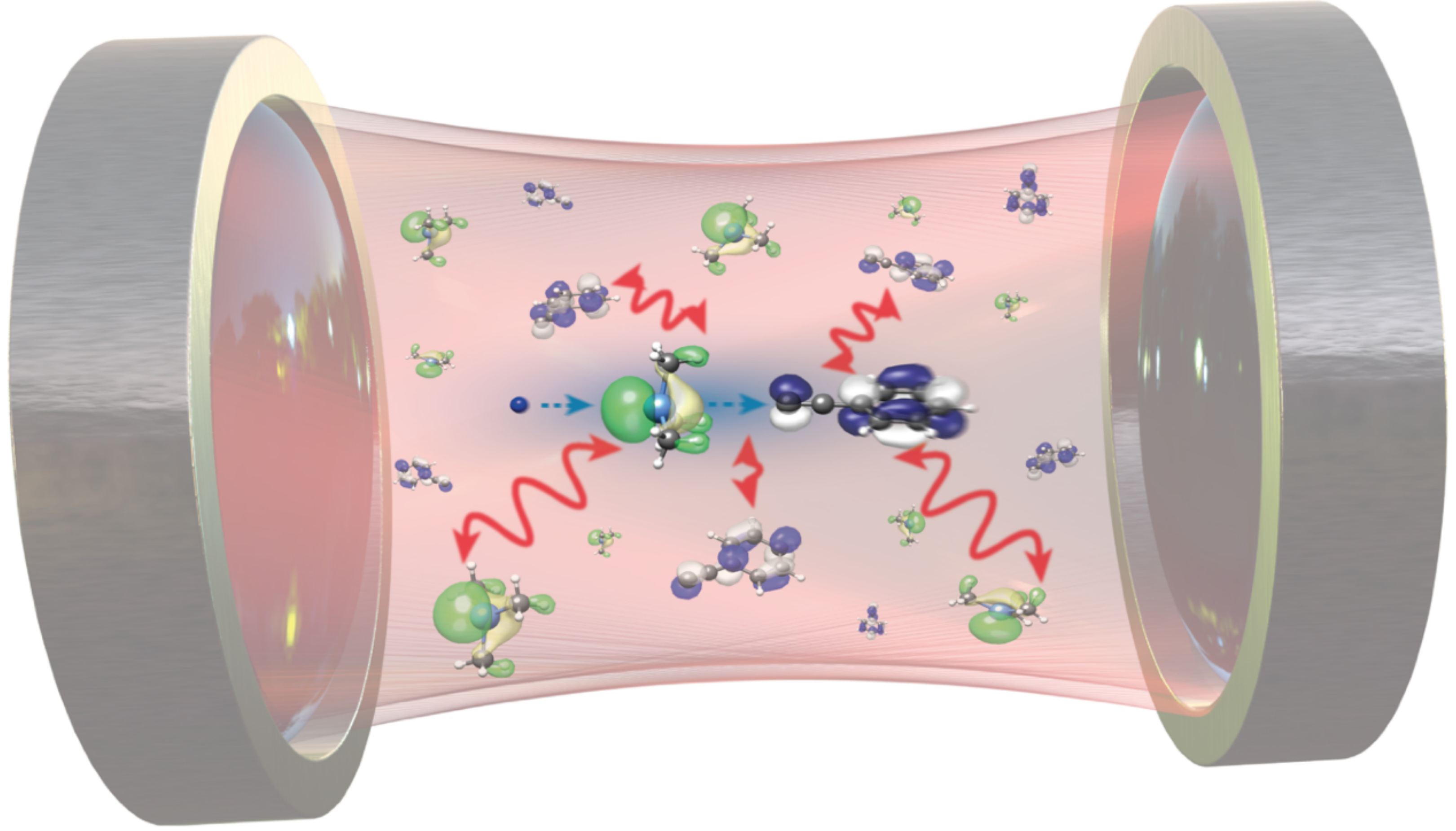
Dominik Sidler, 2025

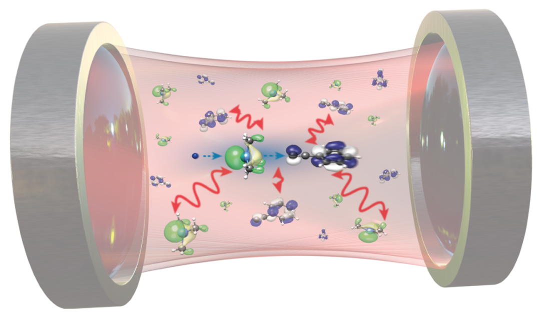
Polaritonic / QED Chemistry

Lecture 9: Literature Talk: Quantum Hall Effect in a Cavity



Max-Planck-Institut für Struktur und Dynamik der Materie
PSI
University of Zurich

