

Recurrent Neural Network-based Music Language Models for Improving Automatic Music Transcription

Abstract

In this paper, we investigate the use of Music Language Models (MLMs) for improving Automatic Music Transcription (AMT) performance. AMT is the process of converting an acoustic music signal into a symbolic notation, and is considered to be a fundamental problem in music signal processing. The MLMs are trained on sequences of symbolic polyphonic music. We train RNN-based models, as they are capable of capturing complex temporal structure present in the symbolic music data. Similar to the function of language models in automatic speech recognition, we use the MLMs to generate a prior probability for the occurrence of a sequence. The acoustic AMT model is based on probabilistic latent component analysis (PLCA), and prior information from the MLM is incorporated into the transcription framework using Dirichlet priors. We test our hybrid models on a dataset of multiple-instrument polyphonic music and report a significant 2.5% improvement in terms of F-measure, when compared to using an acoustic-only model.

1. Introduction

Automatic Music Transcription (AMT) involves automatically generating a symbolic representation of an acoustic musical signal (Benetos et al., 2013a). AMT has is considered to be a fundamental topic in the field of music information retrieval (MIR) and has numerous applications in related fields in music technology, such as interactive music applications and computational musicology. Typically, the output of an AMT system is a *pianoroll* representation, which is a two-dimensional matrix representation of a musical piece where the X-axis represents time quantized into regular intervals, and the Y-axis represents the 88 keys of a piano in increasing pitch. A cell in this matrix is 1 if the key represented by its X-coordinate is sounded at the time

instant represented by its Y-coordinate.

The majority of recent transcription papers utilise and expand *spectrogram factorisation* techniques, such as non-negative matrix factorisation (NMF) (Li & Seung, 1999) and its probabilistic counterpart, probabilistic latent component analysis (PLCA) (Smaragdis et al., 2006). Spectrogram factorisation techniques decompose an input two-dimensional spectrogram of the audio signal into a product of spectral templates (that typically correspond to musical notes) and component activations (that indicate when each note is active at a given time frame). Spectrogram factorisation-based AMT systems include the work by Bertin et al. (Bertin et al., 2010), who proposed a Bayesian framework for NMF, which considers each pitch as a model of Gaussian components in harmonic positions. Benetos and Dixon (Benetos & Dixon, 2012) proposed a convolutional model based on PLCA, which supports the transcription of multiple-instrument music and supports tuning changes and frequency modulations (modelled as shifts across log-frequency).

In terms of connectionist approaches for AMT, Nam et al. (Nam et al., 2011) proposed a method where features suitable for transcribing music are learned using a deep belief network consisting of stacked restricted Boltzmann machines (RBMs). The model performed classification using support vector machines and was applied to piano music. Böck and Schedl used recurrent neural networks (RNNs) with Long Short-Term Memory units for performing polyphonic piano transcription (Bock & Schedl, 2012), with the system being particularly good at recognising note onsets.

There is no doubt that a reliable acoustic model is important for generating accurate symbolic transcriptions of a given music signal. However, since music exhibits a fair amount of structural regularity much like language, it is natural for one to think of the possibility of improving transcription accuracy using a *music language model* (MLM) in a manner akin to the use of a language model to improve the performance of a speech recognizer (Rabiner & Juang, 1993). In (Boulanger-Lewandowski et al., 2012), the predictions of a polyphonic MLM were used to this end, which was further developed in (Boulanger-Lewandowski et al., 2013), where an input/output extension of the RNN-RBM was proposed that learned to map input sequences to out-

put sequences in the context of AMT. Another example of symbolic information which can improve the performance of acoustic models are *score informed* approaches, which have been applied in music research tasks such as source separation (Ewert & Müller, 2012), voice separation (Ewert & Müller, 2011) and tonic identification (Sentürk et al., 2013).

In the present work, we make use of the predictions made by a Recurrent Neural Network-Neural Autoregressive Distribution Estimator (RNN-NADE) based polyphonic MLM proposed in (Boulanger-Lewandowski et al., 2012) to refine the transcriptions of a PLCA-based AMT system (Benetos & Dixon, 2012; Benetos et al., 2013b). Information from the MLM is incorporated into the PLCA-based acoustic model as priors for pitch activations during the parameter estimation stage. It was observed that combining the two models in this way boosts transcription accuracy to 100.00% on the Bach10 dataset of multiple-instrument polyphonic music (Duan et al., 2010), where the existing state-of-the-art accuracy is 99.00%.

