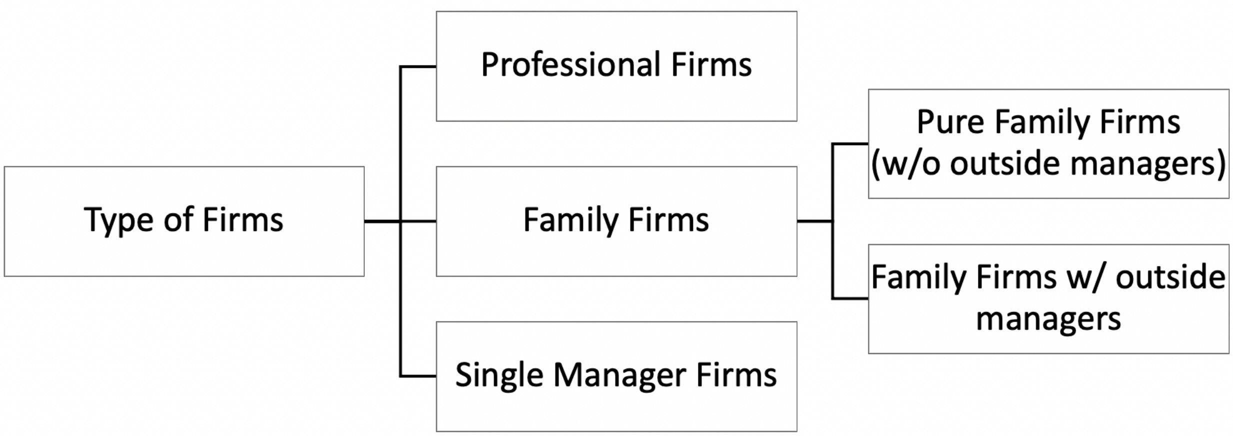
- Perfectly-sorted firm

$$f(\mathbf{z}, \mathbf{l}) = n^{\alpha} z l^{\theta}$$

- When there are no gains from specialization, i.e. $\alpha = 1$ or absent complementarities, i.e. $\rho = 1$:

$$f(\mathbf{z}, \mathbf{l}) = \sum_{i=1}^{n} z_i l_i^{\theta}$$

  In particular, note that there is no need to form a firm and the model simplifies to the standard span of control model.



We now describe various types of firms possible in this economy.

2.1 Perfectly-Sorted Non-Family Firm

The complementarity between managers implies that organizations that only employ non-family managers will be perfectly sorted[1]. We assume that managers can steal a fraction ϕ/m of the output of the organization, where ϕ is a parameter and m is the per-manager monitoring effort. We assume that monitoring effort has a cost c . Therefore, payments to each individual manager τ must be at least as large as the income obtained

[1] see Appendix B.4

by stealing the output,

$$\tau \geq \min\left\{1, \frac{\phi}{m}\right\} n^{\alpha} z, \tag{1}$$

and they must also be greater or equal to the market wage for their type of managers

$$\tau \geq w(z). \tag{2}$$

Noting that τ would never be greater than w(z), the problem of an organization with managers of talent z the level of monitoring per manager, m, and the total number of managers, n, is to maximize

$$\max_{n,l} n^{\alpha} z l^{\theta} - cnm - nw(z) - wnl \tag{3}$$

$$\text{s.t.}$$

$$w(z) \geq \frac{\phi}{m}(n^{\alpha} z l^{\theta} - wnl) \tag{4}$$

Noting that the constraint would hold with equality, substituting m from the constraint in the objective function

$$\max_{n,l} \left[1 - \frac{cn\phi}{w(z)}\right] \left(n^{\alpha} z l^{\theta} - wnl\right) - nw(z) \tag{5}$$

The first order conditions of this problem are

$$\alpha n^{\alpha-1} z l^{\theta} - w(z) - wl - c\frac{\phi}{w(z)}(\alpha+1) n^{\alpha} z l^{\theta} + \frac{c\phi}{w(z)} 2nlw = 0, \tag{6}$$

$$\theta n^{\alpha} z l^{\theta-1} = wn, \tag{7}$$

Using zero profit entry condition, we know that equilibrium wage has to be,

$$w(z) = \left[1 - \frac{cn\phi}{w(z)}\right] \left(n^{\alpha-1} z l^{\theta} - wl\right) \tag{8}$$

Solving (see Appendix B.1), we get,

$$l = \left(\frac{n^{\alpha-1} z\theta}{w}\right)^{\frac{1}{1-\theta}} \tag{9}$$

$$n = \left[\frac{(1-\theta)^2(\alpha-1)}{(\alpha-\theta)^2\phi c}\right]^{\frac{1-\theta}{2-\theta-\alpha}} \left(\frac{w}{\theta}\right)^{\frac{-\theta}{2-\theta-\alpha}} (z)^{\frac{1}{2-\theta-\alpha}} \tag{10}$$

$$w(z) = \frac{(\alpha-\theta)}{(\alpha-1)}\phi c \left[\frac{(1-\theta)^2(\alpha-1)}{(\alpha-\theta)^2\phi c}\right]^{\frac{1-\theta}{2-\theta-\alpha}} \left(\frac{w}{\theta}\right)^{\frac{-\theta}{2-\theta-\alpha}} (z)^{\frac{1}{2-\theta-\alpha}} \tag{11}$$

Note that wage function of professional managers is convex in z.

2.2 Family Firms

This section describes two types of family firms possible in this economy.

1. Pure Family Firms

We assume that family members cannot steal from the family firms. A pure family firm is therefore only constrained by the number of family members.

$$\max_{l} n^{\alpha} \left\{ \frac{1}{n} \left[\sum_{i=1}^{n_f} (z_i l_i^{\theta})^{\rho} \right] \right\}^{1/\rho} - \sum_{i=1}^{n_f} l_i w$$

Solving for optimal labor allocation,

$$n^{\alpha-1} \left\{ \frac{1}{n} \left[\sum_{i=1}^{n_f} (z_i l_i^{\theta})^{\rho} + (n - n_f)(z l^{\theta})^{\rho} \right] \right\}^{1/\rho - 1} \theta l_i^{\rho\theta-1} z_i^{\rho} = w \tag{12}$$

Which is nothing but the standard result of equalization of marginal product in such problems. Note in particular, that we can simplify this to get,

$$l_i = \left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}} z_i^{\frac{\rho}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{1-\rho}{\rho(1-\theta)}} n^{\frac{\alpha-1}{1-\theta}} \tag{13}$$

Where $\bar{z}(z,n) = \frac{1}{n}\sum_{i=1}^{n} z_i^{\frac{\rho}{1-\theta\rho}}$

Boundary case: One Person Family Firm

A special case of pure family firms is one person family firm or where $n = 1$

$$\pi^{1FF} = \max_{l} z_i l_i^{\theta} - l_i w$$

Thus, effective market wage for professional managers that the family firms with outside managers will take as given becomes:

$$w^e(z) = max(\pi^{1FF}, w(z)) \tag{14}$$

This gives us a simple cutoff z^{e*} between professional managers and one person firms. More on this later.