Outline



PHYSICAL REVIEW LETTERS **131,** 196602 (2023)

Weakened Topological Protection of the Quantum Hall Effect in a Cavity

Vasil Rokaj,[1,2,*] Jie Wang,[2,3,†] John Sous,[4,5,6] Markus Penz,[7] Michael Ruggenthaler,[8] and Angel Rubio[8,9,‡]
[1]*ITAMP, Center for Astrophysics | Harvard & Smithsonian, Cambridge, Massachusetts 02138, USA*

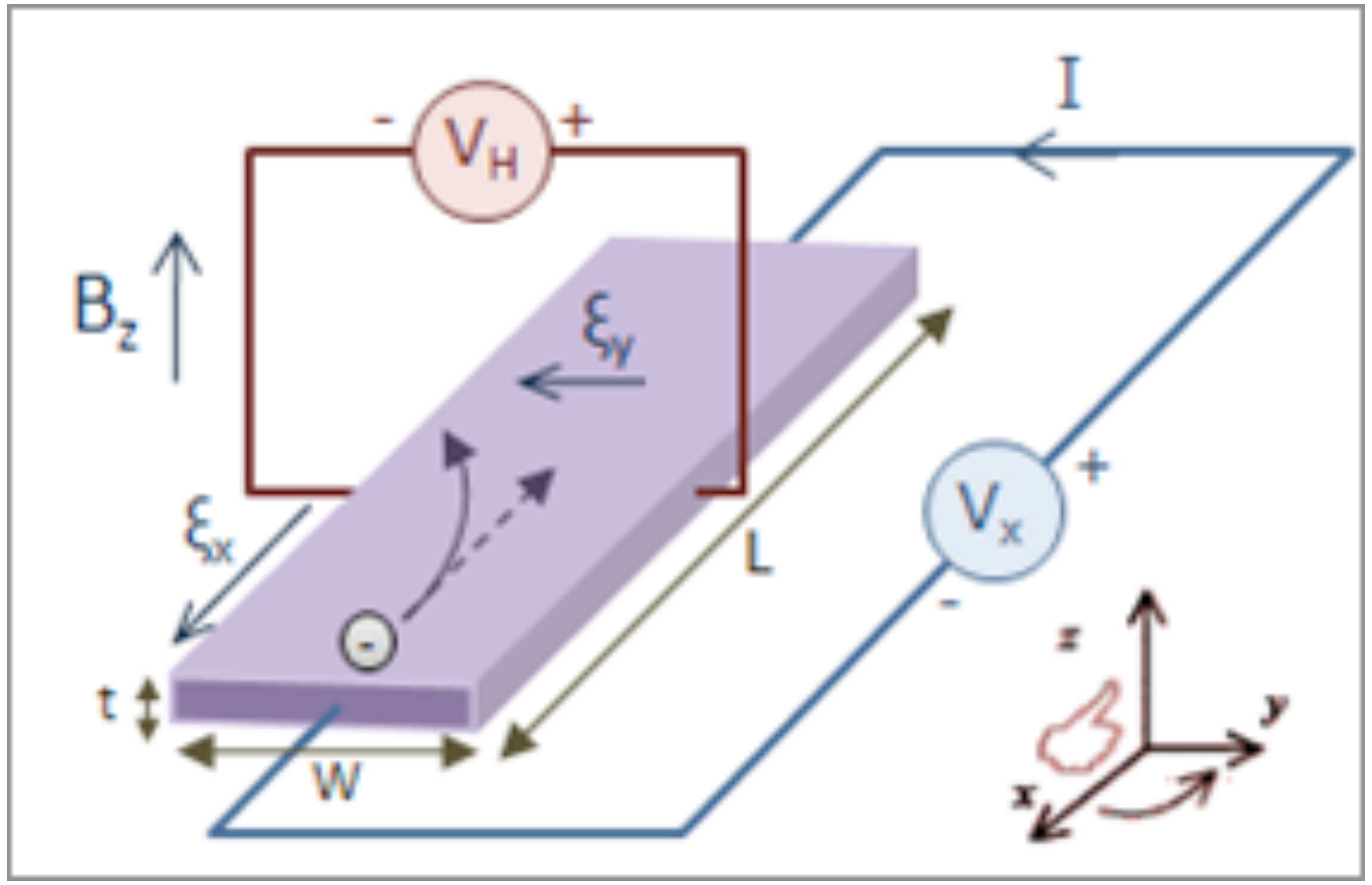
RESEARCH

QUANTUM OPTICS

Breakdown of topological protection by cavity vacuum fields in the integer quantum Hall effect

Felice Appugliese[1*], Josefine Enkner[1], Gian Lorenzo Paravicini-Bagliani[1†], Mattias Beck[1], Christian Reichl[2], Werner Wegscheider[2], Giacomo Scalari[1], Cristiano Ciuti[3], Jérôme Faist[1*]