The outline of this paper is as follows. The PLCA-based transcription system is presented in Section 2. The RNN-RBM-based polyphonic music prediction system that is used as a music language model is described in Section 3. The combination of the two aforementioned systems is presented in Section 4. The employed dataset, evaluation metrics, and experimental results are shown in Section 5; finally, conclusions are drawn and future directions are indicated in Section 6.

2. Automatic Music Transcription System

For combining acoustic and music language information in an automatic transcription context, we employ the transcription model of (Benetos & Dixon, 2012), which supports the transcription of multiple-instrument polyphonic music and also supports pitch deviations or frequency modulations. The model of (Benetos & Dixon, 2012) is based on probabilistic latent component analysis (PLCA), which is a latent variable analysis method which has been used for decomposing spectrograms (Shashanka et al., 2008) and can be viewed as a probabilistic version of non-negative matrix factorization (Li & Seung, 1999). For computational efficiency purposes, we employ the fast implementation from (Benetos et al., 2013b), which utilized pre-extracted note templates that are also pre-shifted across log-frequency, in order to account for frequency modulations or tuning changes. In addition, as was shown in (Smaragdis & Mysore, 2009), PLCA-based models can utilise priors for estimating unknown model parameters, which will be useful in this paper for informing the acoustic transcription system with symbolic information.

The transcription model takes as input a normalised log-frequency spectrogram $V_{\omega,t}$ (ω is the log-frequency index and t is the time index) and approximates it as a bivariate probability distribution $P(\omega, t)$. $P(\omega, t)$ is decomposed into a series of log-frequency spectral templates per pitch, instrument, and log-frequency shifting (which indicates deviation with respect to the ideal tuning), as well as probability distributions for pitch, instrument, and tuning.

The model is formulated as:

$$P(\omega, t) = P(t) \sum_{p,f,s} P(\omega|s, p, f) P_t(f|p) P_t(s|p) P_t(p) \quad (1)$$

where p denotes pitch, s denotes the musical instrument source, and f denotes log-frequency shifting (which indicates tuning/pitch deviations). $P(t)$ is the energy of the log-spectrogram, which is a known quantity. $P(\omega|s, p, f)$ denote pre-extracted log-spectral templates per pitch p and instrument s , which are also pre-shifted across log-frequency. The pre-shifting operation is made in order to account for pitch deviations, without needing to formulate a convolutive model across log-frequency. $P_t(f|p)$ is the time-varying log-frequency shifting distribution per pitch, $P_t(s|p)$ is the time-varying source contribution per pitch, and finally, $P_t(p)$ is the pitch activation, which essentially is the resulting music transcription. As a time-frequency representation in the log-frequency domain we use the constant-Q transform (CQT) with a log-spectral resolution of 60 bins/octave (Schörkhuber & Klapuri, 2010).

The unknown model parameters ($P_t(f|p)$, $P_t(s|p)$, $P_t(p)$) can be iteratively estimated using the expectation-maximisation (EM) algorithm (Dempster et al., 1977). For the *Expectation* step, the following posterior is computed:

$$P_t(p, f, s|\omega) = \frac{P(\omega|s, p, f) P_t(f|p) P_t(s|p) P_t(p)}{\sum_{p,f,s} P(\omega|s, p, f) P_t(f|p) P_t(s|p) P_t(p)} \quad (2)$$

For the *Maximization* step (without using any priors) unknown model parameters are updated using the posterior computed from the Expectation step:

$$P_t(f|p) = \frac{\sum_{\omega,s} P_t(p, f, s|\omega) V_{\omega,t}}{\sum_{f,\omega,s} P_t(p, f, s|\omega) V_{\omega,t}} \quad (3)$$

$$P_t(s|p) = \frac{\sum_{\omega,f} P_t(p, f, s|\omega) V_{\omega,t}}{\sum_{s,\omega,f} P_t(p, f, s|\omega) V_{\omega,t}} \quad (4)$$

$$P_t(p) = \frac{\sum_{\omega,f,s} P_t(p, f, s|\omega) V_{\omega,t}}{\sum_{p,\omega,f,s} P_t(p, f, s|\omega) V_{\omega,t}} \quad (5)$$

We consider the sound state templates to be fixed, so no update rule for $P(\omega|s, p, f)$ is applied. Using fixed templates, 20-30 iterations using the update rules presented in

the present section are sufficient for convergence. The output of the system is a pitch activation which is scaled by the energy of the log-spectrogram:

$$P_t(p) \sum_{\omega} V_{\omega,t} \quad (6)$$

After performing 5-sample median filtering for note smoothing, thresholding is performed on $P(p, t)$ followed by minimum note duration pruning set to 40ms (corresponding to the length of one time frame) in order to convert $P(p, t)$ into a binary piano-roll representation, which is the output of the transcription system, and is also used for evaluation purposes.

