A Contest Model of Talking and Listening*

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Abstract

Advances in communication technology have increased the effective sizes of meetings. Online meetings enable more participants to join and asynchronicity (such as SMS chat groups) amplifies this effect further. Through a simple model in which agents compete in a winner-take-all contest for a prize that scales with the size of the network, I investigate how the cost of communication, network size, and asymmetries in baseline speaking qualities between conversation participants might contribute to participation inequality. I first identify a unique mixed strategy Nash equilibrium in contests where all agents are symmetric in their average speaking quality. I then highlight an equilibrium strategy profile in contests with asymmetric agents whereby agents with the highest baseline speaking qualities will always choose to talk and all others will choose to listen. I conclude by extending the analysis in (Schaffer, 1988) to finite contests with asymmetric agents and present computational results that suggest conditions under which this equilibrium is both unique and evolutionarily stable.

1 Introduction

Simply including many individuals in a meeting does not imply that everyone's voice is heard. Communication involves both *talking* and *listening*, and, in many groups, only a small subset of participants speak. The asymmetry between talking and listening may vary with group size and the cost of communication. An in-person meeting with a few individuals may encourage more equitable participation whereas a larger online meeting or conference may concrete a small number of "talkers". Improvements in communication technologies has enabled meeting sizes to increase: in-person meetings typically limit the group size whereas online meetings allow for a large number of participants (N). Moreover, online meetings involve elements of asynchronicity which facilitate large-N group interactions: for example, a Zoom, Microsoft Teams or Google Meet meeting have associated chat windows where participants can continue to talk asynchronously after the (synchronous) video meeting has ended. Similarly, SMS or message-based group conversations are completely asynchronous while subreddits and online groups can be thought of as asynchronous mini-communities.

While technologies like video calling, social media, and instant messaging offer enhanced social connectivity due to the ease of communication, participation inequality in online groups remains a prominent concern across various academic disciplines (Shaw and Hargittai, 2018; Gasparini et al., 2020). Illustratively, the "90-9-1" rule posits that 90% of online community members do not actively

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engage in discussions or content creation, with 9% responding to existing content and a mere 1% contributing new material (Nielsen, 2006). Although the precise distribution of active members to "lurkers" may vary based on community size and type, such disparities challenge the notion that online communities foster greater diversity compared to their offline counterparts. Additionally, this has important consequences for the direction of innovation, as social forces have been highlighted to be a determinant of whether one paradigm is favored ahead of alternatives (Acemoglu, 2023; Einiö et al., 2019). This is of growing concern as firms continue their transition to online businesses and start to cultivate new ideas from social media and other online platforms. Idea generation and knowledge diffusion are key inputs to the innovation process and when only a few voices are heard, new products and scientific advancements are at risk of benefiting an increasingly smaller subset of people. Understanding the avenues through which differences in online participation arise can therefore help policy makers, firms, and platform designers alike prevent social technology from amplifying existing inequalities and improve the potential impact of new advancements.

In this paper, I model communication within a group as a winner-take-all contest. Agents decide whether to *talk* (at a cost) or to *listen* (which is costless). The agent with the highest realized *speaking quality* (which is stochastic) wins the contest and everyone listens to her. The prize (winning this contest) scales with the size of the group, N. I first consider contests where all agents are symmetric in their baseline speaking qualities. This is a useful benchmark for introducing the model and by construction every agent shares the same propensity to speak in the mixed strategy equilibrium.

The model becomes interesting once I introduce heterogeneity so that agents can be ranked by their baseline speaking quality. In my preliminary analysis I show that in equilibrium agents will either decide to speak or listen with certainty. I focus on an intuitive class of equilibria where only the top agents choose to talk. I present computational results that suggest conditions under which this class of equilibria are both unique and evolutionarily stable. I show that the number of "talkers" increases with N but less than proportionally. Therefore, the proportion of speakers in any group grows increasingly concentrated as the group size grows. This suggests that in the presence of asymmetries, online communication technology may exacerbate participation inequalities. While this model is simple and does not model complex group dynamics (such as multiple rounds of communication or fluctuations in individual influence) as well as different contest structures (such as ones with proportional-payoffs), it provides a motivating example and springboard for later work.

My analysis also extends the literature on contest models and winner-take-all games by expanding to games that include asymmetric agents that interact within a large population and contests of finite size. Similar models where symmetric agents compete in 2 or N-player contests have been explored in the context of games in which agents must decide whether to "enter" the contest or exert a certain amount of effort (Cason et al., 2010; Laferrière et al., 2023; Fischbacher and Thöni, 2008). Additionally, (Cason et al., 2020) explores games where agents are asymmetric in their costs, building off of work in (Fang, 2002) which compares lottery and all-pay auction models in lobbying games of N players with asymmetric valuations. I derive similar theoretical results in both the N-player symmetric and asymmetric cases, but modify the source of the asymmetry by allowing them to arise from players' perceived speaking quality (which can be read as their "ability" in other works). I then expand these results to determine conditions under which the strategy profile where top agents always choose to compete or "enter" the discussion and all others always refrain is evolutionarily stable by drawing inspiration from (Schaffer, 1988). I include extensions to finite groups drawn from the overall population and show that the same general results hold.

The paper is presented as follows: Section 2 introduces the model, Section 3 investigates the game where all agents are symmetric in their average speaking quality, and Section 4 considers equilibria in games where agents are asymmetric in their average speaking quality, introduces conditions for determining the evolutionary stability of the strategy profile with agents that only talk or listen, and presents computational results. A discussion of results, modeling drawbacks, and further analyses is included in Section 5.

2 Contest Model

The game is modeled as a one-shot, winner-take-all game with N agents in a fully-connected network, each denoted as $A_1, A_2, ..., A_N$. Thus, each agent has $N-1$ neighbors. Prior to starting, every agent draws their "speaking quality", q_i, from some unknown distribution, F. I assume for simplicity that these values are drawn i.i.d from the uniform distribution over $[\epsilon_i, \alpha]$, where ϵ_i is a parameter that modulates each agent's *average speaking quality* and $0 < \epsilon_i < \alpha$. I denote the ϵ_i parameter as the agent's *baseline speaking quality.*

Agents must choose between talking and listening in any round. Agent i will choose to talk with probability γ_i and listen with probability $\lambda_i = 1 - \gamma_i$. If an agent chooses to listen, they receive a payoff of 0. If they choose to talk, they enter a lottery with all other agents that have chosen to talk. The winner of the lottery is the agent with the highest realized speaking quality, q_i. An agent's probability of winning (should they choose to talk) is dependent on their realized speaking quality relative to the realized speaking qualities of all other talker. Agents must therefore take into account the set of baseline speaking qualities of all T talkers in the round, denoted as E_T. Should an agent win the round, their payoff is $h(L) - c(N)$ where $h(L)$ is the benefit derived from the number of listeners, L. Since agents can either choose to talk or listen in the network, $L = N - T$, where T is the number of talkers. $c(N)$ is the cost associated with talking, which varies depending on the overall size of the network. Should an agent lose the round, their payoff is $-c(N)$. Thus, agents in the game are incentivized to talk if there are more potential listeners but disincentivized to speak if a large amount of their neighbors ultimately choose to speak as well. The expected payoff to talking and listening for agent i can be represented as

$$\begin{aligned} \pi_i(Talk) &= (h(L) - c(N)) \cdot P(A_i \text{ wins}|E_T) - c(N) \cdot (1 - P(A_i \text{ wins}|E_T)) & (1) \\ \pi_i(Listen) &= 0 & (2) \end{aligned}$$

and the objective function for each agent can be represented as

$$\arg\max_{\gamma_i} \pi_i = \gamma_i \cdot h(L)P(A_i \text{ wins}|E_T) - (1 - \gamma_i) \cdot c(N)(1 - P(A_i \text{ wins}|E_T)) \quad (3)$$

For the rest of this analysis, I consider the simple case where $h(L) = L = N - T$ and $c(N) = \beta$, where $0 < \beta < N - 1$ is a constant.

2.1 Model Interpretation

Participants in this model are assumed to desire an audience when they speak, and therefore view a larger group as yielding a greater potential prize. Individuals' desire for an audience in online groups has been explored in past literature and I take that fact as a given for this model (Toubia and Stephen, 2013). However, since agents must choose between speaking and listening in any round, agents consider the number of potential listeners in the round, and must account for the decrease in the total prize as more agents choose to talk. The winner-take-all objective of the game models the idea that a single agent can be listened to by the network in a single round or time period, and all listeners will choose to allocate their attention towards individuals that have the highest speaking quality.

The speaking quality of agents is chosen to be a random variable to model the fact that while certain agents command greater influence on most conversations, the nature of discussion is variable, allowing others to potentially realize a higher speaking quality depending on the topic at hand. However, each agent also has a personal baseline speaking quality, ϵ_i, which modulates their average speaking quality. Sociological and behavioral literature has highlighted factors that induce participation inequalities in groups such as perceived status and individual member and group characteristics (Skvoretz and Fararo, 1996; Bonito and Hollingshead, 1997). This baseline parameter can therefore also be viewed as an agents' perceived status or ability relative to the rest of the group to reflect the social factors and potential stratification that might contribute to participation inequality.