Hall Effect



Quantum Hall Effect

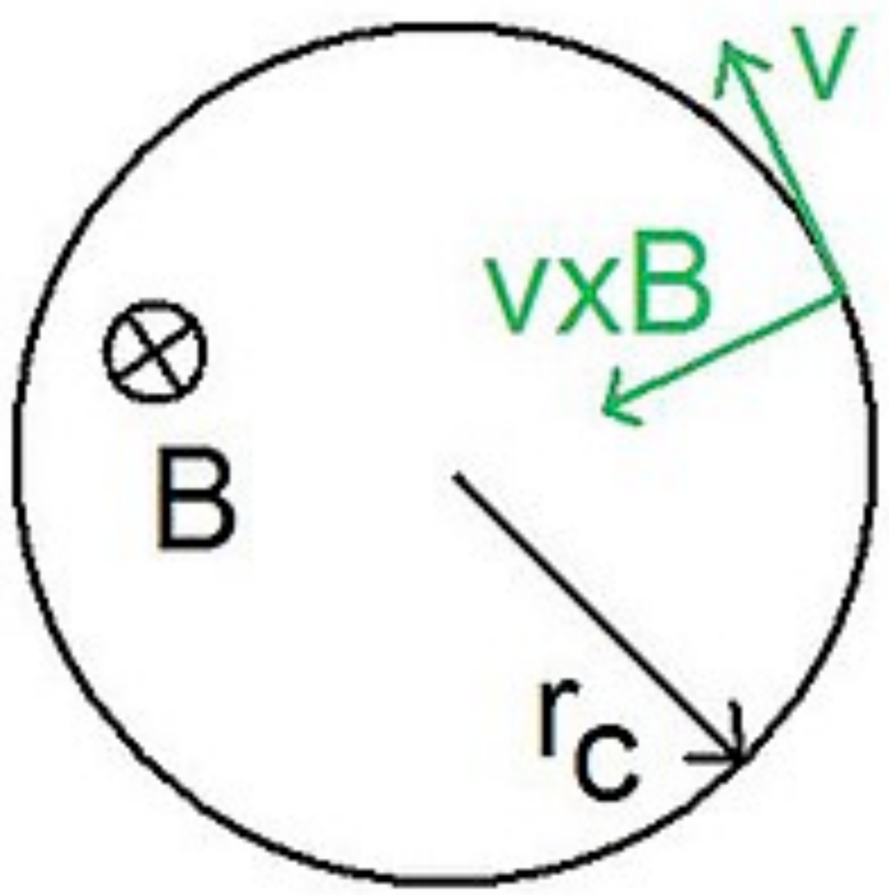
Introduction to Landau Levels in (free) electron gas

Free electron (gas) in x-y plane (2D) with perpendicular magnetic field B_z.

$$\hat{H} = \frac{1}{2m}\left(\hat{\mathbf{p}} - q\hat{\mathbf{A}}\right)^2$$

Landau Gauge choice:

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ B \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} 0 \\ B \cdot x \\ 0 \end{pmatrix} \qquad \hat{H} = \frac{\hat{p}_x^2}{2m} + \frac{1}{2m}\left(\hat{p}_y - qB\hat{x}\right)^2 + \frac{\hat{p}_z^2}{2m}$$



Cyclotron frequency: $\omega_c = qB/m$

Harmonic oscillator problem with shifted minimum of potential, which does not affect energies (but the wave-function). => Degenerate states with respect to quantum number k_y.

$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \frac{1}{2}m\omega_c^2\left(\hat{x} - \frac{\hbar k_y}{m\omega_c}\right)^2 + \frac{\hat{p}_z^2}{2m}$$

Degenerate Landau Level n: $E_n = \hbar\omega_c\left(n + \frac{1}{2}\right) + \frac{p_z^2}{2m}$