3. Polyphonic Music Prediction System

Taking inspiration from speech recognition, it has been known that a good statistical model of symbolic music can help the transcription process (Cemgil, 2004). However there are 2 main reasons for the use of MLMs in AMT not being more common. 1. Training models that capture the temporal structure and complexity in symbolic polyphonic music is not an easy task. In speech recognition, often simple language models like n-grams work extremely well. However, symbolic music has more complex structure and simple statistical models like n-grams and HMMs fail to model these characteristics accurately (Boulanger-Lewandowski et al., 2012). 2. There is no consensus on how to incorporate this prior information within the transcription system. Though, recently there have been some successful attempts at using this prior information to improve the accuracy on AMT tasks (Boulanger-Lewandowski et al., 2012; 2013).

In this section we discuss the details of the music prediction system, the models used and the methods used to train them. In the next section we discuss how we incorporate the predictions from these models in a PLCA-based music transcription system.

3.1. Recurrent Neural Network

A recurrent neural network (RNN) is a powerful model for time-series data which is known to account for long-term temporal dependencies, over multiple time-scales when trained effectively. Given a sequence of inputs v_1, v_2, \dots, v_T each in \mathbb{R}^n , the network computes a sequence of hidden states $\hat{h}_1, \hat{h}_2, \dots, \hat{h}_T$ each in \mathbb{R}^m , and a sequence of predictions $\hat{y}_1, \hat{y}_2, \dots, \hat{y}_T$ each in \mathbb{R}^k by iterating the equations

$$h_t = e(W_{\hat{h}x}v_t + W_{\hat{h}\hat{h}}\hat{h}_{t-1} + b_{\hat{h}}) \quad (7)$$

$$\hat{y}_t = g(W_{y\hat{h}}\hat{h}_t) \quad (8)$$

where $W_{y\hat{h}}$, $W_{\hat{h}x}$, $W_{\hat{h}\hat{h}}$ are the weight matrices and $b_{\hat{h}}$, b_y

are the biases and e and g are activation functions which are typically non-linear and applied element-wise.

In theory, a recurrent neural network can be easily trained using the gradient-based Back-Propagation Through Time algorithm (Werbos, 1990) using the exactly computable error gradients in the network. However, 1st order gradient methods fail to correctly train RNNs for many real-world problems. This difficulty has been associated with what is known as the *vanishing/exploding gradients* phenomenon (Bengio et al., 1994), where the errors exhibit exponential decay/growth as they are back-propagated through time.

However, the research being carried out by the neural network and deep learning community has led to several improvements in gradient based optimization methods that make training of RNNs on real-world data possible. Most notably, the Hessian Free (HF) optimization algorithm has been used to train RNNs successfully on several real world datasets, including symbolic polyphonic music data (Martens & Sutskever, 2011). Apart from second order methods like HF, several modifications to first-order gradient based methods exist that currently form the state of the art in training RNNs (Bengio et al., 2012).

3.2. Recurrent Neural Network-based models

One of the drawbacks of using RNNs to predict polyphonic symbolic music is that the outputs of the network, \hat{y}_T at any time step T , are conditionally independent given the sequence of input vectors v_1, v_2, \dots, v_T . This is a severe constraint when used for modelling polyphonic music, where notes often appear in very correlated patterns within a frame. In order to overcome this limitation, models derived from RNNs have been proposed which model high-dimensional conditional distributions at every time step (Sutskever et al., 2008; Boulanger-Lewandowski et al., 2012).

The first RNN-based model that tried to model high-dimensional distributions at every time-step was the Recurrent Temporal Restricted Boltzmann Machine (RTRBM) (Sutskever et al., 2008). The RTRBM is a conditional RBM, where the RBM parameters at time t are a function of the visible and hidden hidden units till time $\tau < t$. The hidden states of the RTRBM are updated as if they belonged to a regular RNN. However a major constraint with the RTRBM was that the hidden states were responsible for conveying both temporal information and modelling the conditional distribution at each time-step. The RTRBM model was later extended to the more general RNN-RBM model (Boulanger-Lewandowski et al., 2012). In this model, there is a separate hidden state which conveys the temporal information. The hidden state of the RBM is then free to model only the distribution at that time step. It was found that the RNN-RBM performs bet-

ter than the RTRBM at modelling high-dimensional conditional distributions (Boulanger-Lewandowski et al., 2012).

