Assignment 1

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Question

Given:

- $N(t)$ is the population density observed at time t
- K is the rate of reproduction per unit time
- The differential equation in terms of population density at time t and the time of calculation is given by,

$$\frac{dN(t)}{dt} = KN(t)$$

The condition is to solve the above equation in terms of N(t) and t and obtain the time taken to double the population.

Assumption:

Consider the value of K is independent wih respect to time t

Solution

Method - 1: From the given condition,

$$\frac{dN(t)}{dt} = KN(t)$$

$$L\left(\frac{dN(t)}{dt}\right) = L(K.N(t))$$

Applying Laplace Transform on both sides,

$$sN(s) - N(0) = KN(s)$$

$$sN(s) - KN(s) = N_0$$

$$N(s)(s - K) = N_0$$

$$N(s) = \frac{N_0}{(s - K)}$$

Applying Inverse Laplace Transform on both sides,

$$N(t) = N_0.e^{Kt} \qquad \text{(a)}$$

Solution

Method - 1: The time taken to double the population density w.r.t. initial population density is,

$$N(t) = 2N_0$$
$$\frac{N(t)}{N_0} = 2$$

From (a),

$$e^{Kt} = 2$$
$$Kt = \log_e 2$$
$$t = \frac{\log_e 2}{K} \tag{b}$$

Solution

Method - 2: From the given condition,

$$\frac{dN(t)}{dt} = KN(t)$$
$$\frac{dN(t)}{N(t)} = Kdt$$

Integrating on both the sides,

$$\int_{N_0}^{N(t)} \frac{dN(t)}{N(t)} = \int_0^t Kdt$$
$$\log_e N(t)\Big|_{N_0}^{N(t)} = K.(t-0)$$
$$\log_e N(t) - \log_e N_0 = K.t$$
$$\log_e \left(\frac{N(t)}{N_0}\right) = K.t$$
$$\left(\frac{N(t)}{N_0}\right) = e^{Kt}$$
$$N(t) = N_0.e^{Kt} \qquad \text{(a)}$$

Solution

Method - 2: The time taken to double the population density w.r.t. initial population density is,

$$N(t) = 2N(0)$$
$$\frac{N(t)}{N(0)} = 2$$

From (a),

$$e^{Kt} = 2$$
$$Kt = \log_e 2$$
$$t = \frac{\log_e 2}{K} \qquad \text{(b)}$$

Answers

Therefore the answers for (a) and (b) of the question are,

$$N(t) = N_0.e^{Kt} \tag{a}$$

$$t = \frac{\log_e 2}{K} \tag{b}$$