2. Family Firm with Outside Managers

A family firm with n_f family members with (generalized) average ability z_f chooses the total number of managers $n \geq n_f$, the level of monitoring for outside managers m, and productivity of these managers z along with labor l_i to maximize,

$$\max_{m,n \geq n_f, z} n^{\alpha} \left\{ \frac{1}{n} \left[\sum_{i=1}^{n_f} (z_i l_i^{\theta})^{\rho} + (n - n_f)(z l^{\theta})^{\rho} \right] \right\}^{1/\rho} - \sum_{i=1}^{n_f} l_i w - (n - n_f) l w - (n - n_f) c m - (n - n_f) w^e(z) \tag{15}$$

s.t.

$$w^e(z) \geq \frac{\phi}{m} \left(n^{\alpha} \left\{ \frac{1}{n} \left[\sum_{i=1}^{n_f} (z_i l_i^{\theta})^{\rho} + (n - n_f)(z l^{\theta})^{\rho} \right] \right\}^{1/\rho} - \sum_{i=1}^{n_f} l_i w - (n - n_f) l w \right) \tag{16}$$

Following the argument for professional firms described earlier, the objective function becomes

$$\left(1 - \frac{c(n - n_f)\phi}{w^e(z)}\right) \left(n^{\alpha} \left\{ \frac{1}{n} \left[\sum_{i=1}^{n_f} (z_i l_i^{\theta})^{\rho} + (n - n_f)(z l^{\theta})^{\rho} \right] \right\}^{1/\rho} - \sum_{i=1}^{n_f} l_i w - (n - n_f) l w \right) - (n - n_f) w^e(z) \tag{17}$$

In addition to the incentive compatibility (henceforth IC) constraint for outside managers, family firms with outside managers are also subject to the family incentive compatibility constraint. The family IC constraint is that the family's joint profit when they don't steal has to be greater than a fraction of the profit when they collective steal and do not renegade the profit to outside managers. More formally, λ/m_f is the fraction they can collectively steal,

$$\pi^c \geq \frac{\lambda}{m_f}\pi^{nc}$$

where,

$$\pi^c = \left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\left(n^\alpha\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^\theta)^\rho + (n-n_f)(zl^\theta)^\rho\right]\right\}^{1/\rho} - \sum_{i=1}^{n_f} l_i w - (n-n_f)lw\right) \\ -(n-n_f)w^e(z) - n_f c m_f \quad (18)$$

$$\pi^{nc} = \left(n^\alpha\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^\theta)^\rho + (n-n_f)(zl^\theta)^\rho\right]\right\}^{1/\rho} - \sum_{i=1}^{n_f} l_i w - (n-n_f)lw\right) \quad (19)$$

Let $\pi^f = \left(1 - \frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc} - (n-n_f)w^e(z)$.
Thus, the problem becomes,

$$\max_{n,z,l} \frac{\pi^f(n,z,l) + \sqrt{\pi^f(n,z,l)^2 - 4\lambda\pi^{nc}(n,z,l)cn_f}}{2} \quad (20)$$

The domain of the above problem is restricted to: ${\pi^f}^2 \geq 4\lambda\pi^{nc}(n,z,l)cn_f$ and $\pi^f \geq 0$
It can be easily shown that the labor choice is not distorted because of family IC (see appendix B.2). Thus, we get,

$$l_i = \left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}} z_i^{\frac{\rho}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{1-\rho}{\rho(1-\theta)}} n^{\frac{\alpha-1}{1-\theta}} \quad (21)$$

Let the solution to max 20 be $n^*(n_f, z_f)$ and $z^*(n_f, z_f)$ which we are going to obtain numerically.

$$\pi^{nc} = \left(\frac{1}{w}\right)^{\frac{\theta}{1-\theta}} \bar{z}(z,n)^{\frac{1-\rho\theta}{(1-\theta)\rho}} n^{\frac{\alpha-\theta}{1-\theta}} \left(\theta^{\frac{\theta}{1-\theta}} - \theta^{\frac{1}{1-\theta}}\right) \tag{22}$$

Where, $\bar{z}(z,n) = \left\{ \frac{n_f}{n} z_f^{\frac{\rho}{1-\theta\rho}} + \frac{n-n_f}{n} z^{\frac{\rho}{1-\theta\rho}} \right\}$ where $z_f = \left(\frac{1}{n_f} \sum_{i=1}^{n_f} z_i^{\frac{\rho}{1-\theta\rho}} \right)^{\frac{1-\theta\rho}{\rho}}$

2.3 Policy Functions

Professional Firms

The optimal number of managers in professional firms can be described as

$$n^{e*}(z) = \begin{cases} 1, & z \leq z^{e*} \\ n^*(z), & z > z^{e*} \end{cases} \tag{23}$$

A professional firm with z such that $z \leq z^{e*}$, becomes a single manager firm. On the other hand, professional firms operating with high z such that $z > z^{e*}$ have more than one managers. This is described in figure 2. Optimal workers per manager is analogous to optimal number of managers as shown in figure 3. As one would expect, optimal managers and workers per manager is an increasing function of productivity.

Figure 2: Policy Function: Professional Firms $n^{e*}(z)$

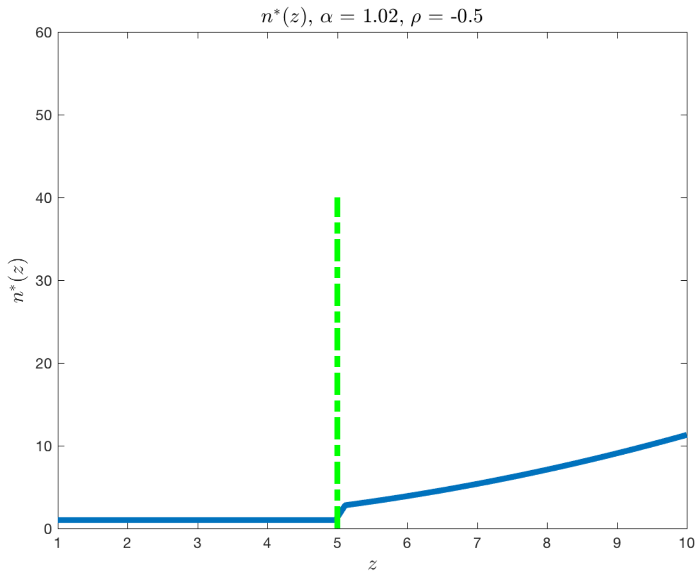
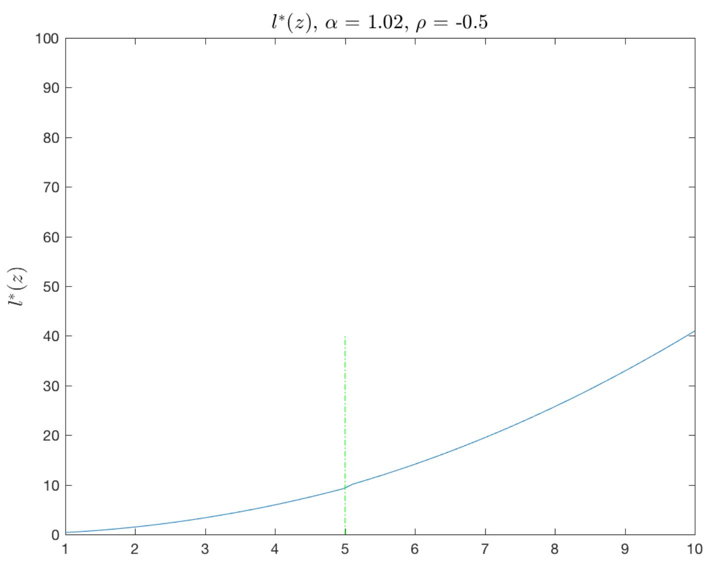


Figure 3: Policy Function: Professional Firms $l^{e*}(z)$



Policy Functions: Family Firms

The optimal choice of outside managers,$n^*(n_f, z_f) - n_f$, is shown in 4 and their productivity $z^*(n_f, z_f) - z_f$ is shown in figure 5. If the effective z_f of the family firm is low, they do not hire any outside managers and are not affected by the incentive compatibility constraint that the family firms with outside managers will be subject to. For high enough z_f they hire outside managers and the number of outside managers is increasing in z_f. The number of managers they decide to hire increases marginally with n_f. Given complementarity in the production function and the wages that the firms face for professional managers, they hire managers with similar productivity as the family as clear from figure 5.

