**Algorithms-1 - CS21003**
**(Class Test II)**
**Date: 09 – October – 2021**

Maximum marks: 30 Duration: 1 hour

**File naming convention: e.g., 18CS3004_G3_CT2.pdf (or any other extension).**
**In case of multiple files, use _1, _2 etc at the end.**
**Submission is via Moodle only. Email submissions will NOT be accepted. Please manage your time well keeping in mind that Internet and power disruptions are a new normal!**
**No clarifications from the TAs today. You can make any assumption as long as it is rational and you clearly state the same while solving the problem.**
**Plagiarism, in any form (including Internet source) will be severely penalized.**
**Whenever pseudocodes are asked, you can write C/C++ style code/pseudocode**

## Question 1

Dorothy has started a new beauty parlour. They are doing well but can do even better if they can plan the placement of the different sections like spa, saloon etc. in a better way. She can have $n$ plans to upgrade her parlour's facilities. Each plan $i$ has an associated cost $c_i$, and an amount of yearly profit $p_i$ that her parlour would earn after using that plan (currently the parlour organization is based on plan 1). Dorothy is trying to develop a plan for the growth of her parlour over the next $m$ years. Initially, her parlour starts with \$0 in savings. Each year, her parlour can do one of the two things. Either the parlour can earn money, adding $p_i$ to it's savings if the current plan $i$ is being used. Alternatively, she can pick some other plan $j$ and pay $c_j$ from her parlour's savings (this is only possible of course if she currently has at least that much in savings) in order to re-org her parlour to plan $j$ (her parlour will not be able to make money that year as the re-org will be disruptive). Design an algorithm to devise a plan for how Dorothy can run her parlour over the course of the next $m$ years in such a way as to maximize the total amount of savings that her parlour has at the end of that period. You should provide an algorithm that runs in time $O(nm)$ or better. Write a pseudocode. **[10 marks]**

## Question 2

(a) Perform a BFS of the following graph (Figure 1), where 1 is the starting node. Give your answer in two ways (i) use an anticlockwise ordering from the top (position 12 o'clock) and (ii) a clockwise ordering from the top.

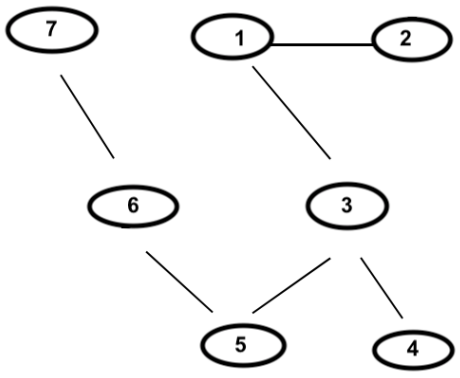


Figure 1: Example graph.

(b) Perform a depth-first search of the same graph as in (a), with 3 as the starting node. Once again give the answer in both (i) anticlockwise and (ii) clockwise ordering.

**[2.5x2=5 marks]**

## Question 3

(a) Prove or disprove: An AVL Tree of height $l$ contains strictly more nodes than an AVL tree of height $l-1$.

(b) Suppose you insert elements $\{x_1, x_2, \ldots, x_n\}$ in a Binary Search Tree in this order. You find that the resultant BST is of the worst possible height. Prove or disprove that the elements in the list have to be in the sorted order.

(c) Consider the following AVL Tree (Figure 2). Draw the resultant AVL tree after inserting 8 and 18 (only final tree, no intermediate steps). How many rotations did you need?

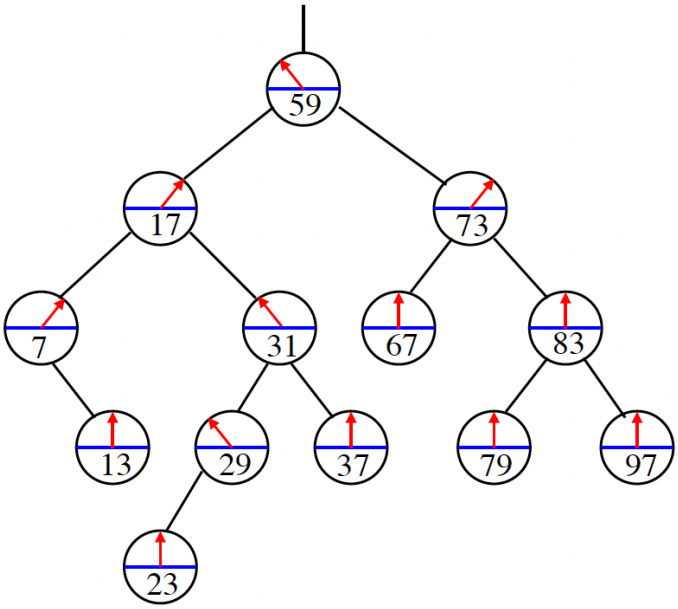


Figure 2: AVL Tree.

**[3+3+4=10 marks]**

## Question 4

Suppose this is the convocation time. There are $n$ students and $n$ gowns have been ordered. You know the height of each student, as well as length of each gown. As an organizer, your task is to create a mapping so as to **minimize** the *average absolute difference* between the height of the student, and the size of the gown, assigned to the student. For instance, if there are two students with height 1m and 5m, and two gowns with size 1.5m and 4.5m, a possible assignment is that student with 1m height gets 1.5m gown and other student gets the other gown. So, the average absolute difference would be $\frac{|1-1.5|+|5-4.5|}{2} = 0.5m$. You can verify that this is the optimal solution.

You have two greedy choices:

**Choice 1:** Sort the students by their heights, and gowns by their sizes. Assign in that order.

**Choice 2:** Find the person and gown with the minimum height difference. Assign the gown to that person. Repeat until everyone has a gown.

Which one is optimal? For each of these choices, provide a counter-example if these are not optimal. If the choice is optimal, just state that, no need to prove.

**[5 marks]**