For our prediction system, we make use of a variant of the RNN-RBM, called the RNN-NADE. The only difference being that the conditional distributions at each step are modelled by a Neural Autoregressive Distribution Estimator (NADE) (?) as opposed to an RBM. As discussed in the next section, to combine the predictions with the transcription system, we need individual pitch activation probabilities at each time-step. Obtaining these probabilities from an RBM is intractable as it requires summing over all possible hidden states. However the NADE is a tractable distribution estimator and we can easily obtain these probabilities from the NADE. The NADE models the probability of occurrence of a vector v as:

$$p(v) = \prod_{i=1}^D p(v_i | \mathbf{v}_{<i}) \quad (9)$$

where $v \in \mathbb{R}^D$.

In our system we utilise each of the conditional probabilities $p(v_i | \mathbf{v}_{<i})$ as probabilities of the pitch activations. Although the pitch activation probabilities are only conditioned on $\mathbf{v}_{<i}$, we hope that this would be a better model than the RNN, where the pitch activation probabilities are completely independent. Another motivation for using the NADE is that the gradients can be computed exactly, and therefore we can employ HF optimization for training the RNN-NADE.

3.3. Training

We trained the RNN to predict the next frame of symbolic music given a sequence. We trained the RNN using both gradient descent and HF optimization. The models were trained on sequences of length 200. For the NADE we employed a similar strategy and trained it using both gradient descent and HF optimization. The models were trained separately on data from the Nottingham dataset and the JSB Chorales dataset. We validate our models, by computing prediction accuracies as described in the next section.

4. Combining Transcription and Prediction

In this section, we describe the process for combining the acoustic model with the music language model for deriving an improved transcription. Firstly, the input music signal is transcribed using the process described in Section 2. The resulting piano-roll representation of the transcription system is considered to be a sequence v_1, v_2, \dots, v_T that is placed as input to the MLM presented in Section 3. We compute the probability $p(v_i = 1 | v_{\text{parents}(i)})$ for all time frames, and use that as prior information for the combined model (the prior information will be denoted as

$P_{MLM}(p, t)$).

As shown in (Smaragdis & Mysore, 2009), PLCA-based models use multinomial distributions; since the Dirichlet distribution is conjugate to the multinomial, a Dirichlet prior can be used to enforce structure on the pitch activation distribution $P_t(p)$. Following the procedure of (Smaragdis & Mysore, 2009), we define the Dirichlet hyperparameter for the pitch activation as:

$$\alpha(p|t) = \frac{P(p|t)P_{MLM}(p, t)}{\sum_p P(p|t)P_{MLM}(p, t)} \quad (10)$$

where $\alpha(p|t)$ essentially is a pitch activation probability which is filtered through a pitch indicator function computed from the symbolic prediction step (the denominator is simply for normalisation purposes).

The recording is then re-transcribed, using as additional information the prior computed from the transcription step. The modified update for the pitch activation which replaces (5) is given by:

$$P_t(p) = \frac{\sum_{\omega, f, s} P_t(p, f, s | \omega) V_{\omega, t} + \kappa \alpha(p|t)}{\sum_{p, \omega, f, s} P_t(p, f, s | \omega) V_{\omega, t} + \kappa \alpha(p|t)} \quad (11)$$

where κ is a weight parameter expressing how much the prior should be imposed; as in (Smaragdis & Mysore, 2009), the weight decreases from 1 to 0 throughout the iterations. In a larger context, the transcription creates a symbolic prediction, which in turn improves the subsequent re-transcription of the music signal. An overview of the complete transcription-prediction system architecture can be seen in Fig. 4.

5. Evaluation

5.1. Dataset

For testing the transcription system, we employ the Bach10 dataset (Duan et al., 2010), which is a freely available multi-track collection of multiple-instrument polyphonic music, suitable for multi-pitch detection experiments. It consists of ten recordings of J.S. Bach chorales, performed by violin, clarinet, saxophone, and bassoon. Pitch ground truth for each instrument is also provided. Due to the tonal and homogenous content of the dataset, it is suitable for testing the incorporation of music language models in a transcription system. For training the transcription system, pre-extracted and pre-shifted spectral templates are extracted for the instruments present in the dataset, using isolated note samples from the RWC database (Goto et al., 2003).