Figure 4: Policy Function: $n^*(n_f, z_f) - n_f$

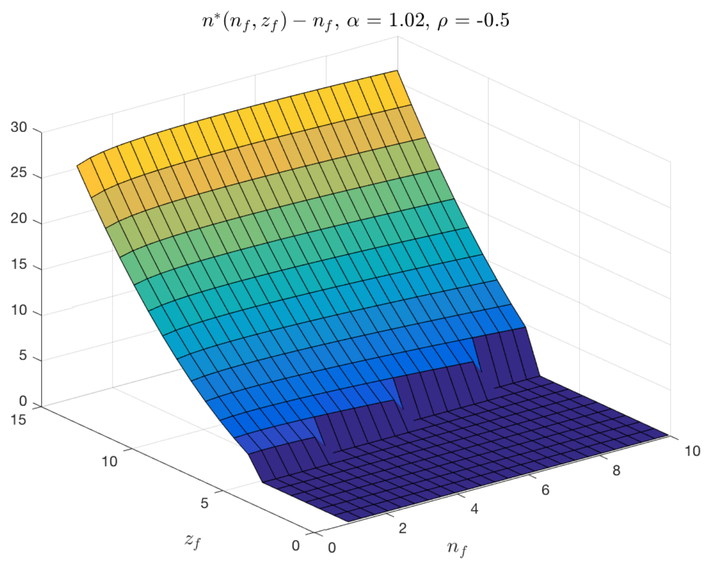
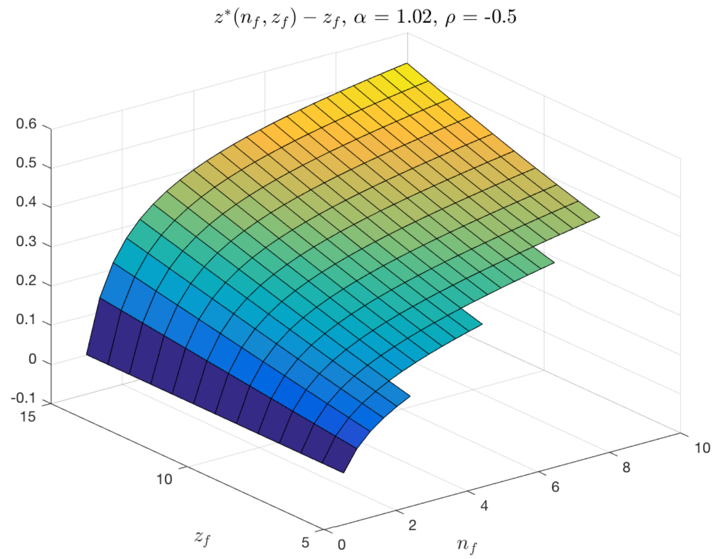


Figure 5: Policy Function: $z^*(n_f, z_f) - z_f$



2.4 Occupational Choice

Given the family size, n_f and draws of z's $(z_1, z_2, ..., z_{n_f})$, individuals decide where to be a worker, professional manager, self-employed/ one manager firm or form a firm with other members of the family. They further decide whether to hire outside managers and workers to the firm.

Illustration: Family of Size 2

For a family of size 2 with z_1, z_2, the figure **??** illustrates the choices: if both z's are low they are both workers, if one is low while the other is high, the low z is a worker and the high z is a professional manager or managers a one person firm. If z's are along the diagonal, they decide to form a family firm, this result is because of complementarity in the production function. At this point, it is important to highlight the key tension in the model: while family members want to benefit from increasing returns technology and get around the contract enforcement constraint, they are constrained by the endowment of the family members- given the complementarities in the production function, they benefit from the increasing returns technology only if the cousins have similar productivity. When $z \leq z^{e*}$, individuals choose to run a one person firm. For clear exposition, we only provide four broad categories in the figure.

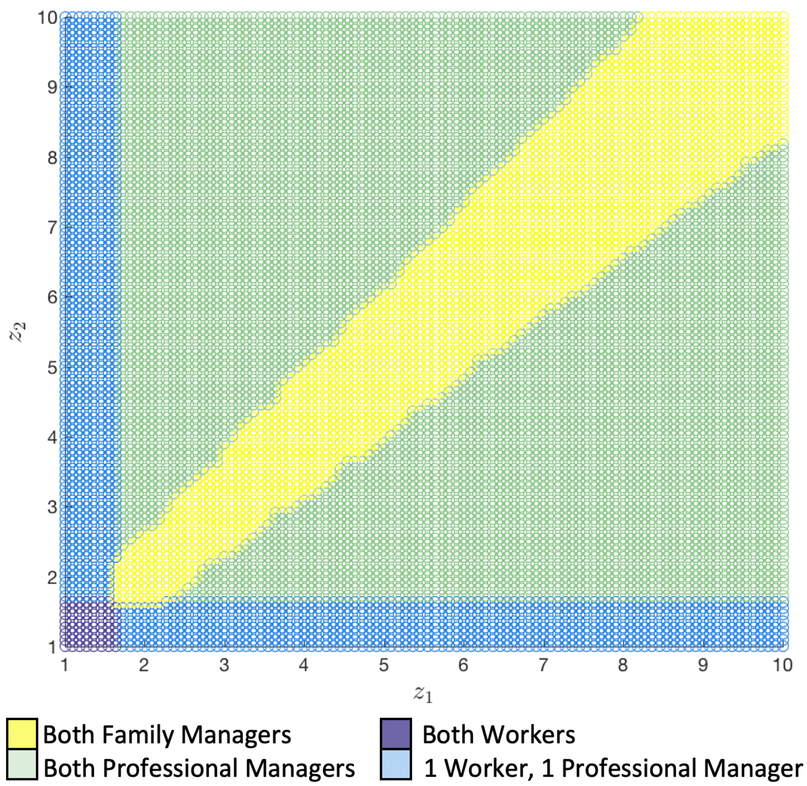


Illustration: Family of Size 3

Figure 7 represents the 3-D figure of occupational choice at the family level for family of size three. We also present a slice along the parallel line ($z_1 + z_2 + z_3 = 24$) in figure 6, this is a view along the diagonal of the cube. Note that for clear representation/ visualization of family firms, we club together workers, professional managers and one-person firms in this figure. When $z's$ along the main diagonal of the cube, they form a family firm of size 3 - policy function described in the previous section informs whether they hire outside managers or not: for low productivity along the main diagonal, they do not hire outside managers while along the top end of the main diagonal, they hire outside managers. Let's focus on the side formed by $z_2 = 1$ plane: as we saw in the family of size two, if the productivity is along this side diagonal, they form family firm of size two and the low productivity z_2 becomes a worker. Figure 7 illustrates this further- while along main diagonal there are family firms of size 3, if the productivity of one manager is doesn't align well with the other two family managers, the family is better off with her being a professional manager or running a firm by herself while the other two members form a family firm together. If the productivity of the other two managers also doesn't align with each other, the family is better off with all three being professional managers or running a one-person firm each (unless one's productivity is below the worker-professional cutoff in which case he opts to be a worker).

Figure 6: Occupational Choice: $n_f = 3$ (Simple)

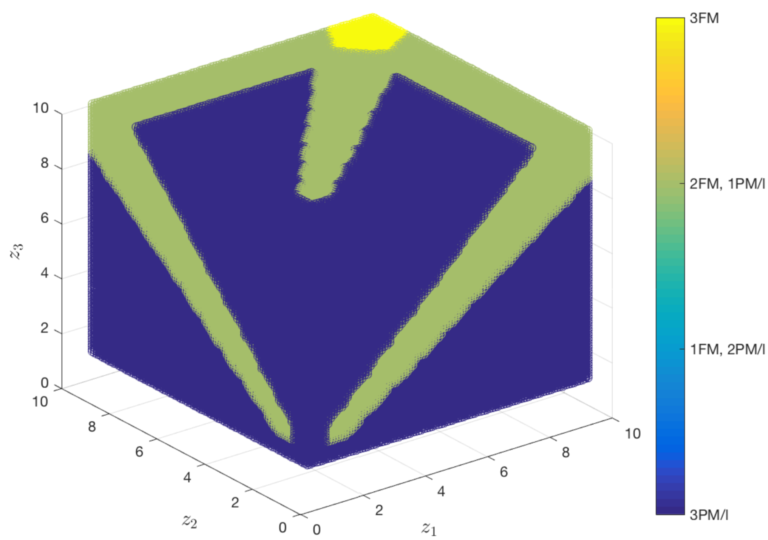
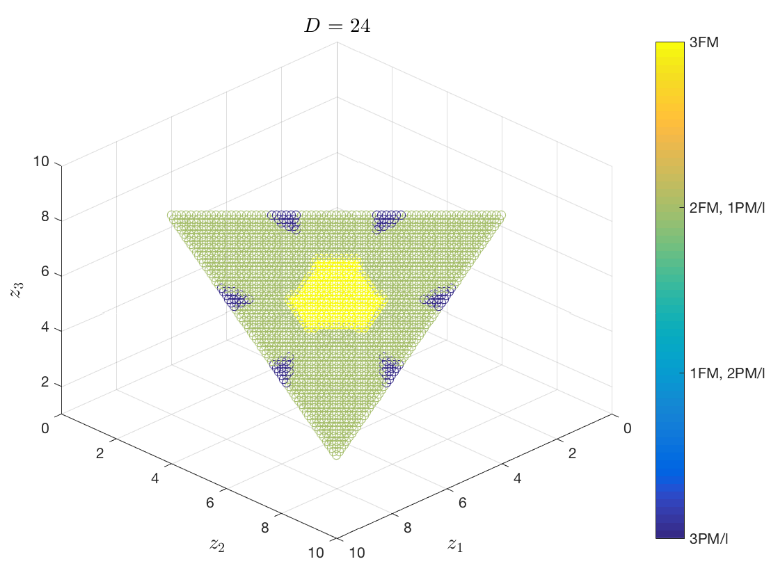