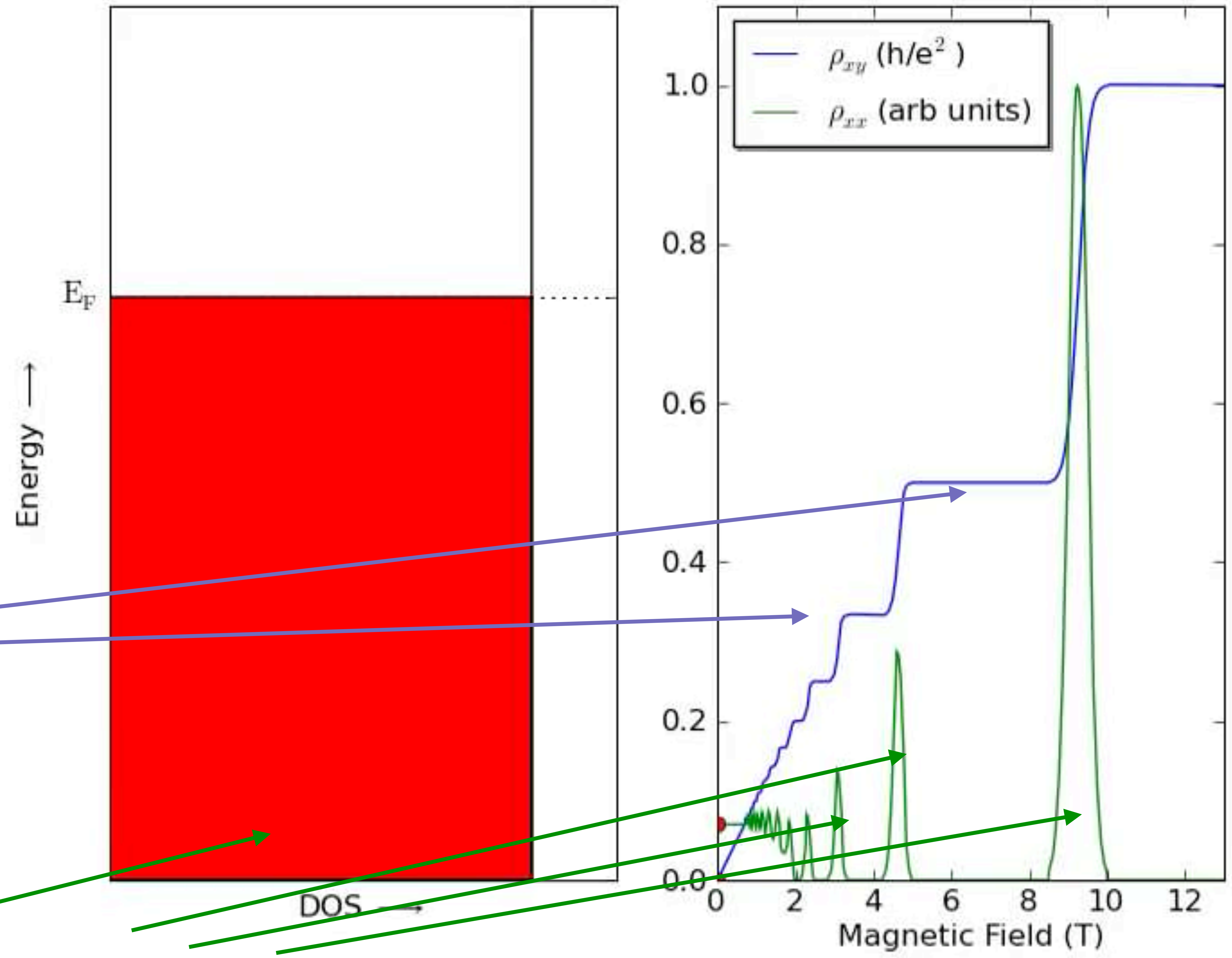
Quantum Hall Effect

Quantized Resistance

Transverse Hall resistance: $\rho_{xy} = \frac{V_{\text{Hall}}}{I_{\text{channel}}} = \frac{h}{\nu e^2}$

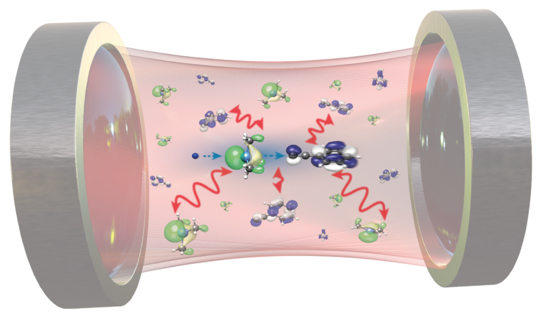
Integer QHE: $\nu = 1,2,3,...$

Striking feature: persistence of quantization while electron density or magnetic field is varied (**Hall plateaus**).
Attributed to **topological protection** (beyond the scope of this lecture), i.e., the continuously varying magnetic field does „not" change topology, it only deforms it. Thus transverse Hall resistance is discrete (integer quantized).



Quantized Landau levels

Quantum Hall Effect in a Cavity



Theory

$$\mathbf{B} = \nabla \times \mathbf{A}_{\text{ext}}(\mathbf{r}) = B\mathbf{e}_z \qquad \mathbf{A}_{\text{ext}}(\mathbf{r}) = -\mathbf{e}_x By$$

$$H = \sum_{i=1}^{N} \frac{(\boldsymbol{\pi}_i + e\mathbf{A})^2}{2m} + \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) + \sum_{i<j} W(\mathbf{r}_i - \mathbf{r}_j),$$

