CHEME 7770: Advanced Biomolecular Engineering
Homework #4

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Question 1

1.a.

At steady state, $\frac{dR^*}{dt} = 0 = k_{on}[R][L] - k_{off}[R^*]$

Therefore, $\frac{k_{off}}{k_{on}[L]} = \frac{[R]}{[R^*]} = ([R_T] - [R^*])/[R^*]$

i.e. $\kappa_D = \frac{[R_T]}{[R^*]} - 1 = \frac{1}{\theta_B} - 1$

Therefore, $\theta_B = \frac{1}{1+\kappa_D}$

Now,

We know $\frac{d[X^*]}{dt} = -\frac{d[X]}{dt} = 0 = \frac{V_1[X]}{K_1+[X]} - \frac{V_2[X^*]}{K_2+[X^*]}$

By mathematical manipulation and substituting values, we get $\frac{\gamma_1\theta_B R_T}{V_2} = \frac{\kappa_1+1-x^*}{\kappa_2+x^*} * \frac{x^*}{1-x^*}$

Similarly, we know $\frac{d[Y^*]}{dt} = -\frac{d[Y]}{dt} = 0 = \frac{V_3[Y]}{K_3+[Y]} - \frac{V_4[Y^*]}{K_4+[Y^*]}$

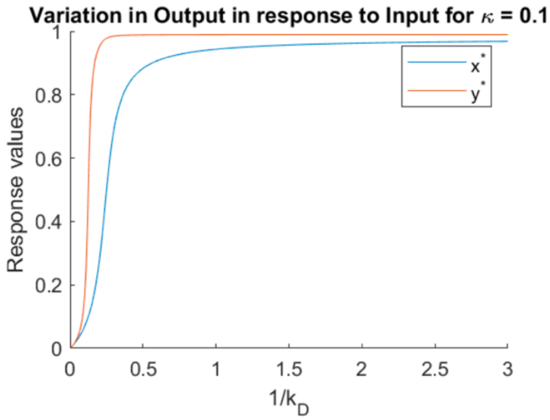
By mathematical manipulation and substituting values, we get $\frac{\gamma_3 x^* X_T}{V_4} = \frac{\kappa_3+1-y^*}{\kappa_4+y^*} * \frac{y^*}{1-y^*}$

With these equations, we have a relation between y* and $1/\kappa_D$.

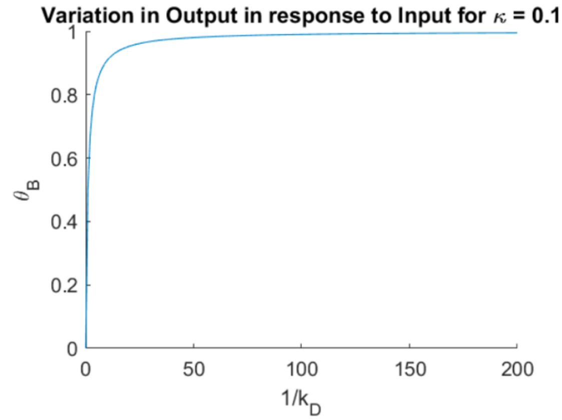
1.b.

See Matlab file Prob 1_b.m for code.

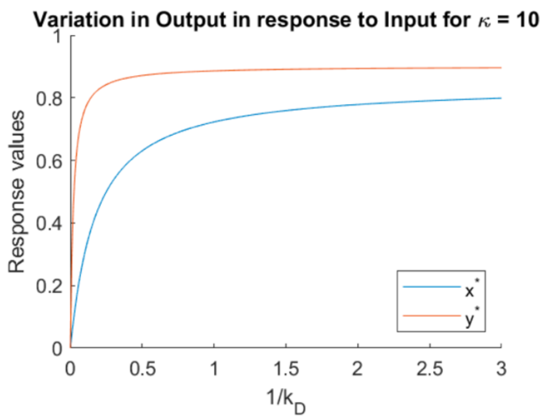
The plots for variation of x*, y*, θB vs $1/\kappa_D$ are given below for two cases κ = 0.1 and 10.



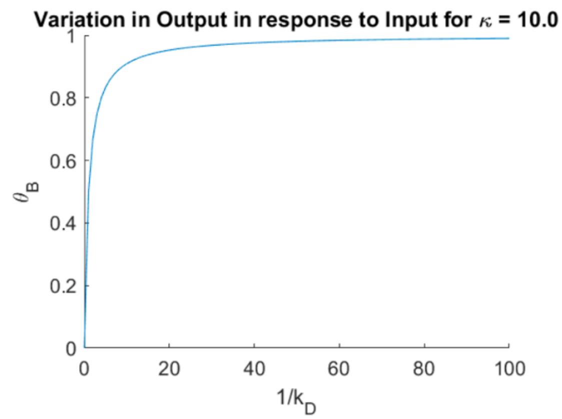
(a) Response curve for x^* and y^*



(b) Response curve for θ_B



(a) Response curve for x^* and y^*



(b) Response curve for θ_B

1.c.