Figure 7: Occupational Choice: $n_f = 3$ (Simple) Slice along ($z_1 + z_2 + z_3 = D$)



Equilibrium

The equilibrium can be described in the standard way. For given prices, i.e. worker wage w, effective manager wage w(z), aggregate labor demand equals supply from occupational choice at the family level. Note that we don't need to clear the managers market separately as long as there is a positive fraction of professional managers or one-person managers in the whole distribution above worker cutoff.
Proof: TBA

Algorithm: Labor Market Clearing

1. Start with a guess of w
2. Obtain $w^e(z)$ (explicitly, from professional firms and one person firm problem)
3. Solve (numerically) for $n^*(z_f, n_f), z^*(z_f, n_f)$ (FF with outside managers and family IC)
4. Obtain the number of families for each size, $n(n_f)$ from data. Simulate multivariate pareto $z's$ for $n(n_f)$.
5. Solve occupational choice of the Family problem and obtain labor supply and family firm labor demand by keeping track of family firms
6. Obtain labor demand from professional firms by calculating $\int \frac{p(z)}{n(z)} n(z) l(z) F(dz)$, where $p(z)$ is fraction of professional managers among z types. We can calculate this by $p(z) = \frac{\sum_{n_f=1}^{n_f=15} n(n_f) \text{ Number of professional managers } (z, n_f)}{\sum_{n_f=1}^{n_f=15} n(n_f) \text{ Number } (z, n_f)}$
7. Obtain the demand for professional manager from family firms. If the demand for professional managers is less than supply, excess supply professional managers come together to form a professional firm.
8. If the demand is greater than the professional managers supply, market wage $w(z)$ schedule adjusts wherein the managers for such z types seek rent from the family firms.

3. Parameterization

We pick some parameters to describe the working of the model in table 1. For the family size distribution, we use working individuals in household distribution from India described in table 5.

Table 1: Illustrative Parameters

Parameter	FF
α	1.31
ϕ	0.53
ρ	-4.90
θ	0.27
μ	2.96
σ	0.85

Table 2: Key Moments

Moment	Model
Worker Compensation to Value Added bottom 40-ptile	0.38
Worker Compensation to Value Added Top 5-ptile	0.46
Top 10-percentile employment share	0.39
Average Firm Size	2.43
Average Number of Managers	1.19
Fraction FF top 1-ptile	0.28
Fraction households in FF	0.93

Under the parameters described, we'll describe some of the moments generated by the model. The model generates a large number lot of one person firms situated in the lower end of the distribution primarily as depicted in the in figure **??** which shows the size distribution of firms. This is because if a manager is productive enough ($z \geq z^{e*}$), his income as manager working in professional firm or family firms exceeds the profit

Table 3: Baseline Model Patterns- I

	Baseline Model
Average Firm Size	2.448
Average Managers	1.134
Fraction SE Small Family	0.465
Top 10-ptile Employment Share	0.367
Fraction Manager in Family Firms with Outside Managers	0.153
Fraction Manager in Single Manager Firms	0.308
Fraction Family Firms without Outside Managers	0.162
Fraction Family Firms with Outside Managers	0.009
Fraction Professional Firms	0.028
Fraction Single Manager Firms	0.801
Fraction from Family Firm Households in Firm	0.846
Average Family Members in Family Firms	2.176
Average Outside Managers in Family Firms	0.013
Fraction of Family Firms with Outside Managers	0.054
Average Outside Managers in Family Firms with Outside Managers	2.694
Output per Employee	7.996
Worker Wage	0.986

Notes:

earned by running a single firm. Family firms exist throughout the distribution and are on average less productive than one person firms and subsequently hire lower number of managers and workers than professional firms. A small number of professional firms exist in the upper end of the distribution but they are on average thrice as much productive compared to one person or family firms and employ multiple-folds more individuals per firm compared to their counterparts. It is also worthwhile to note that most family firms do not hire outside managers but the ones that hire are of very high productivity and they hire 2.7 managers on average as tabulated in 4.

Table 4: Baseline Model Patterns - II

	Baseline Model: Family Firms vs Professional Firms		
	Family Firm	Professional Firm	1 Person PF
Average employment	4.559	18.849	1.624
Average Workers	2.238	14.532	0.624
Average managers	2.321	4.317	1.000
Average z	3.248	8.042	2.825
Fraction of Firms	0.171	0.028	0.801

Notes:

Role of Family Firms

In order to understand the role of family firm in the economy, we take away the option to form a family firm and show the implications for size distribution and income distribution. For the parameters described in the previous section, we show that if we take away the option to form a family firm, it leads to a missing middle from the firm size distribution implied by the model.

Figure 8: With FF

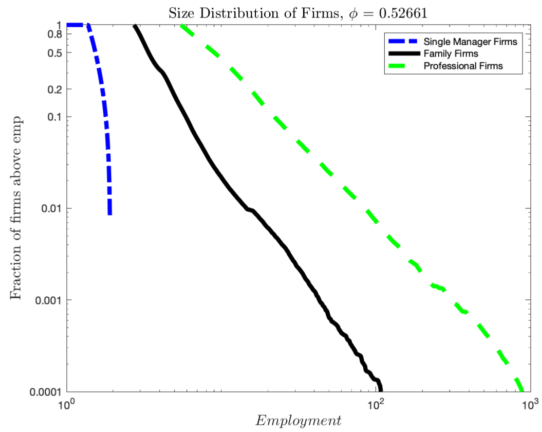
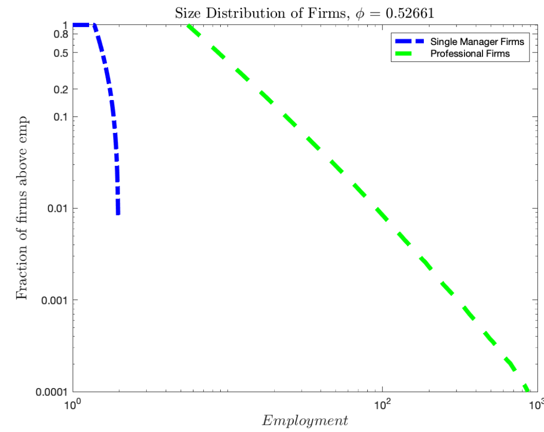


Figure 9: Without FF



Family firms also could have a potentially large distributional impact: for family income per person distribution, the p99/p1 is 14.6 w/o family firms and it goes down to 12.1 w/ family firms, therefore family firms could help lowering the income inequality. In this exercise, wealthy small families gain 20% w/o family firms while poor small families lose 8%. The biggest losers are large wealthy families lose 10-30% w/o family firms who benefit most from the existence of family firms in presence of delegation frictions.