Finally, the cost of talking can be thought of as the cost of propagating a message to any individual while the cost of listening can be thought of as the cost of processing a message from another individual. These costs can depend on a variety of factors including, but not limited to, financial cost, time, and mental effort. In this view, one can think of the cost of talking as becoming more constant as digital communication technologies have proliferated; the effort expended in talking to multiple people is the same as talking to a single person since the same message can be duplicated and sent instantaneously to all parties. This stands in contrast to communication without such technology, as individuals must exert increasing effort in relaying the same message to multiple individuals when duplication and instant transmission are not possible. Choosing $c(N) = \beta$ models the constant cost that online communication technology provides. Without the presence of such technology, one might instead choose to model $c(N) = \alpha N$, where the cost of talking increases with the size of the network. From a modeling standpoint, $\beta = 1$ reflects the idea that agents derive the same utility from a single listener as the cost they expend speaking to them. Values of $\beta > 1$ indicate that agents exert more effort gaining a single listener while $\beta < 1$ implies that agents find it easier to speak rather than listen. $\beta < N - 1$ since there are at most $N - 1$ listeners in a group and if the cost equals or exceeds this amount, agents will not achieve a positive payoff even if they win the round.

3 Contests with Symmetric Agents

I begin by solving for equilibrium strategies in the symmetric case, where all agents have the same baseline speaking quality. In this case, we see that $\epsilon_1 = \epsilon_2 = ... = \epsilon_N$, but the realized speaking qualities in any given round may differ between agents. Since each agent shares the same distribution over speaking qualities, any agent's chance of winning the lottery, should they choose to compete, is $\frac{1}{T} = \frac{1}{X+1}$, where T is the number of "talkers" in the network and X is the number of Agent i's *neighbors* that choose to talk ($T = X + 1$). Recall that E_T is the set of baseline speaking qualities for all T talkers in the network.

There are no symmetric pure NE in the game. If every agent chooses to listen, any agent is better off deviating to talking and receiving payoff $N - 1 - \beta > 0$. Likewise, if every agent chooses to talk, the expected payoff for all agents is $-\beta$. Therefore, any agent is better off deviating to listening. However, there is a unique, symmetric MSNE where every agent will choose to talk with probability $\gamma \in [0, 1]$. In this case, agents are indifferent between speaking and listening when the expected payoff to talking is equal to the expected payoff to listening. Thus, we can solve for γ^*, which can be viewed as agent i's equilibrium probability of talking in any given round:

$$
\begin{aligned}
E[\pi_i(Talk)] &= E[\pi_i(Listen)] && (4)\\
\sum_{x=0}^{N-1}\left[(N-1-x-\beta)P(A_i \text{ wins}|E_{x+1}) - \beta(1-P(A_i \text{ wins}|E_{x+1}))\right]P(X=x) &= 0 && (5)\\
\sum_{x=0}^{N-1}\left[(N-1-x)P(A_i \text{ wins}|E_{x+1})P(X=x)\right] - \beta &= 0 && (6)\\
\sum_{x=0}^{N-1}(N-1-x)\frac{1}{x+1}\binom{N-1}{x}\gamma^x(1-\gamma)^{N-1-x} &= \beta && (7)\\
\sum_{x=0}^{N-1}(N-(x+1))\frac{1}{x+1}\binom{N-1}{x}\gamma^x(1-\gamma)^{N-(x+1)} &= \beta && (8)\\
\sum_{x=0}^{N-1}(N-(m+1))\frac{1}{m+1}\binom{N-1}{x}\gamma^x(1-\gamma)^{N-(x+1)} &= \beta && (9)\\
\sum_{x=0}^{N-1}\frac{1-\gamma}{\gamma}\binom{N-1}{m+1}\gamma^{x+1}(1-\gamma)^{(N-1)-(x+1)} &= \beta && (10)\\
\frac{1-\gamma}{\gamma}\left[\sum_{z=0}^{N}\binom{N-1}{z}\gamma^z(1-\gamma)^{(N-1)-(z)} - \binom{N-1}{0}\gamma^0(1-\gamma)^{N-1-0}\right] &= \beta && (11)\\
\frac{1-\gamma}{\gamma}[1-(1-\gamma)^{N-1}] &= \beta && (12)
\end{aligned}
$$

Solving (12) provides us with γ^*. Figure 1 plots (12) as a function of γ as well as numerical solutions for different values of N and β. We see that, holding β fixed, as N increases, γ^* stabilizes with higher values of β resulting in lower values of γ^*, and vice-versa. Moreover, we see that as network sizes grow unboundedly large,

$$
\begin{aligned}
\lim_{N\to\infty}\frac{1-\gamma}{\gamma}[1-(1-\gamma)^{N-1}] &= \lim_{N\to\infty}\beta && (13)\\
\frac{1-\gamma}{\gamma} &= \beta && (14)\\
\frac{1}{\gamma} &= \beta+1 && (15)\\
\gamma^* &= \frac{1}{\beta+1} && (16)
\end{aligned}
$$

$\gamma^* = \frac{1}{\beta+1}$ matches the long run behavior we observe in Figure 1. Using this, we find that the expected number of talkers in the network is $\approx N\gamma^* = \frac{N}{\beta+1}$. Implications of this results are intuitive: the propensity of participation by any agent and the expected number of talkers increases with the network size and decreases with the cost of talking. The high amount of participation despite increasingly low odds of winning corroborates findings in (Laferrière et al., 2023) and (Fischbacher and Thöni, 2008) which investigate the puzzling amount excess entry in winner-take-all games where the total prize increases with contest size and the probability of winning decreases with contest size. The equilibrium propensity to speak highlights this result and provides a useful connection back to the literature.

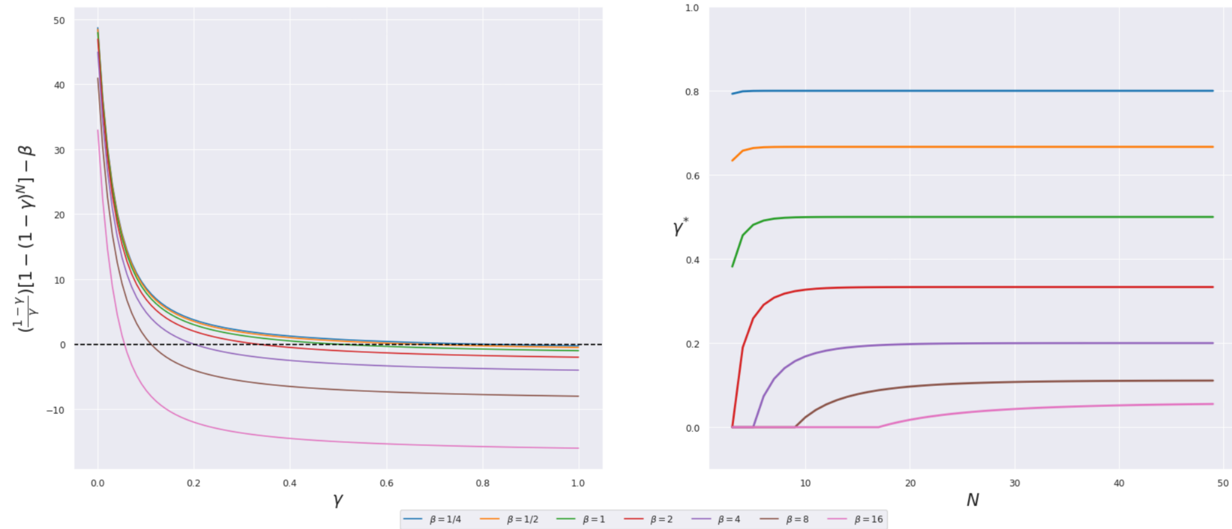

Figure 1: (Left) Equation 12 as a function of $0 < \gamma < 1$. Values calculated with $N = 50$ and $\beta = \{\frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16\}$. **(Right)** γ as a function of N and β. Values for γ are found by solving Eq. (12) for values of $\beta = \{\frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16\}$ and values of $N = \{3, ..., 50\}$.

4 Contests with Asymmetric Agents

I build upon the results in Section 3 by now considering the case where all agents differ in their baseline speaking quality. Therefore, it follows that $\epsilon_i \neq \epsilon_j$ for $j \neq i$. Further, since every agent has a different ϵ-value, we use this to construct an ordering of agents based on their respective distributions. That is, agent i is the agent with the i-th highest ϵ value and $0 < \epsilon_N < \epsilon_{N-1} < ... < \epsilon_1 < \alpha$.

Using this setup, I first identify one equilibrium strategy profile in the game: agents with the highest average speaking values will always choose to speak while their counterparts always choose to listen. I then discuss conditions for finding this equilibrium, which I denote as identifying a "marginal agent" in the network. Sections 4.3 and 4.4 then outline conditions for determining whether this equilibrium strategy profile is evolutionary stable and *how* stable individual strategies are. These sections heavily build off of the work in (Schaffer, 1988), which I include an overview of in Appendix 7.2. Section 4.5 presents computational results.