5.2. Metrics

For evaluating the performance of the proposed system for multi-pitch detection, we employ the precision, recall, and

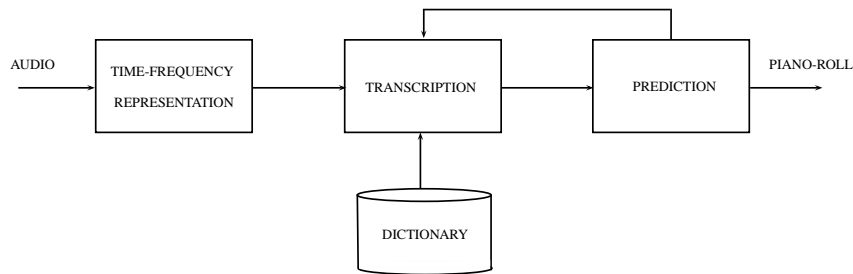


Figure 1. Proposed system diagram.

F-measure metrics, which are commonly used in transcription evaluations (MIR):

$$Pre = \frac{N_{tp}}{N_{sys}}, \quad Rec = \frac{N_{tp}}{N_{ref}}, \quad F = \frac{2 \cdot Rec \cdot Pre}{Rec + Pre} \quad (12)$$

where N_{tp} is the number of correctly detected pitches, N_{sys} is the number of detected pitches, and N_{ref} is the number of ground-truth pitches. As in the public evaluations on multi-pitch detection carried out through the MIREX framework (MIR), a detected note is considered correct is if its pitch is the same as the ground truth pitch and its onset is within a 50ms tolerance interval of the ground-truth onset.

5.3. Results

Multi-pitch detection experiments are performed using the proposed system, with various configurations. A first configuration only considers the transcription system from Section 2. A second configuration takes the output of the transcription system and gives it as input to the prediction system of Section 3, where the final piano-roll is the output of the prediction step. A third configuration is the one presented in Section 4, where the recording is re-transcribed, having the prediction as a prior information for estimating the pitch activations. For the prediction system, experiments were made using both the RNN-NADE and the RNN.

Results using the various system configurations are displayed in Table 1.

For comparison with the method of (?) (where the Bach10 dataset was first introduced), the proposed method using the frame-based accuracy metric reaches 71.4%, whereas the method of (?) reaches 69.7% (with unknown polyphony).

6. Conclusions

In this paper, we proposed a system for automatic music transcription which incorporated prior information from a polyphonic music prediction model based on recurrent neu-

Configuration	F	Pre	Rec
Configuration 1	62.02%	58.51%	66.12%
Configuration 2 - NADE	62.62%	59.70%	65.92%
Configuration 3 - NADE	64.08%	61.96%	66.44%
Configuration 2 - RNN	62.29%	59.08%	65.98%
Configuration 3 - RNN	63.85%	61.14%	66.90%
Configuration 2 - NADE-HFO	%	%	%
Configuration 3 - NADE-HFO	%	%	%

Table 1. Transcription results using various system configurations.

ral networks. The acoustic transcription model was based on probabilistic latent component analysis, and information from the prediction system was incorporated using Dirichlet priors. Experimental results using the Bach10 dataset of multiple-instrument recordings showed that there is a clear and significant improvement (more than 2% in terms of F-measure) by combining a music language model with an acoustic model for improving the performance of the latter.

In the future, we will evaluate the proposed system using language models trained from different sources. We will also investigate different system configurations, by bootstrapping the system for demonstrating that an improved transcription can lead to an improved prediction, and so on. Finally, ++(other prediction models?)

References

- Music Information Retrieval Evaluation eXchange (MIREX). <http://music-ir.org/mirexwiki/>.
- Benetos, E., Dixon, S., Giannoulis, D., Kirchhoff, H., and Klapuri, A. Automatic music transcription: challenges and future directions. *Journal of Intelligent Information Systems*, 41(3):407–434, December 2013a. doi: 10.1007/s10844-013-0258-3.
- Benetos, Emmanouil and Dixon, Simon. A shift-invariant latent variable model for automatic music transcription. *Computer Music Journal*, 36(4):81–94, 2012.