Figure 10: With and Without FF

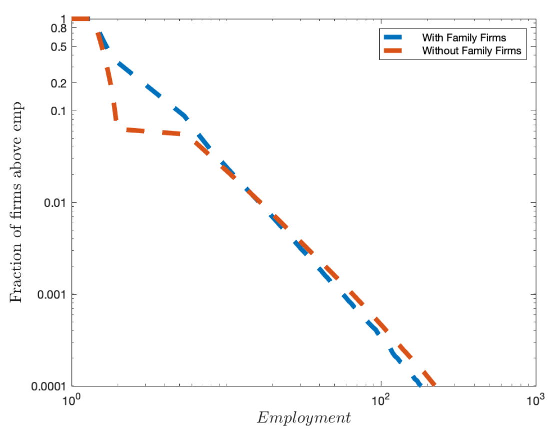
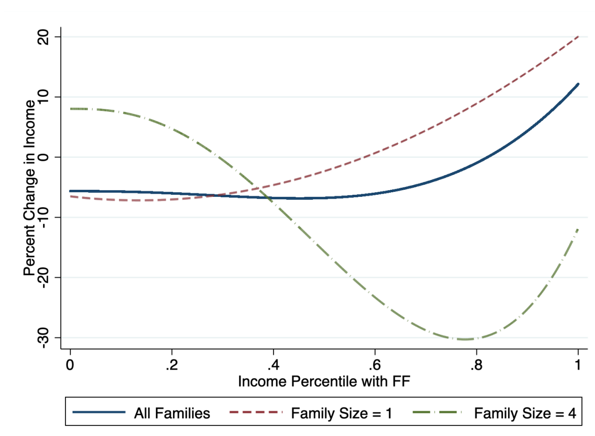
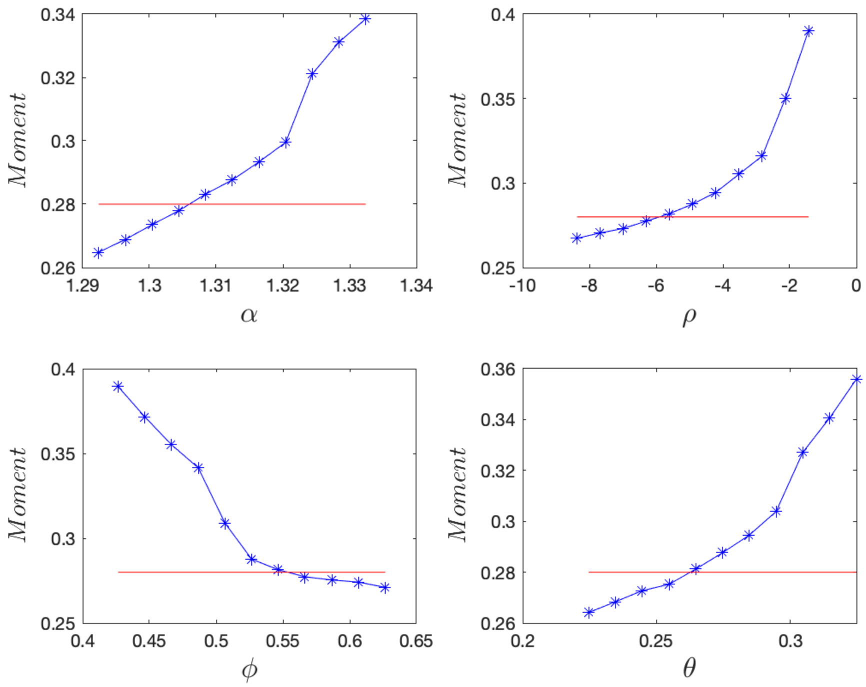


Figure 11: Without FF



Family firms data could be potentially useful in pinning down the production complementarities in standard class of models. In figure 12, we show that fraction of

Figure 12: On ρ: Fraction FF in top 1-ptile



FF in top 1-ptile moves one-to-one with complementarity, ρ and increasing returns, $alpha$.While due to increasing returns individuals may want to form a family firm, if the production function exhibits more complementarity, they only do it with a similar in ability family member, thereby reducing the fraction of family firms.

4. Conclusion

This paper explores the mechanisms for the existence of family firms and its aggregate implications for economic development. We model a firm as a collection of managers who coordinate on joint production. The firm-level production technology features increasing returns to the number of managers and complementarity across managers of heterogeneous skills. Our model gives rise to the existence of a fairly rich set of firms: single person firms, professional firms, family firms without any outside managers and family firms with outside managers. In our model, the size of any non-family firm is limited by the ability of managers to divert a fraction of output, i.e., imperfect enforcement of contracts. On the other hand, size of a family firm is limited by the number of family members and their productivity endowment. In future work, the estimated model can be used to quantify the role of such constraints on the growth of firms and economic development.

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Appendix

A. Tables

Table 5: Working Individuals in Household Distribution

Number	Weighted
1	68.948
2	23.027
3	5.841
4	1.623
5	0.371
6	0.130
7	0.046
8	0.006
9	0.002
10	0.006
N	70942

B. Proofs/ Algebra

B.1 Non-family Firm

$$\max_{n,l} \left[1 - \frac{cn\phi}{w(z)}\right]\left(n^\alpha z l^\theta - wnl\right) - nw(z) \tag{24}$$

The first order conditions of this problem are

$$\alpha n^{\alpha-1} z l^\theta - w(z) - wl - c\frac{\phi}{w(z)}(\alpha+1)n^\alpha z l^\theta + \frac{c\phi}{w(z)}2nlw = 0, \tag{25}$$

$$\theta n^\alpha z l^{\theta-1} = wn, \tag{26}$$

rearranging 26

$$l = \left(\frac{n^{\alpha-1} z\theta}{w}\right)^{\frac{1}{1-\theta}} \tag{27}$$

$$\tag{28}$$

Equilibrium wage has to be,

$$w(z) = \left[1 - \frac{cn\phi}{w(z)}\right]\left(n^{\alpha-1} z l^\theta - wl\right) \tag{29}$$

Subs 29 in 25

$$\begin{aligned}
\alpha n^{\alpha-1} z l^\theta - wl - c\frac{\phi}{w(z)}(\alpha+1)n^\alpha z l^\theta + \frac{c\phi}{w(z)}2nlw - \left[1 - \frac{cn\phi}{w(z)}\right]\left(n^{\alpha-1} z l^\theta - wl\right) &= 0 \\
\alpha n^{\alpha-1} z l^\theta - wl - c\frac{\phi}{w(z)}(\alpha+1)n^\alpha z l^\theta + \frac{c\phi}{w(z)}2nlw - \left[n^{\alpha-1} z l^\theta - wl - \frac{cn\phi}{w(z)}n^{\alpha-1} z l^\theta + \frac{cn\phi}{w(z)}wl\right] &= 0 \\
\alpha n^{\alpha-1} z l^\theta - wl - c\frac{\phi}{w(z)}(\alpha+1)n^\alpha z l^\theta + \frac{c\phi}{w(z)}2nlw - n^{\alpha-1} z l^\theta + wl + \frac{cn\phi}{w(z)}n^{\alpha-1} z l^\theta - \frac{cn\phi}{w(z)}wl &= 0
\end{aligned}$$

$$\alpha n^{\alpha-1} z l^\theta - c\frac{\phi}{w(z)}(\alpha)n^\alpha z l^\theta + \frac{c\phi}{w(z)}nlw - n^{\alpha-1} z l^\theta = 0 \tag{30}$$

Hence,

$$\begin{aligned} \alpha n^{\alpha-1} z l^{\theta} - n^{\alpha-1} z l^{\theta} &= c\frac{\phi}{w(z)} \alpha n^{\alpha} z l^{\theta} - \frac{c\phi}{w(z)} n l w \\ (\alpha-1) n^{-1} z &= c\frac{\phi}{w(z)} \left(\alpha z - n^{1-\alpha} l^{1-\theta} w \right) \end{aligned} \tag{31}$$

Substituting 27,

$$(\alpha-1) n^{-1} z = c\frac{\phi}{w(z)} \left(\alpha z - n^{1-\alpha} \frac{n^{\alpha-1} z \theta}{w} w \right) \tag{32}$$

Hence,

$$\begin{aligned} w(z,n) &= c\frac{\phi n}{(\alpha-1) z} \left(\alpha z - \theta z \right) \\ w(z,n) &= \frac{(\alpha-\theta)}{(\alpha-1)} \phi c n \end{aligned} \tag{33}$$

Subs 33 in 29,

$$\begin{aligned} \frac{(\alpha-\theta)}{(\alpha-1)} \phi c n &= \left[1 - \frac{c n \phi}{\frac{(\alpha-\theta)}{(\alpha-1)} \phi c n} \right] \left(n^{\alpha-1} z l^{\theta} - w l \right) \\ \frac{(\alpha-\theta)}{(\alpha-1)} \phi c n &= \left[1 - \frac{(\alpha-1)}{(\alpha-\theta)} \right] \left(n^{\alpha-1} z l^{\theta} - w l \right) \end{aligned} \tag{34}$$