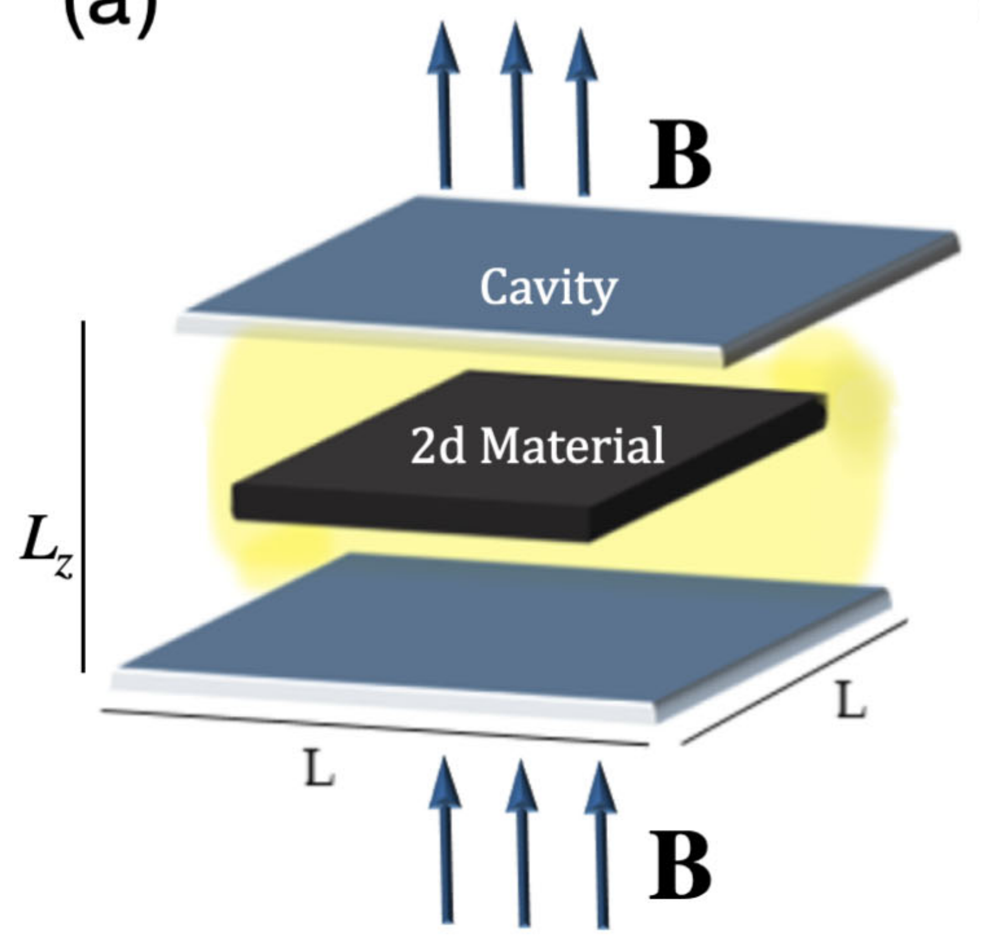
$$\boldsymbol{\pi}_i = i\hbar\nabla_i + e\mathbf{A}_{\text{ext}}(\mathbf{r}_i)$$

Cavity field (velocity gauge)

$$\mathbf{A} = \sqrt{(\hbar/2\epsilon_0 \mathcal{V}\omega)}\mathbf{e}_x(a + a^\dagger)$$

Cavity frequency ω, modevolume $\mathscr{V}$

(a)



For homogeneous system: Center of mass (COM) decouples from relative electron motion.

Look at kinetics of COM. $\mathbf{R} = (X, Y) = \sum_{i=1}^{N} \mathbf{r}_i/\sqrt{N}$

$$H_{\text{c.m.}} = \frac{1}{2m}\left(\boldsymbol{\Pi} + e\sqrt{N}\mathbf{A}\right)^2 + \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) \qquad \boldsymbol{\Pi} = i\hbar\nabla_{\mathbf{R}} + e\mathbf{A}_{\text{ext}}(\mathbf{R})$$

2 coupled quantum harmonic oscillators (Landau levels and photons) can be solved by Hopfield transformation

$$H_{\text{c.m.}} = \boxed{\hbar\Omega_+\left(b_+^\dagger b_+ + \frac{1}{2}\right)} + \hbar\Omega_-\left(b_-^\dagger b_- + \frac{1}{2}\right)$$

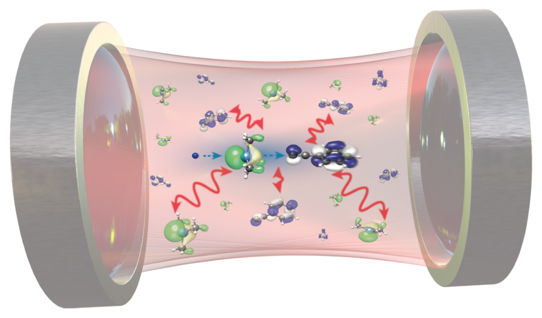
Upper and lower Landau polaritons:

$$\boxed{\Omega_\pm^2 = \frac{\omega^2 + \omega_d^2 + \omega_c^2}{2} \boxed{\pm} \sqrt{\omega_d^2\omega_c^2 + \left(\frac{\omega^2 + \omega_d^2 - \omega_c^2}{2}\right)^2}}$$