4.1 Equilibria

We first note that no mixed strategy Nash equilibria exist in this game. It is clear from the setup of the expected payoff functions (Eq. (1) and Eq. (2)) that the agent with the highest baseline speaking quality (and therefore highest average speaking quality), A_1, has the highest probability of winning and therefore the greatest incentive to speak. Should they choose to speak, all other agents' probability of speaking will decrease since there will be a smaller potential payoff and the chances of winning the round will be decreased. Therefore, since $\beta < N - 1$, A_1 will eventually choose to speak all the time ($\gamma_1 = 1$) since it offers a higher expected payoff than any randomized strategy or choosing to listen. If all subsequent agents choose to randomize between talking and listening, then A_2 can take advantage of the randomization and receive a higher payoff by choosing to speak all the time ($\gamma_2 = 1$), should their expected payoff exceed their expected payoff to listening given that A_1 will always speak. Similarly, A_3 will make a decision to speak all the time ($\gamma_3 = 1$) knowing that if they have a positive incentive to talk, then A_1 and A_2 will also chose to speak all the time as well since they have a greater chance of winning and therefore a greater incentive to speak.

Agents will therefore face the decision to speak all the time in decreasing order of their ϵ values, taking into account that, if they choose to speak all the time, then agents with a higher average speaking quality will also choose to speak all the time as well since they have a greater incentive to do so. This process will continue until an agent, when accounting for the fact that higher agents will speak, does not have an expected payoff to talking that is greater than 0 and will therefore choose to listen. All agents with a lower speaking quality will face the same choice and a lower probability of winning and will also choose to listen all the time ($\gamma_i = 0$). We call the last agent that chooses to speak the *marginal agent*, where all agents up to and including the marginal agent will speak all the time and receive a positive expected payoff and all other agents will choose to listen and receive a payoff of 0. Should the agents $i \leq m$ choose to deviate to listening, they will receive a payoff of 0, which is strictly less than the positive expected payoff they would receive from talking. Further, should agents $i > m$ choose to deviate to speaking, they will receive a negative expected payoff due to the top agents that have already made the decision to speak all the time. Therefore, since no agent has an incentive to deviate from their pure strategy, this is a NE strategy profile with $\gamma_{i \leq m} = 1$ and $\gamma_{i>m} = 0$. This equilibrium is very similar to the one outlined in (Fang, 2002), which shows that the strategy profile in which only the agents with the topmost valuations have positive bids in the presented lottery model is a unique pure strategy equilibrium.

Finally, we note that since an equilibrium strategy profile cannot contain all talkers or all listeners without at least one agent having an incentive to deviate, there are at most $2^N - 2$ (since there cannot be an equilibrium with all talkers or all listeners) potential NE depending on the game setup. Strategy profiles apart from the one highlighted above include a mix of agents always choosing to speak while all others choose to listen. I focus on the case described above and outline steps for determining the marginal agent, but include a discussion of other identified pure strategy NE in Section 4.5. Overall, the results suggest that when the contest size and cost of talking are high enough, the above strategy profile is both unique and *evolutionarily stable*. However, if this is not the case, then other evolutionarily stable or non-evolutionarily stable pure strategy NE may be identified. Preliminary analysis through computational results (as presented in Section 4.5) suggest that this is the case for very small network sizes and costs although analysis is still ongoing. The rest of this section focuses on how to determine the marginal agent, characterizing and constructing conditions for evolutionary stability for this equilibrium, and concluding with computational results that provide motivation for later analytical work.

4.2 Determining the Index of the Marginal Agent

Given the identified equilibrium above, much of the analysis therefore hinges on identifying the index of the "marginal agent", m. In order to determine the index of the marginal agent, we need to determine the chance that the marginal agent will win if they choose to speak all the time. If the marginal agent decides to speak, then we know that all $m - 1$ agents with a higher average speaking quality will also choose to speak all the time. Let A_m be the marginal agent in that the expected payoffs for agents $i = 1, ..., m$ are ≥ 0, but the expected payoff for agents $i = m + 1, .., N$ are less than 0. Since all agents $i \leq m$ can achieve a positive expected payoff to talking, they will always choose to talk while all agents $i > m$ will always choose to listen. Agents $i > m$ do not have any incentive to deviate from listening all the time since their maximum expected payoff to talking will be less than 0. Similarly, all agents $i \leq m$ do not have an incentive to deviate from talking since their expected payoff to talking will be strictly greater than their payoff to listening. Thus, we modify Eq. (1) to only account for the top m agents talking in order to determine the expected payoff to speaking for agent m. That is, $E_T = \{\epsilon_1, ..., \epsilon_m\}$. Then,

$$\begin{aligned} \pi_m(T) &= (N - m - \beta) \cdot P(A_m \text{ wins} | \{\epsilon_1, ..., \epsilon_m\}) - \beta \cdot P(A_m \text{ loses}) & (17) \\ &= (N - m) \cdot P(A_m \text{ wins} | \{\epsilon_1, ..., \epsilon_m\}) - \beta & (18) \end{aligned}$$

We can then find an expression for the probability of agent m winning given that the top m agents (including the marginal agent) talk:

$$\begin{aligned} P(A_m \text{ wins} | \{\epsilon_1, ..., \epsilon_m\}) &= \int_{\epsilon_1}^{\alpha} P(q_m = y) P(\max(\{q_1, ..., q_{m-1}\}) < y) dy & (19) \\ &= \int_{\epsilon_1}^{\alpha} \frac{1}{1 - \epsilon_m} \left(\frac{y - \epsilon_1}{1 - \epsilon_1} \right) \cdots \left(\frac{y - \epsilon_{m-1}}{1 - \epsilon_{m-1}} \right) dy & (20) \\ &= \left[\prod_{i=1}^{m} \frac{1}{1 - \epsilon_i} \right] \int_{\epsilon_1}^{\alpha} (y - \epsilon_1)(y - \epsilon_2) \cdots (y - \epsilon_{m-1}) dy & (21) \end{aligned}$$

Using this expression, we know that A_m will choose to talk all the time if the expected value to talking is greater than or equal to the expected value to listening. That is,

$$\begin{aligned} \pi_m(Talk) &> \pi_m(Listen) & (22) \\ (N - m) \cdot P(A_m \text{ wins} | \{\epsilon_1, ..., \epsilon_m\}) - \beta &> 0 & (23) \\ (N - m) \left(\prod_{i=1}^{m} \frac{1}{1 - \epsilon_i} \right) \int_{\epsilon_1}^{\alpha} (y - \epsilon_1)(y - \epsilon_2) \cdots (y - \epsilon_{m-1}) dy - \beta &> 0 & (24) \end{aligned}$$

Thus, we must find the maximum m such that (24) holds. This value of m will be the index of the "marginal agent." We see that the index of the marginal agent depends on the size of the network (N), the cost of talking β, and perhaps most interestingly, the distribution of the baseline speaking quality (ϵ) amongst all agents. While an analytical solution to (24) is not recovered, I include a discussion of a loose upper and lower bounds on the probability that A_m wins and their implications in Appendix 7.1.

4.3 Evolutionary Stability, Equilibrium Condition

To investigate the evolutionary stability of the equilibrium strategy profile in which agents with the topmost baseline speaking qualities always choose to talk while all others choose to listen, I first outline the equilibrium condition for an evolutionary stable strategy in the game consisting of N agents with asymmetric average speaking quality that engage in contests of size C. Let m denote the index of the marginal agent within any chosen contest, which is determined using (24). In the spirit of (Schaffer, 1988), beginning with a population of only ESS players, I consider the case when one player is removed and replaced with a mutated agent, who plays strategy s^M. Let s_i^{ESS} denote the evolutionary stable strategy of agent i and s_i^M denote the mutated strategy of agent i. I define agent-specific strategies since agents differ in their average speaking quality and will therefore have different expected payoffs depending on whether they choose to talk or listen. Therefore, the ESS strategy will be different for agents $i \leq m$ and $i > m$. It can be seen that the payoff for agent i, should they choose the evolutionarily stable strategy, can be written as

$$\pi_i^{ESS} = \left(1 - \frac{C-1}{N-1}\right)\pi_i(s_i^{ESS}|\vec{s}_{C-1}^{ESS}) + \left(\frac{C-1}{N-1}\right)\pi_i(s_i^{ESS}|\vec{s}_{C-2}^{ESS}, s_{j\neq i}^{M}) \tag{25}$$

Where $\vec{s}_{C-1}^{ESS}$, $\vec{s}_{C-2}^{ESS}$ is the vector of strategies taken by the other $N-1$ or $N-2$ agents in the group, respectively. Should agent i choose the mutated strategy, they will be the lone mutant in the group while all other agents play their ESS strategy. Therefore, their payoff will be

$$\pi_i^M = \pi_i(s_i^M|\vec{s}_{C-1}^{ESS}) \tag{26}$$

(Schaffer, 1988) denotes the equilibrium condition for an evolutionarily stable strategy as $\pi^{ESS} \geq \pi^M$. I extend this definition to an individual agent since we must consider that agents with asymmetric average speaking quality will have different expected payoffs. That is, agent i's strategy is evolutionarily stable if

$$\pi_i^{ESS} \geq \pi_i^M \tag{27}$$

Using this agent-specific equilibrium condition, we can determine when the strategy profile where the top m agents speak and the rest listen satsifies the equilibrium condition. In order to do this, we must separately consider agents $i \leq m$, that choose to speak all the time and agents $i > m$ that chooose to listen all the time.