Using 27, we note, $n^{\alpha-1} z l^{\theta} = \frac{lw}{\theta}$. Hence,

$$\begin{aligned} \frac{(\alpha-\theta)}{(\alpha-1)} \phi c n &= \left[1 - \frac{(\alpha-1)}{(\alpha-\theta)} \right] \left(\frac{wl}{\theta} - w l \right) \\ \frac{(\alpha-\theta)}{(\alpha-1)} \phi c n &= \left[1 - \frac{(\alpha-1)}{(\alpha-\theta)} \right] \left(\frac{1}{\theta} - 1 \right) w l \end{aligned} \tag{35}$$

Substituting 27,

$$
\begin{aligned}
\frac{(\alpha-\theta)}{(\alpha-1)}\phi c n &= \left[\frac{(1-\theta)^2}{(\alpha-\theta)\theta}\right] w \left(\frac{n^{\alpha-1}z\theta}{w}\right)^{\frac{1}{1-\theta}} \\
n &= \left[\frac{(1-\theta)^2(\alpha-1)}{(\alpha-\theta)^2\theta\phi c}\right] w \left(\frac{n^{\alpha-1}z\theta}{w}\right)^{\frac{1}{1-\theta}} \\
n^{1-\frac{\alpha-1}{1-\theta}} &= \left[\frac{(1-\theta)^2(\alpha-1)}{(\alpha-\theta)^2\theta\phi c}\right] w^{1-\frac{1}{1-\theta}}(z\theta)^{\frac{1}{1-\theta}} \\
n^{\frac{2-\theta-\alpha}{1-\theta}} &= \left[\frac{(1-\theta)^2(\alpha-1)}{(\alpha-\theta)^2\phi c}\right] \left(\frac{w}{\theta}\right)^{-\frac{\theta}{1-\theta}} (z)^{\frac{1}{1-\theta}} \\
n &= \left[\frac{(1-\theta)^2(\alpha-1)}{(\alpha-\theta)^2\phi c}\right]^{\frac{1-\theta}{2-\theta-\alpha}} \left(\frac{w}{\theta}\right)^{\frac{-\theta}{2-\theta-\alpha}} (z)^{\frac{1}{2-\theta-\alpha}} \qquad (36)
\end{aligned}
$$

Using 33, we get,

$$
w(z) = \frac{(\alpha-\theta)}{(\alpha-1)}\phi c \left[\frac{(1-\theta)^2(\alpha-1)}{(\alpha-\theta)^2\phi c}\right]^{\frac{1-\theta}{2-\theta-\alpha}} \left(\frac{w}{\theta}\right)^{\frac{-\theta}{2-\theta-\alpha}} (z)^{\frac{1}{2-\theta-\alpha}} \qquad (37)
$$

B.2 Family Firm with Outside Managers: Part I

To show: The objective function leaves labor choice undistorted.

$$
\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^c - (n-n_f)w^e(z) - n_f c m_f \qquad (38)
$$

F.O.C. w.r.to. l_i,

$$
\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\frac{\partial \pi^{nc}}{\partial l_i} - n_f c\frac{\partial m_f}{\partial l_i} = 0 \qquad (39)
$$

m_f solves the following equation

$$\begin{aligned}
\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc}-(n-n_f)w^e(z)-n_fcm_f &= \frac{\lambda}{m_f}\pi^{nc}\\
-\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc}m_f+(n-n_f)w^e(z)m_f+n_fcm_f^2 &= -\lambda\pi^{nc}\\
n_fcm_f^2-\left[\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc}-(n-n_f)w^e(z)\right]m_f+\lambda\pi^{nc} &= 0
\end{aligned} \tag{40}$$

$$m_f=\frac{\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc}-(n-n_f)w^e(z)-\sqrt{\left[\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc}-(n-n_f)w^e(z)\right]^2-4\lambda\pi^{nc}n_fc}}{2cn_f} \tag{41}$$

$$\begin{aligned}
\frac{\partial m_f}{\partial l_i}=\frac{1}{2cn_f}\Bigg\{\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\frac{\partial\pi^{nc}}{\partial l_i}-\frac{1}{2}\Bigg\{\left[\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc}-(n-n_f)w^e(z)\right]^2-4\lambda\pi^{nc}n_fc\Bigg\}^{-1/2}\\
2\left[\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc}-(n-n_f)w^e(z)\right]\left[\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\frac{\partial\pi^{nc}}{\partial l_i}-4\lambda n_fc\frac{\partial\pi^{nc}}{\partial l_i}\right]\Bigg\}
\end{aligned} \tag{42}$$

Substituting in the F.O.C.,

$$\begin{aligned}
\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\frac{\partial\pi^{nc}}{\partial l_i}-\frac{1}{2}\Bigg\{\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\frac{\partial\pi^{nc}}{\partial l_i}-\frac{1}{2}\Bigg\{\left[\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc}-(n-n_f)w^e(z)\right]^2-\\
4\lambda\pi^{c}n_fc\Bigg\}^{-1/2}2\left[\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc}-(n-n_f)w^e(z)\right]\\
\left[\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\frac{\partial\pi^{nc}}{\partial l_i}-4\lambda n_fc\frac{\partial\pi^{c}}{\partial l_i}\right]\Bigg\}=0
\end{aligned}$$

We can take the partial out to write,

$$\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)-\frac{1}{2}\Bigg\{\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)-\frac{1}{2}\Bigg\{\left[\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc}-(n-n_f)w^e(z)\right]^2- \\ 4\lambda\pi^c n_f c\Bigg\}^{-1/2} 2\left[\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc}-(n-n_f)w^e(z)\right] \\ \left[\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)-4\lambda n_f c\right]\Bigg\}\frac{\partial\pi^{nc}}{\partial l_i}=0$$

This implies,

$$\frac{\partial\pi^{nc}}{\partial l_i}=0$$

Thus, the labor choice is not distorted!

B.3 Family Firm with Outside Managers: Part II

Let $\pi^f=\left(1-\frac{c(n-n_f)\phi}{w^e(z)}\right)\pi^{nc}-(n-n_f)w^e(z)$.
Thus, the problem becomes,

$$\max_{n,z,l}\frac{\pi^f(n,z,l)+\sqrt{\pi^f(n,z,l)^2-4\lambda\pi^{nc}(n,z,l)cn_f}}{2} \tag{43}$$

Important note: the domain is restricted to: ${\pi^f}^2\geq 4\lambda\pi^{nc}(n,z,l)cn_f$ and $\pi^f\geq 0$

Solving for optimal labor allocation,

$$n^{\alpha-1}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^\theta)^\rho+(n-n_f)(zl^\theta)^\rho\right]\right\}^{1/\rho-1}\theta l_i^{\rho\theta-1}z_i^\rho=w \tag{44}$$

and

$$n^{\alpha-1}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f}(z_i l_i^\theta)^\rho+(n-n_f)(zl^\theta)^\rho\right]\right\}^{1/\rho-1}\theta l^{\rho\theta-1}z^\rho=w \tag{45}$$

Simplifying 44 and 45, we get,

$$l_i = l_1 \left(\frac{z_i}{z_1}\right)^{\frac{\rho}{1-\rho\theta}} \tag{46}$$

Subs 46 in 44,

$$\begin{aligned}
n^{\alpha-1}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f} z_i^{\rho} l_1^{\theta\rho}\left(\frac{z_i}{z_1}\right)^{\frac{\theta\rho^2}{1-\rho\theta}} + (n-n_f)z^{\rho} l_1^{\theta\rho}\left(\frac{z}{z_1}\right)^{\frac{\theta\rho^2}{1-\rho\theta}}\right]\right\}^{1/\rho-1} \theta l_1^{\rho\theta-1} z_1^{\rho} &= w \\
n^{\alpha-1}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f} z_i^{\frac{\rho}{1-\theta\rho}} l_1^{\theta\rho} z_1^{-\frac{\theta\rho^2}{1-\rho\theta}} + (n-n_f)z^{\frac{\rho}{1-\theta\rho}} l_1^{\theta\rho} z_1^{-\frac{\theta\rho^2}{1-\rho\theta}}\right]\right\}^{1/\rho-1} \theta l_1^{\rho\theta-1} z_1^{\rho} &= w \\
n^{\alpha-1}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f} z_i^{\frac{\rho}{1-\theta\rho}} + (n-n_f)z^{\frac{\rho}{1-\theta\rho}}\right]\right\}^{1/\rho-1} l_1^{\theta-1} z_1^{-\frac{\theta\rho(1-\rho)}{(1-\rho\theta)}+\rho}\theta &= w \\
n^{\alpha-1}\left\{\frac{1}{n}\left[\sum_{i=1}^{n_f} z_i^{\frac{\rho}{1-\theta\rho}} + (n-n_f)z^{\frac{\rho}{1-\theta\rho}}\right]\right\}^{1/\rho-1} l_1^{\theta-1} z_1^{\frac{\rho(1-\theta)}{(1-\rho\theta)}}\theta &= w
\end{aligned} \tag{47}$$