$\{b_\pm^\dagger, b_\pm\}$ are the creation and annihilation operators

https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.131.196602

Quantum Hall Effect in a Cavity



Theory

$$H_{\text{c.m.}} = \hbar\Omega_+\left(b_+^\dagger b_+ + \frac{1}{2}\right) + \hbar\Omega_-\left(b_-^\dagger b_- + \frac{1}{2}\right)$$

$\{b_\pm^\dagger, b_\pm\}$ are the creation and annihilation operators

Upper and lower Landau polaritons:

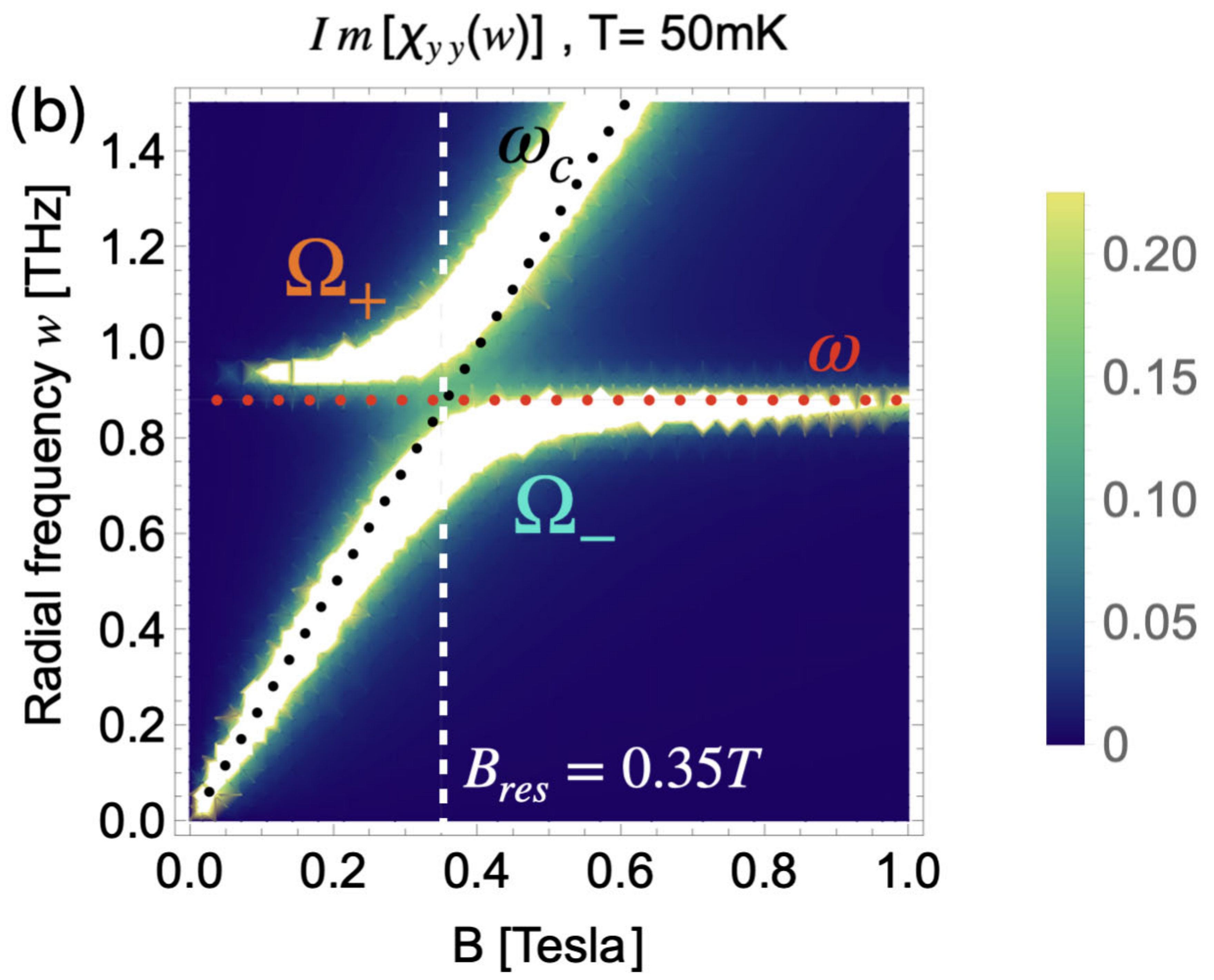
$$\boxed{\Omega_\pm^2 = \frac{\omega^2+\omega_d^2+\omega_c^2}{2} \pm \sqrt{\omega_d^2\omega_c^2 + \left(\frac{\omega^2+\omega_d^2-\omega_c^2}{2}\right)^2}}$$