See Matlab code Prob1_c.m

The Hill equation is given by $Output = \frac{A*Input^{n_h}}{C_{\frac{1}{2}}^{n_h}+Input^{n_h}}$, where n_h is the Hill Coefficient and $C_{\frac{1}{2}}^{n_h}$ is the input at which the output is half the maximum value. For different outputs x*, y*, θ_B and input $\frac{1}{\kappa_D}$, the Hill Coefficient was calculated.

κ	Output	Hill Coefficient
0.1	θ_B	1
0.1	x*	3.5446
0.1	y*	7.4094
10	θ_B	1
10	x*	1.0291
10	y*	1.0538

1.d.

The same code from part (b) was used, changing the κ_D value.

κ	Output	Percentage change
0.1	θ_B	43.4783%
0.1	x*	101.8261%
0.1	y*	402.5798%
10	θ_B	43.4783%
10	x*	27.9659%
10	y*	5.6566%

1.e.

We see that there is zero-order sensitivity for low values of κ since from part (d), we see that for $\kappa = 0.1$, for a small change in κD, the percentage change in output was large. This is also indicated by the higher Hill coefficients for this case, from part (c). Thus, by choosing a small κ value, it is possible to amplify a small change in input to a large change in output.
But we see that this property is lost at higher values of κ, again as seen from parts (c) and (d). Therefore, now the higher κ ends up attenuating the small change in input.

Hence by choosing the parameter value κ, we can amplify or attenuate the input signal using the same circuit.

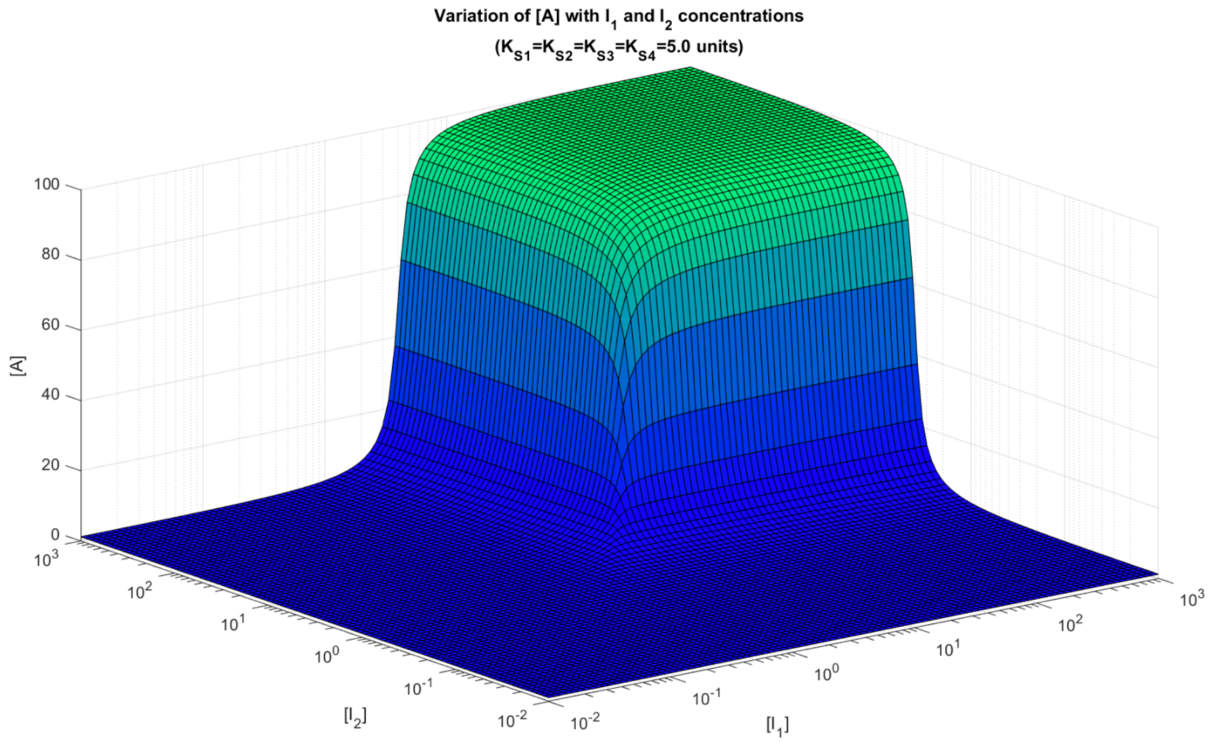
Question 2

2.a.