Let $\bar{z}(z,n) = \left\{\frac{1}{n}\left[\sum_{i=1}^{n_f} z_i^{\frac{\rho}{1-\theta\rho}} + (n-n_f)z^{\frac{\rho}{1-\theta\rho}}\right]\right\}$.
or
$\bar{z}(z,n) = \left\{\frac{n_f}{n}z_f + \frac{n-n_f}{n}z^{\frac{\rho}{1-\theta\rho}}\right\}$ where $z_f = \frac{1}{n_f}\sum_{i=1}^{n_f} z_i^{\frac{\rho}{1-\theta\rho}}$
Thus,

$$n^{\alpha-1}\bar{z}(z,n)^{1/\rho-1} l_1^{\theta-1} z_1^{\frac{\rho(1-\theta)}{(1-\rho\theta)}}\theta \quad = \quad w \tag{48}$$

or

$$\begin{aligned}
l_1^{1-\theta} &= \frac{\theta}{w} z_1^{\frac{\rho(1-\theta)}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{1-\rho}{\rho}} n^{\alpha-1} \\
l_1 &= \left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}} z_1^{\frac{\rho}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{1-\rho}{\rho(1-\theta)}} n^{\frac{\alpha-1}{1-\theta}}
\end{aligned} \tag{49}$$

More generally,

$$l_i = \left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}} z_i^{\frac{\rho}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{1-\rho}{\rho(1-\theta)}} n^{\frac{\alpha-1}{1-\theta}} \tag{50}$$

Let the solution to max 43 be $n^*(n_f, z_f)$ and $z^*(n_f, z_f)$.
Subs 50 in π^{nc}

$$\left(n^\alpha \left\{ \frac{1}{n} \left[\sum_{i=1}^{n_f} (z_i l_i^\theta)^\rho + (n - n_f)(z l^\theta)^\rho \right] \right\}^{1/\rho} - \sum_{i=1}^{n_f} l_i w - (n - n_f) l w \right) \tag{51}$$

$$\begin{aligned}
l_i &= \left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}} z_i^{\frac{\rho}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{1-\rho}{\rho(1-\theta)}} n^{\frac{\alpha-1}{1-\theta}} \\
l_i^\theta &= \left(\frac{\theta}{w}\right)^{\frac{\theta}{1-\theta}} z_i^{\frac{\rho\theta}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{(1-\rho)\theta}{\rho(1-\theta)}} n^{\frac{(\alpha-1)\theta}{1-\theta}} \\
z_i l_i^\theta &= \left(\frac{\theta}{w}\right)^{\frac{\theta}{1-\theta}} z_i^{\frac{1}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{(1-\rho)\theta}{\rho(1-\theta)}} n^{\frac{(\alpha-1)\theta}{1-\theta}} \\
(z_i l_i^\theta)^\rho &= \left(\frac{\theta}{w}\right)^{\frac{\rho\theta}{1-\theta}} z_i^{\frac{\rho}{(1-\rho\theta)}} \bar{z}(z,n)^{\frac{(1-\rho)\theta}{1-\theta}} n^{\frac{(\alpha-1)\theta\rho}{1-\theta}} \\
\frac{1}{n} \left[\sum_{i=1}^{n_f} (z_i l_i^\theta)^\rho + (n - n_f)(z l^\theta)^\rho \right] &= \left(\frac{\theta}{w}\right)^{\frac{\rho\theta}{1-\theta}} \bar{z}(z,n)^{\frac{(1-\rho)\theta}{1-\theta}} n^{\frac{(\alpha-1)\theta\rho}{1-\theta}} \bar{z}(z,n) \\
\frac{1}{n} \left[\sum_{i=1}^{n_f} (z_i l_i^\theta)^\rho + (n - n_f)(z l^\theta)^\rho \right] &= \left(\frac{\theta}{w}\right)^{\frac{\rho\theta}{1-\theta}} \bar{z}(z,n)^{\frac{1-\rho\theta}{1-\theta}} n^{\frac{(\alpha-1)\theta\rho}{1-\theta}} \\
\left\{ \frac{1}{n} \left[\sum_{i=1}^{n_f} (z_i l_i^\theta)^\rho + (n - n_f)(z l^\theta)^\rho \right] \right\}^{1/\rho} &= \left(\frac{\theta}{w}\right)^{\frac{\theta}{1-\theta}} \bar{z}(z,n)^{\frac{1-\rho\theta}{(1-\theta)\rho}} n^{\frac{(\alpha-1)\theta}{1-\theta}} \\
n^\alpha \left\{ \frac{1}{n} \left[\sum_{i=1}^{n_f} (z_i l_i^\theta)^\rho + (n - n_f)(z l^\theta)^\rho \right] \right\}^{1/\rho} &= \left(\frac{\theta}{w}\right)^{\frac{\theta}{1-\theta}} \bar{z}(z,n)^{\frac{1-\rho\theta}{(1-\theta)\rho}} n^{\frac{\alpha-\theta}{1-\theta}}
\end{aligned}$$

Thus,

$$\pi^{nc} = \left(n^\alpha \left\{ \frac{1}{n} \left[\sum_{i=1}^{n_f} (z_i l_i^\theta)^\rho + (n - n_f)(z l^\theta)^\rho \right] \right\}^{1/\rho} - \sum_{i=1}^{n_f} l_i w - (n - n_f) l w \right)$$

$$\begin{aligned}
&= \left(\frac{\theta}{w}\right)^{\frac{\theta}{1-\theta}} \bar{z}(z,n)^{\frac{1-\rho\theta}{(1-\theta)\rho}} n^{\frac{\alpha-\theta}{1-\theta}} - w \left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}} \bar{z}(z,n)^{\frac{1-\rho\theta}{(1-\theta)\rho}} n^{\frac{\alpha-\theta}{1-\theta}} \\
&= \left(\frac{1}{w}\right)^{\frac{\theta}{1-\theta}} \bar{z}(z,n)^{\frac{1-\rho\theta}{(1-\theta)\rho}} n^{\frac{\alpha-\theta}{1-\theta}} \left(\theta^{\frac{\theta}{1-\theta}} - \theta^{\frac{1}{1-\theta}}\right) \qquad (52)
\end{aligned}$$

$\bar{z}(z,n) = \left\{ \frac{n_f}{n} z'_f + \frac{n-n_f}{n} z^{\frac{\rho}{1-\theta\rho}} \right\}$ where $z'_f = \frac{1}{n_f} \sum_{i=1}^{n_f} z_i^{\frac{\rho}{1-\theta\rho}}$

For clear interpretation, lets relabel stuff: $z'_f = (z_f)^{\frac{\rho}{1-\theta\rho}}$ or $z_f = (z'_f)^{\frac{1-\theta\rho}{\rho}}$

Thus, $\bar{z}(z,n) = \left\{ \frac{n_f}{n} z_f^{\frac{\rho}{1-\theta\rho}} + \frac{n-n_f}{n} z^{\frac{\rho}{1-\theta\rho}} \right\}$ where $z_f = \left(\frac{1}{n_f} \sum_{i=1}^{n_f} z_i^{\frac{\rho}{1-\theta\rho}} \right)^{\frac{1-\theta\rho}{\rho}}$