4.3.1 Agents with index $i \leq m$

Consider the case where $i \leq m$. If all players follow the ESS strategy, then agents $i \leq m$ will choose to talk while all others choose to listen. Therefore, the ESS strategy for agent i is to talk while the mutated strategy where the agent chooses to listen, which provides a payoff of 0. Thus,

$$\pi_i^M = 0 \tag{28}$$

When determining the expected payoff to playing the ESS strategy in the face of a mutant player, we must account for the specific mutant player that agent i may face since agents are asymmetric. Let A_j be the mutant. Then, if $j \leq m$, then A_j's ESS is to speak and they will mutate to listening which will improve A_i's probability of winning and therefore expected payoff. Conversely, if $m < j$, then A_j's ESS is to listen and they will mutate to talking which will hurt the expected payoff to talking for A_i. There are $m-1$ potential talkers that may mutate, $N-m$ potential listeners that can mutate, and a $\frac{1}{N-1}$ chance of facing any potential mutant. Thus, we must find the expected payoff to talking for A_i, accounting for which agent actually mutates. We begin by first considering the case when $C = N$.

$$\begin{aligned}
\pi_i^{ESS} &= \pi_i(s_i^{ESS}|\vec{s}_{C-1}^{ESS}) && (29)\\
&= \left(\frac{1}{N-1}\right)\sum_{j\neq i, j\leq m} P(A_i \text{ win}|\{\epsilon_1, ..., \epsilon_m\} \setminus \{\epsilon_j\})(N-m+1) + \\
&\quad \left(\frac{1}{N-1}\right)\sum_{m<j\leq N} P(A_i \text{ win}|\{\epsilon_1, ..., \epsilon_m\} \cup \{\epsilon_j\})(N-m-1) - \beta && (30)
\end{aligned}$$

Thus, in order for agent $i \leq m$ to satisfy the equilibrium condition, we must show that

$$\begin{aligned} \pi_i^{ESS} &\geq \pi_i^M \quad (31) \\ \left(\frac{1}{N-1}\right) \sum_{j\neq i, j\leq m} P(A_i \text{ win}|\{\epsilon_1, ..., \epsilon_m\} \setminus \{\epsilon_j\})(N-m+1) + & \\ \left(\frac{1}{N-1}\right) \sum_{m<j\leq N} P(A_i \text{ win}|\{\epsilon_1, ..., \epsilon_m\} \cup \{\epsilon_j\})(N-m-1) &\geq \beta \quad (32) \end{aligned}$$

Generalizing to contests of size $C < N$, we see that there is a $\frac{C-1}{N-1}$ chance of encountering a mutant agent within the realized group. In this case, we see that the expected payoff the ESS strategy of talking becomes

$$\begin{aligned} \pi_i^{ESS} &= \left(1 - \frac{C-1}{N-1}\right) \pi_i(s_i^{ESS}|\vec{s}_{C-1}^{ESS}) + \left(\frac{C-1}{N-1}\right) \pi_i(s_i^{ESS}|\vec{s}_{C-2}^{ESS}, s_{j\neq i}^M) \quad (33) \\ &= \left(1 - \frac{C-1}{N-1}\right) (C-m) P(A_i \text{ wins}|\{\epsilon_1, ..., \epsilon_m\}) + \\ &\left(\frac{C-1}{N-1}\right) \left[\left(\frac{1}{N-1}\right) \sum_{j\neq i, j\leq m} P(A_i \text{ win}|\{\epsilon_1, ..., \epsilon_m\} \setminus \{\epsilon_j\})(N-m+1) + \right. \\ &\left. \left(\frac{1}{N-1}\right) \sum_{m<j\leq N} P(A_i \text{ win}|\{\epsilon_1, ..., \epsilon_m\} \cup \{\epsilon_j\})(N-m-1) \right] - \beta \quad (34) \end{aligned}$$

Thus, in order for agent $i \leq m$ satisfy the equilibrium condition, we must show that

$$\begin{aligned} \pi_i^{ESS} &\geq \pi_i^M \quad (35) \\ \left(1 - \frac{C-1}{N-1}\right) (C-m) P(A_i \text{ wins}|\{\epsilon_1, ..., \epsilon_m\}) + & \\ \left(\frac{C-1}{N-1}\right) \left[\left(\frac{1}{N-1}\right) \sum_{j\neq i, j\leq m} P(A_i \text{ win}|\{\epsilon_1, ..., \epsilon_m\} \setminus \{\epsilon_j\})(N-m+1) + \right. & \\ \left. \left(\frac{1}{N-1}\right) \sum_{m<j\leq N} P(A_i \text{ win}|\{\epsilon_1, ..., \epsilon_m\} \cup \{\epsilon_j\})(N-m-1) \right] &\geq \beta \quad (36) \end{aligned}$$

We see that, holding C fixed, as $N \to \infty$, this results in the Nash equilibrium where the top m agents speak in any group. However, for finite populations, we must compute (36) for each agent to determine the whether talking satisfies the equilibrium condition. Since agents that with higher baseline speaking qualities have a higher chance of winning, (32) and (36) show that agents that are further away from the marginal agent and therefore have the highest average speaking quality are more likely to satsify the condition whereas agents close to and including the marginal agent may or may not satisfy this condition as the addition of another talker in the mutated agent can affect their expected payoff by lowering their chance of winning if they choose to speak. Finally, as $\frac{C}{N} \to 0$, we see that the condition is more likely to be satisfied as there is less of a chance that the mutated agent will be included in the group that an agent might face. The opposite holds true as $\frac{C}{N} \to 1$.

4.3.2 Agents with index $i > m$

For agents $i > m$, the ESS strategy of listening yields a payoff of 0 while the mutated strategy of talking yields a negative payoff due to the top m agents choosing to talk since all other agents will be playing the evolutionarily stable strategies. Thus,

$$\begin{aligned}
\pi_i^{ESS} &= (1 - \frac{C-1}{N-1})\pi_i(s_i^{ESS}|\vec{s}_{C-1}^{ESS}) + \frac{C-1}{N-1}\pi_i(s_i^{ESS}|\vec{s}_{C-2}^{ESS}, s_{j\neq i}^{M}) && (37)\\
&= 0 && (38)\\
\pi_i^{M} &= \pi_i(s_i^{M}|\vec{s}_{C-1}^{ESS}) && (39)\\
&= (C-m-1)P(\text{Agent } i \text{ wins}|\{\epsilon_1, ..., \epsilon_m, \epsilon_{m+1}\}) - \beta && (40)\\
&< 0 && (41)
\end{aligned}$$

We see that the mutated strategy yields a negative expected payoff since we defined the marginal agent as the agent with the largest m such that (24) holds and since no other agent will mutate. Since $i > m$, we know that the condition cannot be satisfied and therefore the expected payoff to talking if all agents with a higher average speaking quality choose to talk must be negative. Thus, it is clear that $\pi_i^{ESS} > \pi_i^{M}$ and therefore the strategy of listening for agents outside of the top m agents satisfies the equilibrium condition and is guaranteed to hold.

4.4 Evolutionary Stability, Stability Conditions

I next extend the definition of Y-stability outlined in (Schaffer, 1988) to allow for asymmetric agents that may have different strategies. That is, agent i's ESS strategy, s_i^{ESS} is Y-stable if, in a population with a total of up to Y mutant strategists with any mutant strategy $s^M \neq s^{ESS}$,

$$\pi_i^{ESS} > \pi_i^{M} \text{ for all } 2 \leq K \leq Y \qquad (42)$$

The ESS is considered *globally stable* if $Y = N - 1$. Notice how we do not require that the mutant strategists be identical. This allows the definition of stability to match the game setup.