See Matlab code Prob2_a.m
The steady state values were calculated as
[A] = 1.1097 units
[B] = 49.4451 units
[C] = 49.4451 units

2.b.

See Matlab code Prob2_b.m
The 3D plot obtained was



With the inhibitor concentrations plotted on a log scale, and [A] plotted linearly.

2.c.

From above graph we see that only when both [I_1] and [I_2] are high, [A] is high, and in other cases it is low.
This shows that it acts as a AND gate.

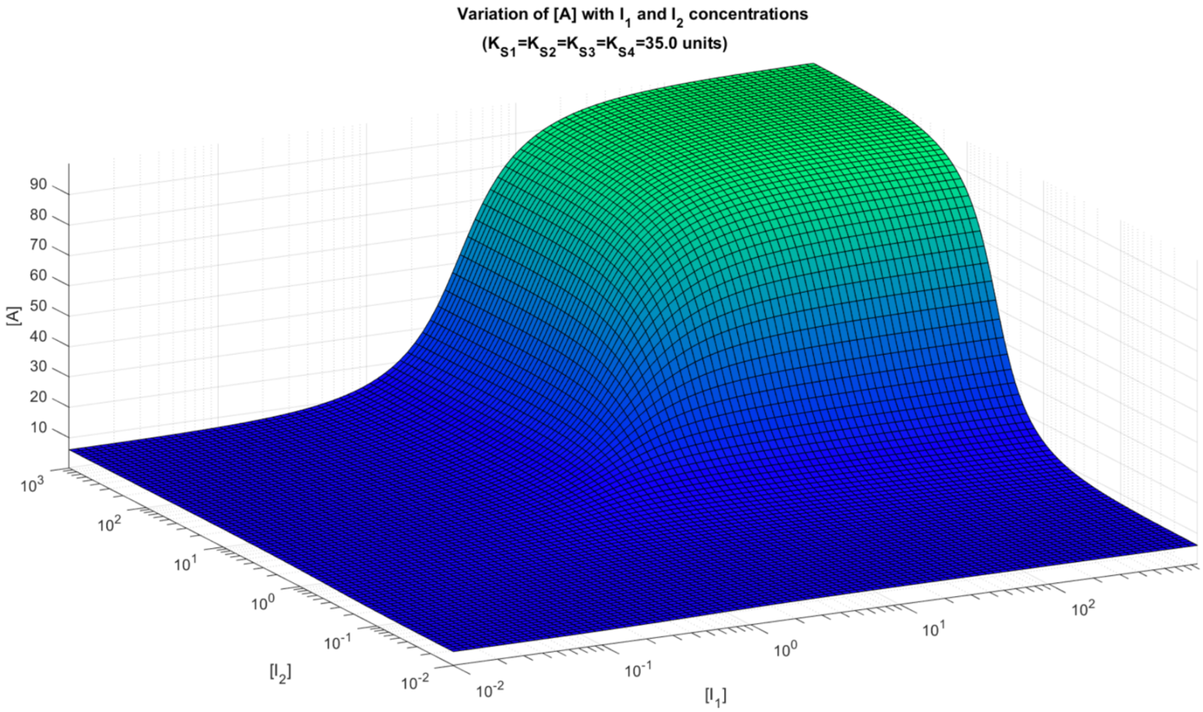
IF [I_1] low, [I_2] low → [A] low
IF [I_1] low, [I_2] high → [A] low
IF [I_1] high, [I_2] low → [A] low
IF [I_1] high, [I_2] high → [A] high

The above can be assumed to be a truth table for an AND gate with [I_1] and [I_2] as inputs and [A] as output.

2.d.

Same matlab file as Prob 2.b. Only values were changed.

The 3D plot obtained:



Again, to note here that [I_1] and [I_2] are plotted on a log scale, while [A] is plotted linearly.

We see here that the rise in [A] is more gradual and not as sharp as in part (b). Also, for the cases where the inputs are mismatched, i.e. {[I_1] low [I_1] high} or {[I_1] high [I_1] low} we now observe that the value of [A] isn't negligibly low anymore. It still roughly follows the same logic, but it is not zero-order sensitive and doesn't have a good switch-like performance. This is why it is termed a "fuzzy operator".

2.e.

For low values of K_S (as in part (b)), the system shows zero-order sensitivity, i.e., for a very small change in input, there is a large change in output. This switch from one level to the other occurs within physiologically significant time intervals helps generate a switch-like response and therefore it acts as a logic gate.

For larger values of K_S (As in part (d)), the system doesn't exhibit zero-order sensitivity, and so it doesn't follow perfect Boolean logic resulting in a "fuzzy operator". This shows that the zero-order sensitivity is a necessary prerequisite for proper functioning of a logic gate.

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