Blueshift (diamagentic shift) from A^2 (electronic strong coupling) $\omega_d = \sqrt{e^2N/m\epsilon_0\mathcal{V}}$

Cyclotron frequency: $\omega_c = qB/m$

Cavity frequency: ω

Resonance condition: $\omega = \omega_c$



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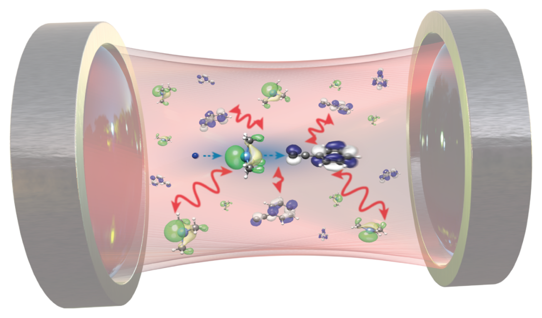
Quantum Hall Effect in a Cavity



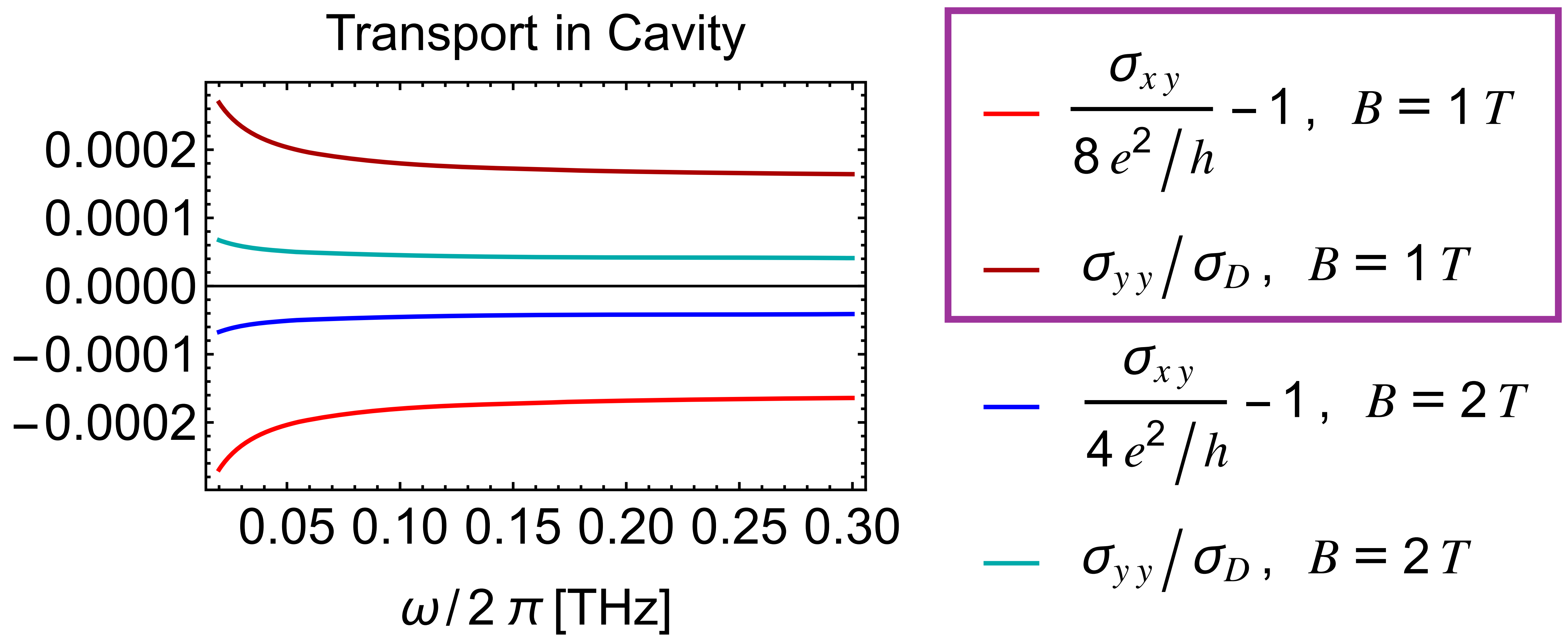
Theory: Breakdown of topological protection

Deviation from perfectly quantized Hall conductance σ_{xy} attributed to finite polariton lifetime $\tau = 1/\delta$. Deviations stronger for small magnetic fields (1T).