4.4.1 Agents with index $i \leq m$

In determining the stability of agents, we want to determine the maximum Y such that (42) is satisfied. For agents with index $i \leq m$, we know that the evolutionarily stable strategy is to talk, while the mutant strategy is to listen. (42) is broken when there are a sufficient number of mutated agents agent i's group such that $\pi_i^{ESS} \leq 0$. Similar to the derivation of the equilibrium condition, agent i's strategy of talking decreases in stability when the $N - m$ listeners mutate increases in stability when the m talkers mutate. Thus, when determining the stability of the ESS strategy, we must take into account the number of mutating listeners and talkers that mutate as well as *which* of these agents choose to mutate since each player has a different baseline speaking quality. This process becomes cumbersome since we must consider all possible combinations of mutating agents, so we can instead note that if there are K total mutants with j mutating listeners and $K - j$ mutating talkers, agent i's stability is minimized when the j listeners with the highest ϵ-values mutate and the $K - j$ talkers with the lowest ϵ-values mutate. Then, we see that in the $C = N$ case,

$$\pi_i^{ESS} \geq \sum_{j=0}^{K} \frac{\binom{N-m}{j}\binom{m-1}{K-j}}{\binom{N-1}{K}}(N - m - j + (K - j))P(\text{Agent } i \text{ wins}|E^K) - \beta \qquad (43)$$

where E^K is the set of ϵ-values for all agents that choose to talk. For example, if we are determining the expected payoff for the marginal agent given j and K, $E^K = \{\epsilon_1, ..., \epsilon_m\} \cup \{\epsilon_{m+1}, ..., \epsilon_{m+j}\} \setminus \{\epsilon_{m-(K-j)-1}, ..., \epsilon_{m-1}\}$. Then, we want to show that the expected payoff to talking for agent i is *strictly greater than* the expected payoff to listening:

$$\begin{aligned} \pi_i^{ESS} &> \pi_i^M && (44) \\ \sum_{j=0}^{K} \frac{\binom{N-m}{j}\binom{m-1}{K-j}}{\binom{N-1}{K}}(N-m-j+(K-j))P(\text{Agent } i \text{ wins}|E^K) &> \beta && (45) \end{aligned}$$

When $C < N$, we must now consider the number of mutated agents that are included in the group. Once again, however, we can bound the expected payoff by noting that the payoff is minimized when the bottom-most talkers and top-most listeners in the group mutate to get

$$\begin{aligned} \pi_i^{ESS} &> \pi_i^M && (46) \\ \sum_{l=0}^{C-1} \frac{\binom{K}{l}\binom{N-1-K}{C-1-l}}{\binom{N-1}{C-1}} \sum_{j=0}^{l} \frac{\binom{C-m}{j}\binom{m-1}{l-j}}{\binom{C-1}{l}}(C-m-j+(l-j))P(\text{Agent } i \text{ wins}|E^l) &> \beta && (47) \end{aligned}$$

Where E^l is the set of talker ϵ-values in the group after accounting for the l in-group mutations.

Similar to the equilibrium condition, agents further away from the marginal agent are more stable while ones closer to the marginal agent are less stable. Perhaps more noteworthy is that the stability for agents in this setup is more susceptible to the number of in-group mutations but less susceptible to the number of out-of-group mutations since strategies are confined to the contest and not the overall population. Thus, agents may be able to withstand more overall mutations in the population as long as the size of the population is large even though they are more "vulnerable" if the mutated agents are added to their contest. Finally, if we hold C fixed and let $N \to \infty$, we see that the stability of strategies for all agents $i \leq m$ increases since there is an increasingly lower chance of multiple in-group mutations.

4.4.2 Agents with index $i > m$

Similar to the case for talking agents, we want to consider the case when there are j agents that mutate to talking and $K - j$ agents that mutate to listening. Additionally, however, we need to account for the fact that agent i, should they mutate, will decide to talk. Thus, we must show that (42) must hold for all values of K such that $2 \leq K \leq Y$ in order for agent i's strategy of listening to be Y-stable. Again, however, we can simplify calculation of the expected payoff by considering the upper-bound case where the bottom j listeners and the top $K - j$ talkers deviate. In the $C = N$ case, we see that

$$\pi_i^M \leq \sum_{j=0}^{K-1} \frac{\binom{N-m-1}{j}\binom{m}{K-j}}{\binom{N-1}{K-1}}(N-m-j+(K-1-j)-1)P(\text{Agent } i \text{ wins}|E^K) - \beta \tag{48}$$

and we must show that

$$\beta > \sum_{j=0}^{K-1} \frac{\binom{N-m-1}{j}\binom{m}{K-j}}{\binom{N-1}{K-1}}(N-m-j+(K-1-j)-1)P(\text{Agent } i \text{ wins}|E^K) \tag{49}$$

Generalizing to contests of size $C < N$, we must instead show that

$$\beta > \sum_{l=0}^{C-1} \frac{\binom{K-1}{l}\binom{N-1-K}{C-1-l}}{\binom{N-1}{C-1}} \sum_{j=0}^{l} \frac{\binom{N-m-1}{j}\binom{m}{K-j}}{\binom{N-1}{K-1}} (N - m - j + (l-j) - 1) P(\text{Agent } i \text{ wins}|E^l) - \beta \quad (50)$$

We see that the RHS is maximized when $i = N$ and minimized when $i = m + 1$. Again, this means that the agent with the lowest speaking quality is the most stable in their strategy to listen all the time. This stability decreases as we consider agents that are closer to the marginal agent. Furthermore, we see that as we fix C and let $N \to \infty$, the stability of strategies for all agents $i > m$ strategies increases since there is a lower chance of facing a mutated agent in the group.

4.5 Computational Results

To supplement the analysis of the conditions presented in previous sections, I constructed computational results under various game setups. These computational results serve as starting points for later theoretical work in which I aim to prove the both the evolutionary stability of the highlighted strategy profile and characterize the growth of "always talkers". For each game setup, I assumed that agents' ϵ-values were evenly distributed on $[0.01, 0.99]$, $\alpha = 1$, and $C = N$.

4.5.1 Index of the Marginal Agent and Growth in the Number of "Always Talkers"

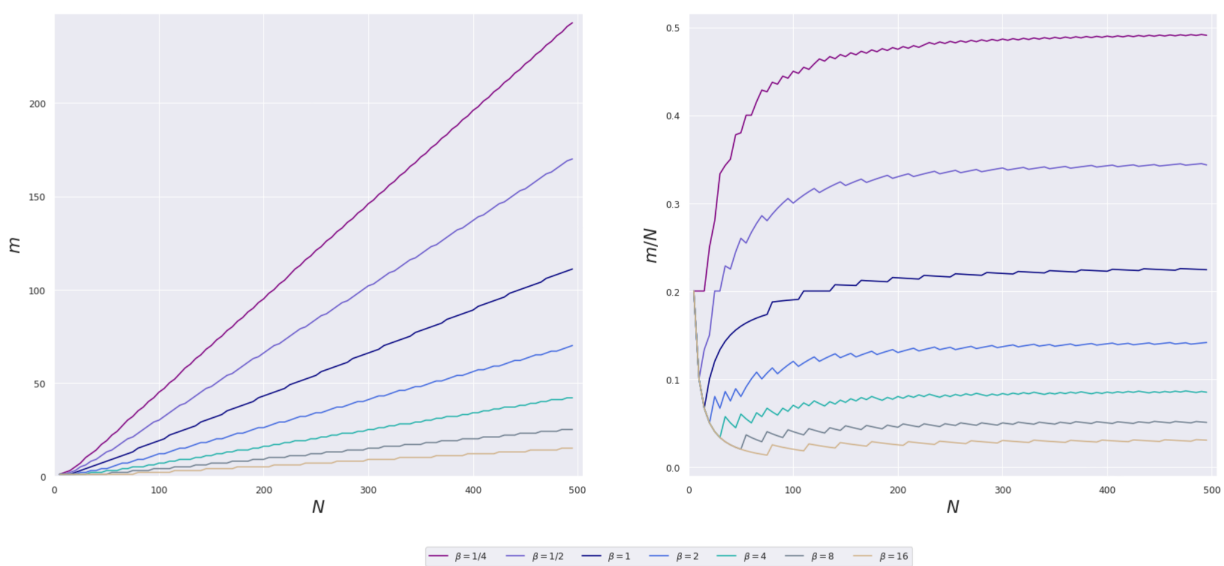

Figure 2: (Left) Plot of the index of the marginal agent, m against content size, N. **(Right)** Plot of the proportion of talkers in the contest, $\frac{m}{N}$ against contest size, N. Results computed with ϵ-values evenly distributed on $[0.01, 0.99]$, $\alpha = 1$, $N = \{3, ..., 500\}$ and $\beta = \{\frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16\}$.

Figure 2 plots the index of the marginal agent as well as the proportion of the number of "always talkers" in the contest ($\frac{m}{N}$). Values for the index of the marginal agent were found by solving (24) for fixed values of N, α, ϵ, and β. We see that the index of the marginal agent grows with network size while the "cost", β acts as a sort of rotating parameter that affects the rate of growth in this context. Turning our focus to the share of agents with $\gamma = 1$ in the contest, we see that the growth in the proportion of talkers is roughly logarithmic for low cost values ($\beta \leq 2$) and decreasing at first and then relatively constant at higher cost levels. Taken together, these results suggest that while the number of talkers may grow with the network size, the growth in the share of talkers is sublinear and therefore talking is more concentrated in larger networks. It is clear that the cost parameter determines the extent to which the network is concentrated, with networks with lower costs having a more equitable share of talkers to listeners.

