Assignment 4
MA/FIM 548 Monte Carlo Methods for FM
Spring 2022

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Deadline: 3 pm, April 22, 2022

Instructions:

- Please submit one work per group.
- It counts for 10% of your final grade.
- Submit your work as single pdf file on Moodle before 3 pm on April 22.
- Late assignments will not be accepted.
- NO interactions between groups.
- Each group member must make a substantial contribution to each part of the assignment.
- It is not acceptable, e.g., to divide the assignments amongst the team members.
- You can use any software (Matlab, Python, C/C++, R etc.)

1. We consider fifth-to-default swap on $N = 10$ assets. The interest rate is $r = 4\%$, the maturity is $T = 5$ years. We have 5 annual protection payments with size s .If the fifth default occurs before T , these payments cease and the protection seller makes a payment $1 - R$ to the protection buyer, where $R = r_i$ is the recovery rate of firm i given that it was fifth default. Assets have constant default intensity rates (0.05, 0.01, 0.05, 0.05, 0.01, 0.1, 0.01, 0.09, 0.1, 0.02). The recovery rates are (0.1, 0.1, 0.3, 0.1, 0.3, 0.1, 0.2, 0.2, 0.1, 0.1). They are correlated and the correlation is modeled through normal copula with matrix Σ , that have $\Sigma_{ii} = 1$, $i = 1, \ldots, 10$, and $\Sigma_{i,j} = 0.2$, $i \neq j$.

   Apply MC method to estimate the value of s that ensures zero value of CDS

   $$\mathbb{E}[V(\tau_1, \ldots, \tau_N)] = 0$$

   where

   $$V(\tau_1, \ldots, \tau_N) = V_{\text{value}}(\tau_1, \ldots, \tau_N) - V_{\text{prot}}(\tau_1, \ldots, \tau_N)$$

   with

   $$V_{\text{value}}(\tau_1, \ldots, \tau_N) = (1 - R)e^{-r\tau} I(\tau \leqslant T)$$

   and

   $$V_{\text{prot}} = \begin{cases} \sum_{i=1}^{j} s e^{-rT_i} + s e^{-r\tau} \frac{\tau - T_j}{T_{j+1} - T_j}, & \text{if } T_j \leqslant \tau \leqslant T_{j+1} \\ \sum_{i=1}^{m} s e^{-rT_i}, & \text{if } \tau > T \end{cases}$$

   and where τ is the time of the fifth default.

2. Let us suppose that a unicorn is raising \$100 million of new VC investment at \$1 per share in a Series B round with a post–money valuation of \$1 billion. In the past, this company raised \$50 million of VC investment in a Series A round with a post–money valuation of \$450, and pari passu seniority with, the newly issued shares. Using subscripts to denote the different rounds, $P_A = 450$, $P_B = 1,000$, $I_A = 50$, and $I_B = 100$ (in millions). After the current round, if all shares convert, the new investor owns 10% of the total shares, the old investor owns 10%, and the current common shareholders own the remaining 80%.

   We assume that the asset value follows geometric Brownian motion. Parameters are $r = 0.025$, $\sigma = 0.9$.

   Exit time T is random (independent of X) and has the following pdf

   $$f(x) = \frac{1}{18}x, \quad 0 \leq x \leq 6.$$

   If $T < 6$, the exit payoff is the sum of the payoff in an IPO, $f^{IPO}(X(T))$, and the payoff in M&A or liquidation, $f^{M\&A}(X(T))$, weighted by the probability of each outcome conditional on the exit value, $p^{IPO}(X(T))$ and $1 - p^{IPO}(X(T))$

   $$f(X(T)) = p^{IPO}(X(T)) f^{IPO}(X(T)) + \left(1 - p^{IPO}(X(T))\right) f^{M\&A}(X(T))$$

   We assume that IPOs happen for all exits above \$1 billion and all other exits are M&A.

If $T = 6$, let us suppose that the exit is $M\&A$, i.e.,

$$f(X(T)) = f^{M\&A}(X(T))$$

The seniority rule is pari passu with the old investor, and Series B investor's payout in M&A is

$$f_B^{M\&A}(X) = \max\left\{\min\left\{\frac{I_B}{I_A + I_B}X, I_B\right\}, X \times \frac{I_B}{P_B}\right\}.$$

The payout to Series B investor in an IPO is the converted payoff

$$f_B^{IPO}(X) = X \times \frac{I_B}{P_B}.$$

Make use of the MC method to estimate the fair value $X(0)$ of company so that

$$I_B = \mathbb{E}\left[e^{-rT} f\left(X(0)e^{\left(r-\sigma^2/2\right)T+\sigma\sqrt{T}Z}\right)\right].$$

Also calculate overvaluation $100(P_B - X(0))/X(0)$ in %.