$$\sigma_{xy} = \frac{e^2\nu}{h(1+\Lambda^2)}\left[\frac{\Lambda(\Lambda+\eta)}{\Omega_-^2/\omega_c^2+\delta^2/\omega_c^2} + \frac{1-\eta\Lambda}{\Omega_+^2/\omega_c^2+\delta^2/\omega_c^2}\right],$$

$$\eta = \omega_d/\omega_c \qquad \nu = n_{2\mathrm{D}}h/eB$$

Mixing of light and electrons
$$\Lambda = \alpha - \sqrt{1+\alpha^2}$$
$$\alpha = (\omega_c^2 - \omega^2 - \omega_d^2)/2\omega_d\omega_c,$$



Standard Hall conductance for $\delta \to 0$
$$\sigma_{xy} = e^2\nu/h,$$

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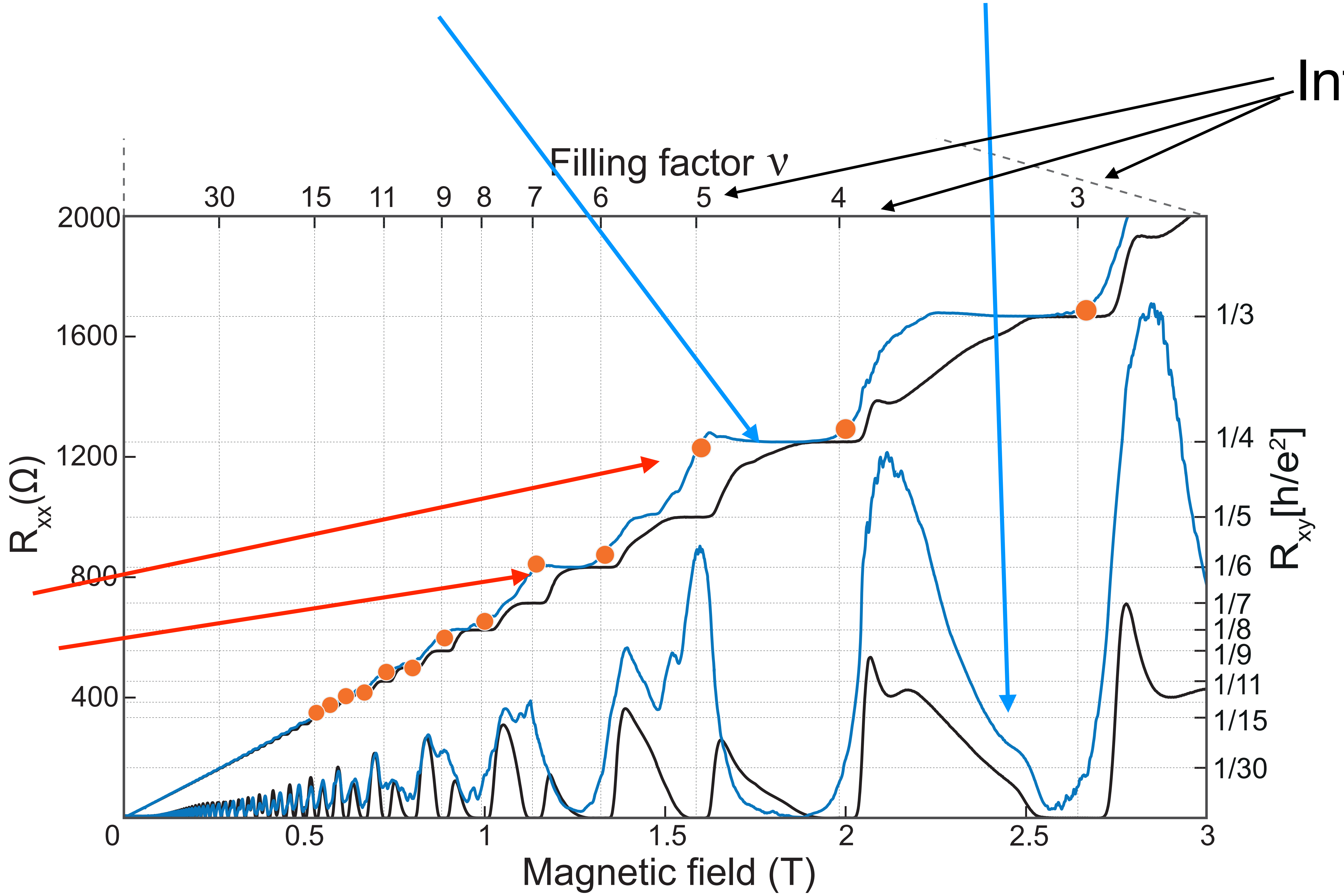
Quantum Hall Effect in a Cavity

Experiment: Transversal R_{xy} and Longitudinal R_{xx} Hall Resistivity

Cavity

No cavity

Breakdown of „topological protection" in a cavity



https://www.science.org/doi/epdf/10.1126/science.abl5818