4.5.2 Evolutionary Stability of the Marginal Agent Equilibrium

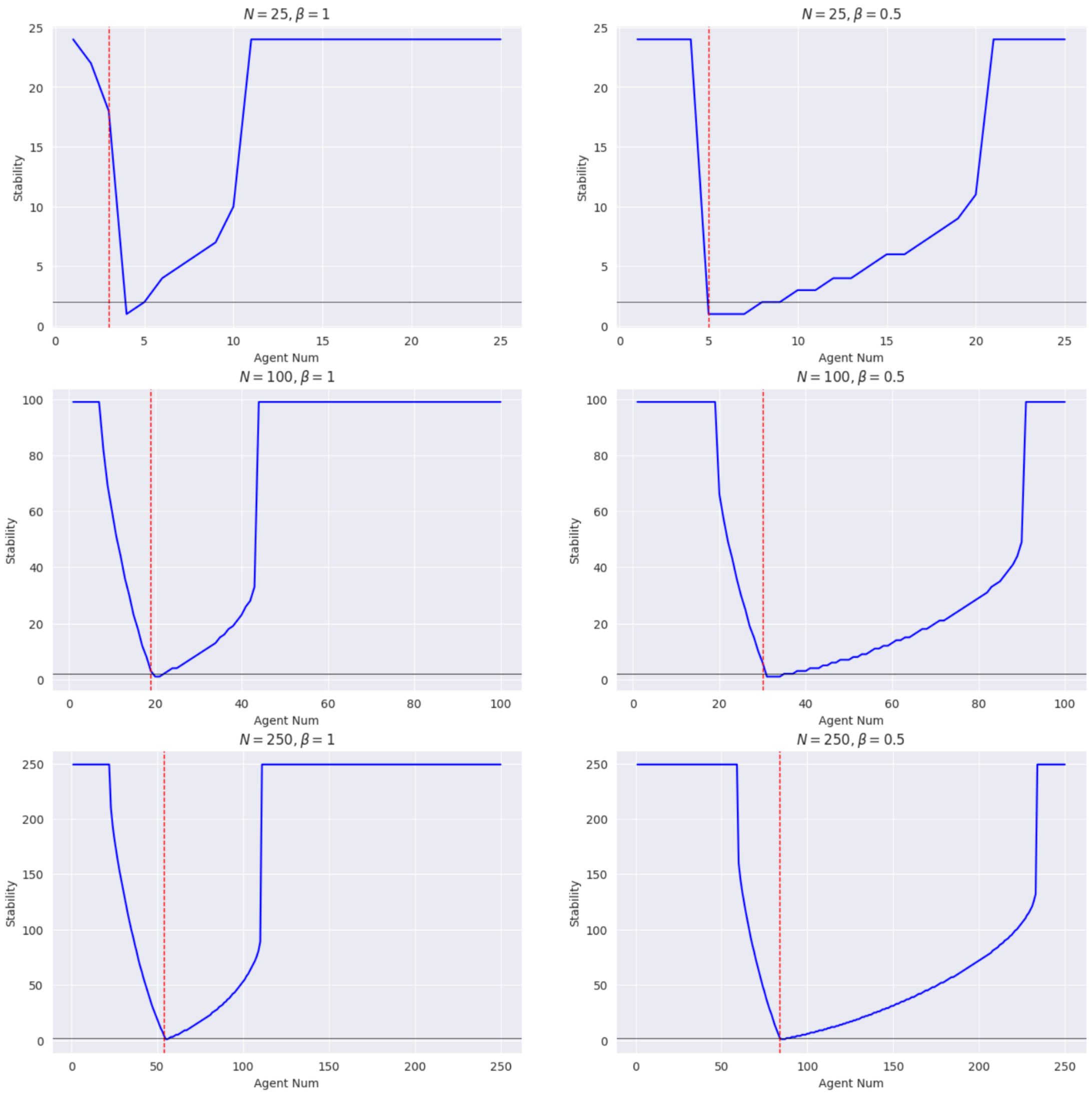

Figure 3: Y-Stability calculated for each agent in games with $N = \{25, 100, 250\}$ and $\beta = \{\frac{1}{2}, 1\}$. ϵ-values are evenly spaced on the interval $[0, 1]$ and $C = N$. The red dashed line indicates the marginal agent and the black line indicates $Y = 2$.

To test whether the equilibrium condition holds under various game setups, I first determined the index of the marginal agent in each game using (24) and then computed (36). Game setups with $\alpha = 1$, $C = N$, $\beta = \{\frac{1}{10000}, \frac{1}{1000}, \frac{1}{100}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16\}$ and $N = \{5, 10, ..., 500\}$ were considered. ϵ-values were considered to be equally spaced on the interval $[0, 1]$. Results showed that the equilibrium conditions were satisfied for all agents (although it was guaranteed to hold for the "listener" group) for all contest sizes of 20 agents or greater. The only contests that failed to satisfy the equilibrium conditions were contests with the following (N, β) pairs: $(5, 4), (5, 8), (5, 16), (10, 8), (10, 16), (15, 16)$. These results suggest that the marginal agent NE is

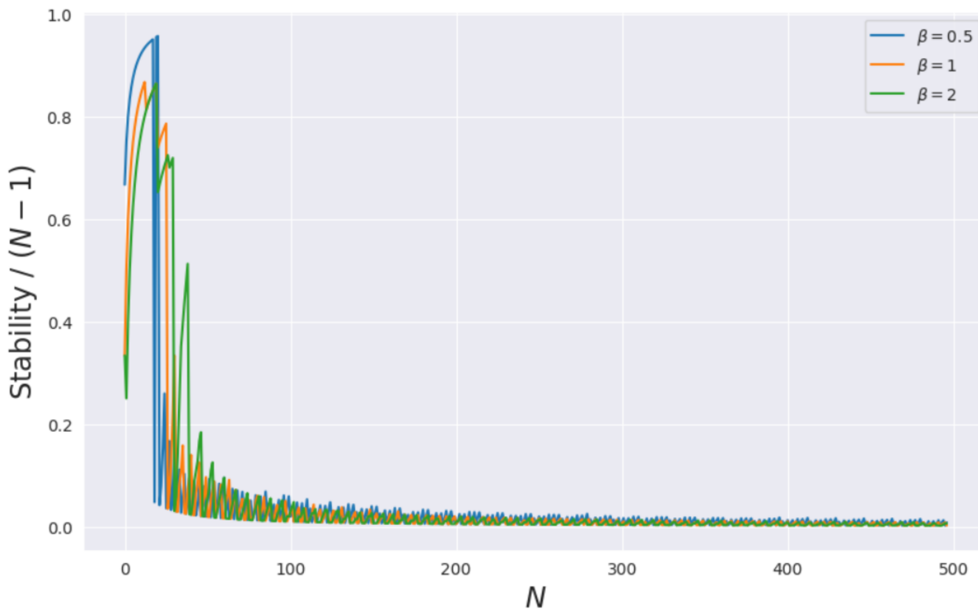

Figure 4: Plot of the ratio of Y-stability to the number of potential mutations $(N-1)$ for the marginal agent. Computed with $N = \{5, 10, ..., 500\}$ and $\beta = \{\frac{1}{2}, 1, 2\}$. ϵ-values evenly distributed on $[0.01, 0.99]$, $\alpha = 1$, and $C = N$.

robust to low values in the cost of talking, but fails to satisfy the conditions for evolutionary stability for some agent strategies as the cost approaches or exceeds the size of the network as noted in Section 2. Intuitively, in the presence of mutations, agents that choose to speak all the time ($i \leq m$) will choose to listen due to the high costs of speaking and are therefore susceptible to mutations. However, as the size of the network increases holding cost fixed, all strategies are evolutionarily stable.

Figure 3 plots the Y-stability for all agents in 6 game setups with $N = \{25, 100, 250\}$ and $\beta = \{\frac{1}{2}, 1\}$. We see that the intuition from the conditions holds: agents that are closer to the marginal agent are less stable in their evolutionary strategies while agents further away are more stable. Interestingly, agents that have sufficiently high or low baseline speaking qualities are *globally stable*, meaning that their strategies are robust to every agent in the contest mutating. We do see, however, that while the equilibrium conditions for evolutionary stable strategies might be satisfied for all agents in these games (at minimum 1-stability), agents closer to the marginal agent may not be stable in their strategies in the presence of 2 or more mutations. Turning our attention to the slope of ascent in stability for "listening" agents (right of the red line), we see that the cost of talking modifies the slope of Y-stability. This is because listening agents have a greater incentive to switch to speaking when costs are lowered and are therefore less stable in their strategies when there is an increasingly higher number of mutations in the overall network.

Figure 4 plots the ratio of Y-stability to the number of potential mutations (of which there are $N-1$) for the *marginal agent* in similar games with $N = \{25, 100, 250\}$ and $\beta = \{\frac{1}{2}, 1\}$. While the strategy of talking for the marginal agent is increasingly likely to satisfy the equilibrium condition as the network size increases, we see that the Y-stability of the agent doesn't necessarily increase and it never reaches global stability in its ESS strategy. In fact, the marginal agent's stability relative to the size of the network is highest in smaller networks and decreases sharply regardless of the magnitude of the cost of speaking.