B.4 Professional Firms

Lets say the professional firm of type z wants to deviate and hire an ϵ number of managers of type z_1.

$$\pi(\epsilon) = \left(1 - \frac{n\phi}{w(z)} - \frac{\epsilon\phi}{w(z_1)}\right)\left((n+\epsilon)^{\alpha}\left\{\frac{1}{n+\epsilon}\left[nz^{\rho} + \epsilon z_1^{\rho}\right]\right\}^{1/\rho}\right) - nw(z) - \epsilon w(z_1) \tag{53}$$

$$n = \left[\frac{\alpha-1}{\alpha^2}\frac{1}{c\phi}z\right]^{\frac{1}{2-\alpha}} \tag{54}$$

$$\begin{aligned} \frac{\partial \pi(\epsilon)}{\partial \epsilon} = {} & \frac{-\phi}{w(z_1)}(n+\epsilon)^{\alpha-\frac{1}{\rho}}\left[nz^{\rho} + \epsilon z_1^{\rho}\right]^{1/\rho} - w(z_1) + \left(1 - \frac{n\phi}{w(z)} - \frac{\epsilon\phi}{w(z_1)}\right) \\ & \left[\alpha - \frac{1}{\rho}(n+\epsilon)^{\alpha-\frac{1}{\rho}-1}[nz^{\rho} + \epsilon z_1^{\rho}]^{1/\rho} + (n+\epsilon)^{\alpha-\frac{1}{\rho}}\frac{z^{\rho}}{\rho}[nz^{\rho} + \epsilon z_1^{\rho}]^{1/\rho-1}\right] \end{aligned} \tag{55}$$

$$\begin{aligned} \frac{\partial \pi(\epsilon)}{\partial \epsilon} = {} & (n+\epsilon)^{\alpha-\frac{1}{\rho}}\left[nz^{\rho} + \epsilon z_1^{\rho}\right]^{1/\rho}\left\{\frac{-\phi}{w(z_1)} + \left(1 - \frac{n\phi}{w(z)} - \frac{\epsilon\phi}{w(z_1)}\right)\right. \\ & \left.\left[\left(\alpha - \frac{1}{\rho}\right)(n+\epsilon)^{-1} + \frac{z_1^{\rho}}{\rho}[nz^{\rho} + \epsilon z_1^{\rho}]^{-1}\right]\right\} - w(z_1) \end{aligned} \tag{56}$$

$$\begin{aligned} \lim_{\epsilon \to 0} \frac{\partial \pi(\epsilon)}{\partial \epsilon} = {} & n^{\alpha-\frac{1}{\rho}}\left[nz^{\rho}\right]^{1/\rho}\left\{\frac{-\phi}{w(z_1)} + \left(1 - \frac{n\phi}{w(z)}\right)\right. \\ & \left.\left[\left(\alpha - \frac{1}{\rho}\right)n^{-1} + \frac{z_1^{\rho}}{\rho}[nz^{\rho}]^{-1}\right]\right\} - w(z_1) \end{aligned} \tag{57}$$

$$\lim_{\epsilon\to 0}\frac{\partial\pi(\epsilon)}{\partial\epsilon} = n^{\alpha}z\Bigg\{\frac{-\phi}{w(z_1)} + \left(1-\frac{n\phi}{w(z)}\right)$$
$$\left[\left(\alpha-\frac{1}{\rho}\right)n^{-1} + \frac{z_1^{\rho}}{\rho}[nz^{\rho}]^{-1}\right]\Bigg\} - w(z_1) \tag{58}$$

$$\lim_{\epsilon\to 0}\frac{\partial\pi(\epsilon)}{\partial\epsilon} = n^{\alpha}z\left\{\frac{-\phi}{w(z_1)} + \left(1-\frac{n\phi}{w(z)}\right)\left[\left(\alpha-\frac{1}{\rho}\right)\frac{1}{n} + \frac{1}{\rho n}\left(\frac{z_1}{z}\right)^{\rho}\right]\right\} - w(z_1) \tag{59}$$

Subs n and $w(z)$ and noting $k = \frac{\alpha-1}{\alpha}\frac{1}{\phi}$

$$\lim_{\epsilon\to 0}\frac{\partial\pi(\epsilon)}{\partial\epsilon} = k^{\frac{\alpha}{2-\alpha}}z^{\frac{2}{2-\alpha}}\left\{-\alpha\phi k^{\frac{\alpha-1}{\alpha-2}}z_1^{\frac{1}{\alpha-1}} + \frac{k^{\frac{1}{\alpha-2}}z^{\frac{1}{\alpha-2}}}{\alpha}\left[\left(\alpha-\frac{1}{\rho}\right) + \frac{1}{\rho}\left(\frac{z_1}{z}\right)^{\rho}\right]\right\} - \frac{1}{\alpha}k^{\frac{\alpha-1}{2-\alpha}}z_1^{\frac{1}{2-\alpha}} \tag{60}$$

Let $\lambda = \frac{z_1}{z}$

$$\lim_{\epsilon\to 0}\frac{\partial\pi(\epsilon)}{\partial\epsilon} = k^{\frac{\alpha-1}{2-\alpha}}z^{\frac{1}{2-\alpha}}\left\{-\alpha\phi k\lambda^{\frac{1}{\alpha-2}} + \frac{1}{\alpha}\left(\alpha-\frac{1}{\rho}+\frac{\lambda^{\rho}}{\rho}-\lambda^{\frac{1}{2-\alpha}}\right)\right\} \tag{61}$$

$$\lim_{\epsilon\to 0}\frac{\partial\pi(\epsilon)}{\partial\epsilon} = k^{\frac{\alpha-1}{2-\alpha}}z^{\frac{1}{2-\alpha}}\left\{1+\frac{\lambda^{\rho}}{\alpha\rho} - \left(\frac{1}{\alpha\rho}+\frac{\lambda^{\frac{1}{2-\alpha}}}{\alpha}+\alpha\phi k\lambda^{\frac{1}{\alpha-2}}\right)\right\} \tag{62}$$

$$\lim_{\epsilon\to 0}\frac{\partial\pi(\epsilon)}{\partial\epsilon} = k^{\frac{\alpha-1}{2-\alpha}}z^{\frac{1}{2-\alpha}}\left\{1+\frac{\lambda^{\rho}}{\alpha\rho} - \left[\frac{1}{\alpha\rho}+\frac{\lambda^{\frac{1}{2-\alpha}}}{\alpha}+\left(1-\frac{1}{\alpha}\right)\lambda^{\frac{1}{\alpha-2}}\right]\right\} \tag{63}$$

Taking the term inside brackets,

$$\pi_1(\lambda) = \left\{1+\frac{\lambda^{\rho}}{\alpha\rho} - \left[\frac{1}{\alpha\rho}+\frac{\lambda^{\frac{1}{2-\alpha}}}{\alpha}+\left(1-\frac{1}{\alpha}\right)\lambda^{\frac{1}{\alpha-2}}\right]\right\} \tag{64}$$

Now, we want to show that $\pi_1(\lambda) \leq 0$. If we find the λ_{max} associated with max of

$\pi_1(\lambda)$ and show that at that $\pi_1(\lambda_{max}) \leq 0$, we are done(?)

$$\frac{\partial \pi_1(\lambda)}{\partial \lambda} = \frac{\lambda^{\rho-1}}{\alpha} - \frac{\lambda^{\frac{\alpha-1}{2-\alpha}}}{\alpha(2-\alpha)} - \frac{(\alpha-1)\lambda^{\frac{3-\alpha}{\alpha-2}}}{\alpha(\alpha-2)} \tag{65}$$

$\frac{\partial \pi_1}{\partial \lambda}\Big|_{\lambda=1} = 0$ and $\frac{\partial^2 \pi_1}{\partial \lambda^2}\Big|_{\lambda=1} < 0$

$$\pi_1(1) = \left\{1 + \frac{1}{\alpha\rho} - \left[\frac{1}{\alpha\rho} + \frac{1}{\alpha} + \left(1 - \frac{1}{\alpha}\right)\right]\right\} = 0 \tag{66}$$

Thus,

$$\lim_{\epsilon \to 0} \frac{\partial \pi(\epsilon)}{\partial \epsilon} \leq 0 \tag{67}$$