- Benetos, Emmanouil, Cherla, Srikanth, and Weyde, Tillman. An efficient shiftinvariant model for polyphonic music transcription. In *6th International Workshop on Machine Learning and Music*, 2013b.
- Bengio, Yoshua, Simard, Patrice, and Frasconi, Paolo. Learning long-term dependencies with gradient descent is difficult. *Neural Networks, IEEE Transactions on*, 5 (2):157–166, 1994.
- Bengio, Yoshua, Boulanger-Lewandowski, Nicolas, and Pascanu, Razvan. Advances in optimizing recurrent networks. 2012.
- Bertin, N., Badeau, R., and Vincent, E. Enforcing harmonicity and smoothness in bayesian non-negative matrix factorization applied to polyphonic music transcription. *IEEE Transactions on Audio, Speech, and Language Processing*, 18(3):538–549, March 2010.
- Bock, S. and Schedl, M. Polyphonic piano note transcription with recurrent neural networks. In *2012 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pp. 121–124, March 2012.
- Boulanger-Lewandowski, N., Bengio, Y., and Vincent, P. Modeling temporal dependencies in high-dimensional sequences: Application to polyphonic music generation and transcription. In *29th International Conference on Machine Learning*, Edinburgh, Scotland, UK, 2012.
- Boulanger-Lewandowski, N., Bengio, Y., and Vincent, P. High-dimensional sequence transduction. In *2013 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pp. 3178–3182, May 2013.
- Cemgil, A. T. *Bayesian Music Transcription*. PhD thesis, Radboud University of Nijmegen, 2004.
- Dempster, A. P., Laird, N. M., and Rubin, D. B. Maximum likelihood from incomplete data via the EM algorithm. *Journal of the Royal Statistical Society*, 39(1): 1–38, 1977.
- Duan, Z., Pardo, B., and Zhang, C. Multiple fundamental frequency estimation by modeling spectral peaks and non-peak regions. *IEEE Transactions on Audio, Speech, and Language Processing*, 18(8):2121–2133, November 2010.
- Ewert, Sebastian and Müller, Meinard. Score-informed voice separation for piano recordings. In *ISMIR*, pp. 245–250, 2011.
- Ewert, Sebastian and Müller, Meinard. Score-informed source separation for music signals. In *Multimodal Music Processing*, pp. 73–94, 2012.
- Goto, M., Hashiguchi, H., Nishimura, T., and Oka, R. RWC music database: music genre database and musical instrument sound database. In *International Conference on Music Information Retrieval*, Baltimore, USA, October 2003.
- Larochelle, Hugo and Murray, Iain. The neural autoregressive distribution estimator. *Journal of Machine Learning Research*, 15:29–37, 2011.
- Li, D. D. and Seung, H. S. Learning the parts of objects by non-negative matrix factorization. *Nature*, 401:788–791, October 1999.
- Martens, James and Sutskever, Ilya. Learning recurrent neural networks with hessian-free optimization. In *Proceedings of the 28th International Conference on Machine Learning (ICML-11)*, pp. 1033–1040, 2011.
- Nam, J., , Ngiam, J., Lee, H., and Slaney, M. A classification-based polyphonic piano transcription approach using learned feature representations. In *12th International Society for Music Information Retrieval Conference*, pp. 175–180, Miami, Florida, USA, October 2011.
- Rabiner, Lawrence and Juang, Biing-Hwang. Fundamentals of speech recognition. 1993.
- Schörkhuber, C. and Klapuri, A. Constant-Q transform toolbox for music processing. In *7th Sound and Music Computing Conf.*, Barcelona, Spain, July 2010.
- Sentürk, Sertan, Gulati, Sankalp, and Serra, Xavier. Score informed tonic identification for makam music of turkey. In *14th Int. Soc. for Music Info. Retrieval Conf. Curitiba, Brazil.(to appear)*, volume 195, pp. 206, 2013.
- Shashanka, M., Raj, B., and Smaragdis, P. Probabilistic latent variable models as nonnegative factorizations. *Computational Intelligence and Neuroscience*, 2008. Article ID 947438.
- Smaragdis, P. and Mysore, G. Separation by “humming”: user-guided sound extraction from monophonic mixtures. pp. 69–72, October 2009.
- Smaragdis, P., Raj, B., and Shashanka, Ma. A probabilistic latent variable model for acoustic modeling. In *Neural Information Processing Systems Workshop*, Whistler, Canada, December 2006.
- Sutskever, Ilya, Hinton, Geoffrey E, and Taylor, Graham W. The recurrent temporal restricted boltzmann machine. In *Advances in Neural Information Processing Systems*, pp. 1601–1608, 2008.
- Werbos, Paul J. Backpropagation through time: what it does and how to do it. *Proceedings of the IEEE*, 78(10): 1550–1560, 1990.