4.5.3 Consideration of other Pure Strategy NE

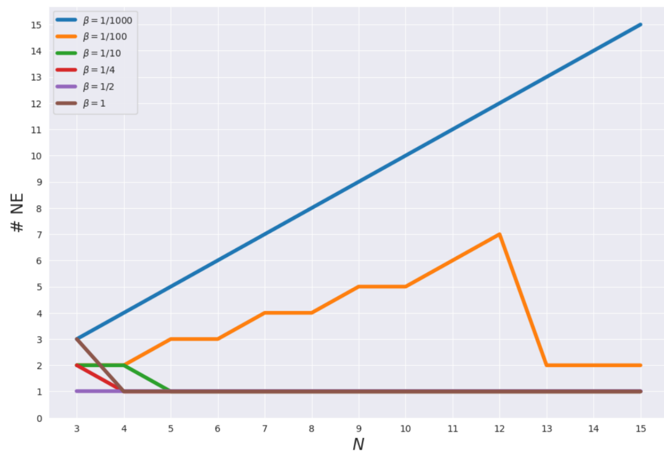

Figure 5: Plot of the number of identified pure strategy Nash Equilibria against contest size for $N = \{3, ..., 15\}$, $\beta = \{\frac{1}{1000}, \frac{1}{100}, \frac{1}{10}, \frac{1}{4}, 1\}$, and $\alpha = 1$.

Figure 5 plots the number of identified pure strategy Nash Equilibria in games with $\beta = \{\frac{1}{1000}, \frac{1}{100}, \frac{1}{10}, \frac{1}{4}, \frac{1}{2}, 1\}$ and $N = \{3, ..., 15\}$. Since identifying all possible equilibria means exploring $2^N - 2$ combinations and therefore imposes a computational constraint, I restrict the results to contests with a maximum size of 15 agents. We see that for values of $N \leq 4$ there are multiple identified equilibria, which levels out into a single unique equilibrium for values of $\beta \geq \frac{1}{10}$ as the network size increases. The unique NE in these cases "marginal agent" strategy profile with $\gamma_{i \leq m} = 1$ and $\gamma_{i > m} = 0$, as described above. For lower values of β or contests of size $N = 3$ or $N = 4$, however, we see that multiple NE are identified even as the size of the network grows. In these cases, identified equilibrium strategy profiles include the marginal agent strategy profile along with other strategy profiles that are composed of a mix of agents speaking and all others listening.

To explore the additional NE that are not the "marginal agent" equilibrium, I separate them into two groups: "saturated" and "unsaturated" equilibria. In the saturated case, there are $N - 1$ agents that choose to talk and 1 agents that choose to listen. The intuition behind these saturated equilibria is that the cost of talking is low enough as to encourage every agent to speak. However, since there is still a nonzero cost, one agent must choose to listen all the time in order to not break the equilibrium. In the unsaturated case, there is at most a combination of $N-2$ agents that choose to talk and at least 2 agents that choose to listen. For values of $\beta = \frac{1}{1000}$ (the blue line in Figure 5), every identified equilibrium is a saturated equilibrium. However, for values of $\beta = \frac{1}{100}$ (orange line), identified equilibria are both saturated and unsaturated for $N \leq 12$ and only a saturated equilibrium is identified other than the "marginal" agent equilibrium for $N \geq 13$.

I then recompute (32) for each agent and strategy profile. I find that in the strategy profiles in the unsaturated case do not satisfy the equilibrium condition for ESS strategies in the game (i.e, they have at least one agent that will deviate in the presence of a mutation), which allows for an equilibrium refinement using the conditions outlined in Section 4.3. However, in the saturated equilibria, I find that all agent strategies do indeed satisfy the equilibrium condition. This is due to the fact that since there are $N-1$ talkers and only 1 listener, there is a $\frac{N-2}{N-1}$ chance of talker deviating (and therefore increasing the expected payoff of other talkers) whereas there is only a $\frac{1}{N-1}$ chance of the listener deviating and destroying payoffs for all agents. Therefore, talkers in the contests are secure in their strategy and likely safe against one mutation while the sole listener is guaranteed a negative expected payoff should they choose to mutate. Thus, the saturated equilibrium presents a particular challenge to studying equilibrium refinements in this game. However, it must be noted that if a saturated equilibrium is not the "marginal agent" equilibrium, then the sole listener must be a top agent that would normally speak with $\gamma = 1$. In this case, the Y-stability of the agent that chooses to listen is very low since they will have a greater incentive to speak once talkers start to mutate, which is guaranteed in populations with at least 2 mutations. This provides a motivating step for future analyses that may rely on the comparisons in the average Y-stability of agent strategies when comparing across potential equilibria. Nevertheless, as suggested by Figure 5, with a high enough cost of talking and moderate network sizes, such results can be avoided and we recover the unique pure strategy NE in which only the topmost agents in the group choose to speak with probability 1.

5 Discussion

The model and subsequent analyses presented in this paper consider a game where N players in a fully-connected network decide whether to compete in a contest for a prize that scales with the size of the network. The model is meant to reflect the objectives and outcomes of online communication, with a specific focus on a single round of messaging. The results show that in communication networks where agents are symmetric in their average speaking quality, participation increases with the size of the network and decreases with the cost of talking. In networks where agents differ in their baseline speaking qualities, however, there exists multiple pure strategy equilibria whereby a subset of agents will always choose to talk and the rest will always choose to listen. I focus on equilibria where the agents with the highest baseline speaking qualities always talk and all other listen and combine extended conditions from (Schaffer, 1988) with computational results to confirm that this strategy profile is evolutionarily stable under certain conditions.

The evolutionarily stable strategy conditions presented suggest that agents further away from the "marginal agent" in any group are more likely to satisfy the equilibrium condition and are are either very stable or possibly globally stable in their strategies. However, agents closer to and including the marginal agent may not satisfy the equilibrium condition (although computational results suggest that it holds for games with a sufficient number of agents) and may not be robust to 2 or more mutations in the population. However, if we consider contests of finite size drawn from the overall population, equilibrium conditions are more likely to be satisfied and agents are more stable in their strategies as $\frac{C}{N} \to 0$ due to the lower probability of facing a mutant in any group. Conversely, if we let $\frac{C}{N} \to 1$, it is not so clear that these conditions will hold and computational results must determine whether or not this is the case. Finally, in large networks (N) where the cost of talking is moderate (β), computational results suggest that the "marginal agent" equilibrium is both unique and evolutionarily stable. However, as costs approach 0, the game may achieve any of N evolutionarily stable "saturated equilibria" in which $N-1$ agents will always choose to talk and 1 agent will always choose to listen. Such results present a challenge to the analysis, but they may be addressed by comparing the Y-stability of identified equilibria and computational results suggest they do not exist in games with higher costs and network sizes.

Before drawing inferences from the model results to online conversation networks, we must first consider some major drawbacks to the model. The model does not capture time-varying group dynamics such as increasing individual influence over time or changes in the *direction* of conversation that might empower or marginalize certain agents. Further, additional cost functions outside of the constant cost function are not included in the analysis, which is necessary to make comparisons to conversation networks that might exist without the presence of online communication technology. Perhaps most importantly, I do not consider changes in the payoff structure to listening. Individuals may benefit from increased conversation and the enhanced information diffusion offered by large networks which may alter their objectives and subsequent strategies in any given conversation. A more complete model of participation in online groups should take such payoffs into consideration.

Despite the limitations of the model, however, the results from this analysis offer a multiple avenues for further research. The marginal agent strategy profile suggests that constant communication costs in groups of asymmetric agents results in increasing participation inequality as network sizes grow. Further, the symmetric equilibrium strategy profile suggests that there will be high amounts of participation in online groups where status or speaking differences between individuals is not as salient. Empirical analyses may therefore include investigations into how factors such as online anonymity or user privacy protections alter participation inequalities and the quality of shared content. Additionally, studies on the richness of communication media (i.e. video calling versus instant messaging) and their effects may include consideration of how they alter have on the cost of talking and to what extent the cost curve is truly constant. Furthermore, alternative prize structures such as those with proportional payoffs can model how users derive utility by being part of the conversation despite not having the greatest speaking quality. For example, websites such as StackOverflow and Reddit reward multiple users on a given post in the form of votes and users can still benefit despite not being the original poster or the top comment. Finally, I plan to extend the results in this paper by designing a behavioral experiment in which individuals participate in conversations through different communication mediums. By making use of automatic speech-to-text transcription algorithms, I can track the share of speaking by any individual across multiple treatments. This work is still ongoing. Ultimately, my hope is that the arguments outlined in this paper provide an ample springboard for later research into related games as well as online communication technologies and their effects on participation inequalities.

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7 Appendix

7.1 Upper and Lower Bounds on $P(A_m$ wins)

To find a bounds for the marginal agent's index, we must first find bounds on the probability that A_m wins the lottery. We begin with the lower bound, assuming that $E_T = \{\epsilon_1, ..., \epsilon_m\}$.

$$
\begin{aligned}
P(A_m \text{ wins}|E_T) &= \int_{\epsilon_m}^{\alpha} P(q_m = y)P(\max(\{q_1, ..., q_{m-1}\}) < y)dy && (51)\\
&= \int_{\epsilon_1}^{\alpha} P(q_m = y)P(\max(\{q_1, ..., q_{m-1}\}) < y)dy && (52)\\
&= \int_{\epsilon_1}^{\alpha} \frac{1}{\alpha-\epsilon_m}\left(\frac{y-\epsilon_1}{1-\epsilon_1}\right)\cdots\left(\frac{y-\epsilon_{m-1}}{1-\epsilon_{m-1}}\right)dy && (53)\\
&= [\prod_{i=1}^{m}\frac{1}{\alpha-\epsilon_i}]\int_{\epsilon_1}^{\alpha}(y-\epsilon_1)(y-\epsilon_2)\cdots(y-\epsilon_{m-1})dy && (54)\\
&\geq \frac{1}{(\alpha-\epsilon_m)^m}\int_{\epsilon_1}^{\alpha}(y-\epsilon_1)(y-\epsilon_2)\cdots(y-\epsilon_{m-1})dy && (55)\\
&\geq \frac{1}{(\alpha-\epsilon_m)^m}\int_{\epsilon_1}^{\alpha}(y-\epsilon_1)^{m-1}dy && (56)\\
&= \frac{1}{(\alpha-\epsilon_m)^m}\left[\frac{(y-\epsilon_1)^m}{m}\right]_{\epsilon_1}^{\alpha} && (57)\\
&= \frac{1}{(\alpha-\epsilon_m)^m}\left[\frac{(\alpha-\epsilon_1)^m}{m}-\frac{(\epsilon_1-\epsilon_1)^m}{m}\right] && (58)\\
&= \frac{1}{(\alpha-\epsilon_m)^m}\cdot\frac{(\alpha-\epsilon_1)^m}{m} && (59)
\end{aligned}
$$

$$= \frac{1}{m}\left(\frac{\alpha-\epsilon_1}{\alpha-\epsilon_m}\right)^m \tag{60}$$

An upper bound on $P(A_m \text{ wins}|E_T)$ can also be found,

$$\begin{aligned}
P(A_m \text{ wins}|E_T) &= \int_{\epsilon_m}^{\alpha} P(q_m = y)P(\max(\{q_1,...,q_{m-1}\}) < y)dy && (61)\\
&= \int_{\epsilon_1}^{\alpha} P(q_m = y)P(\max(\{q_1,...,q_{m-1}\}) < y)dy && (62)\\
&= \int_{\epsilon_1}^{\alpha} \frac{1}{\alpha-\epsilon_m}\left(\frac{y-\epsilon_1}{1-\epsilon_1}\right)\cdots\left(\frac{y-\epsilon_{m-1}}{1-\epsilon_{m-1}}\right)dy && (63)\\
&= [\prod_{i=1}^{m}\frac{1}{\alpha-\epsilon_i}]\int_{\epsilon_1}^{\alpha}(y-\epsilon_1)(y-\epsilon_2)\cdots(y-\epsilon_{m-1})dy && (64)\\
&\le \frac{1}{(\alpha-\epsilon_1)^m}\int_{\epsilon_1}^{\alpha}(y-\epsilon_1)(y-\epsilon_2)\cdots(y-\epsilon_{m-1})dy && (65)\\
&\le \frac{1}{(\alpha-\epsilon_1)^m}\int_{\epsilon_1}^{\alpha}(y-\epsilon_m)^{m-1}dy && (66)\\
&= \frac{1}{(\alpha-\epsilon_1)^m}\left[\frac{(y-\epsilon_m)^m}{m}\right]_{\epsilon_1}^{\alpha} && (67)\\
&= \frac{1}{(\alpha-\epsilon_1)^m}\left[\frac{(\alpha-\epsilon_m)^m}{m}-\frac{(\epsilon_1-\epsilon_m)^m}{m}\right] && (68)\\
&= \frac{1}{m}\left[\left(\frac{\alpha-\epsilon_m}{\alpha-\epsilon_1}\right)^m-\left(\frac{\epsilon_1-\epsilon_m}{\alpha-\epsilon_1}\right)^m\right] && (69)
\end{aligned}$$

Assuming that ϵ-values are evenly distributed on the interval $[\sigma, \alpha-\sigma]$ for some small σ, We see each epsilon value will be separated by intervals of $\frac{\alpha-2\sigma}{N}$ length. Therefore, we see that 60 becomes

$$\frac{1}{m}\left(\frac{\alpha-(\alpha-\frac{\alpha-2\sigma}{N})}{\alpha-(\alpha-\frac{m(\alpha-2\sigma)}{N})}\right)^m = \frac{1}{m}\left(\frac{\frac{\alpha-2\sigma}{N}}{\frac{m(\alpha-2\sigma)}{N}}\right)^m = \frac{1}{m}\left(\frac{1}{m}\right)^m = \left(\frac{1}{m}\right)^{m+1} \tag{70}$$

Similarly, 69 becomes

$$\frac{1}{m}\left[\left(\frac{\frac{m(\alpha-2\sigma)}{N}}{\frac{\alpha-2\sigma}{N}}\right)^m-\left(\frac{\frac{(m-1)(\alpha-2\sigma)}{N}}{\frac{\alpha-2\sigma}{N}}\right)^m\right] = \frac{1}{m}\left[m^m-(m-1)^m\right] \tag{71}$$

As we can see, when taken together, the bounds on the marginal agent are extremely loose due to the exponential terms in the expressions above. However, the lower bound on the probability for the marginal agent tells us that, in the worst case, the index of the marginal agent grows with the network size. However, this growth is extremely slow and implies that the concentration of talkers in the network increases with the size of the group, which is reflected in the computational results. To see this, we plug the lower bound into 24 to see that the following must be satisfied:

$$(N-m)\left(\frac{1}{m}\right)^{m+2} - \beta \quad > \quad 0 \tag{72}$$

7.2 Standard Evolutionary Stable Strategies and Extensions to Contests in Finite Populations

The standard conditions for evolutionary stable strategies were proposed for two-player contests in a symmetric and infinite population in (Smith and Price, 1973) and (Smith, 1982). That is, in an infinitely large population of players, assume that most players play the ESS strategy, s^{ESS} and all others play a mutant strategy, s^M. Let the payoffs for these agents be denoted by π^{ESS} and π^M, respectively. s^{ESS} satisfies the *equilibrium condition* for ESS strategies if

$$\pi(s^{ESS}|s^{ESS}) \geq \pi(s^M|s^{ESS}) \tag{73}$$

where $\pi(s^X|s^Y)$ is the payoff for an agent playing strategy X in a contest against a player playing strategy Y. Further, s^{ESS} satisfies the *stability condition* for ESS strategies if

$$\pi(s^M|s^{ESS}) = \pi(s^{ESS}|s^{ESS}) \tag{74}$$

then,

$$\pi(s^{ESS}|s^M) > \pi(s^M|s^M) \tag{75}$$

Thus, a strategy in such a game is defined to be an evolutionarily stable strategy if it satisfies both the equilibrium and stability conditions. The intuition for such a strategy is that it is protected to a degree from an "invasion" of mutant strategies that might arise.

(Schaffer, 1988) builds upon work by (Riley, 1979) and (Vickery, 1987) that shows that this definition does not hold up for finite populations. Instead, the author proposes an alternative, more general definition that considers a population of size N with players that engage in contests of size C. In this view, the standard conditions above can be thought of as a special case of this general definition with $N = \infty$ and $C = 2$. In this general game, we consider a population composed of $N-1$ ESS players and 1 mutant player. If an agent plays an ESS strategy, there is a $\frac{C-1}{N-1}$ chance that they play the mutant player in the contest. If the agent plays the mutant strategy, then they will be the sole mutant player and only play ESS agents. Let $\vec{s_Y^X}$ be a vector representing Y agents that are playing strategy X. Then, we can write the payoffs for ESS and mutant strategies:

$$\pi^{ESS} = \left(1 - \frac{C-1}{N-1}\right) \pi(s^{ESS}|\vec{s}^{ESS}_{C-1}) + \left(\frac{C-1}{N-1}\right) \pi(s^{ESS}|\vec{s}^{ESS}_{C-2}, s^M) \tag{76}$$

$$\pi^M = \pi(s^M|\vec{s}^{ESS}_{C-1}) \tag{77}$$

Then, the equilibrium condition is satisfied when

$$\pi^{ESS} \geq \pi^M \tag{78}$$

Which intuitively means that the ESS strategy is still preferred when played against mutant strategies. The author builds upon the standard stability condition and defines an ESS strategy as Y-stable if, in a population with up to Y identical mutant agents, then

$$\pi^{ESS} > \pi^M, \text{ for all } 2 \leq k \leq Y \tag{79}$$

Further, the ESS is globally stable is $Y = N - 1$. Thus, depending on the payoff structure and potential strategies of agents, the stability of an evolutionary stable strategy may differ. Notice, however, that the conditions outlined above all rely on the assumption that agents are *symmetric* in their payoff structure and therefore strategies. I build upon the above work and determine similar equilibrium and stability conditions for our game consisting of N agents with *asymmetric* average speaking quality that engage in contests of size C.