Prob-Stat/QUIZ/1/SLOT-B

Fill in the blanks (Numerical)

Date : 6th Sep, 2021

Time : 11:05 am to 11:55 am

Duration : 45min

No of questions: 10 out of 20 questions

Type: Random-sequential (navigation NOT allowed)

Each question carries 4 marks

September 20, 2021

B:Q11. A random experiment has two mutually exclusive and exhaustive possible outcomes, the first occurs with probability $P_1 = p^2 + \frac{p}{4}$ and the second occurs with probability $P_2 = \frac{3-p}{4}$, where p is a suitable real number. Find the sum of all possible values of the product P_1P_2 satisfying the above conditions.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.34375
ERROR RANGE: 0.005

B:Q12. Three coupons are drawn randomly without replacement from 61 coupons numbered consecutively. Find the probability that the numbers on the drawn coupons form an arithmetical progression with positive common difference.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.025
ERROR RANGE: 0.005

B:Q13. A discrete random variables X has the following probability mass function $p_X(1) = 2k, p_X(2) = 3k, p_X(3) = 2k^2, p_X(4) = 4k, p_X(5) = 9k^2 + k.$

Find the $P(2 < X < 5)$.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.5454545 CORRECTED TO 0.3801
ERROR RANGE: 0.005

B:Q32. Let A_1, A_2, A_3, A_4 be independent events with $P(A_i) = p_i, i = 1, \ldots, 4$, where $p_1 = 0.5$ and $p_{i+1} = 0.5p_i \quad$ for $i = 1, 2, 3$. Find the probability that at least two of events $A_i's$ will occur.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER: 0.2178
ERROR RANGE: 0.005

B:Q35. Suppose Ramesh tosses a biased coin with probability of head as p. He gets Rs. 2000 if a head appears and loses Rs. 1000 if a tails appears. What is the smallest value of p so that expected gain by Ramesh is nonnegative?

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER: 0.3334
ERROR RANGE: 0.005

B:Q36. Let a discrete random variable X have probability mass function defined as follows:

$$f(x) = \frac{k}{2^x} \quad \text{for } x = 1, 2, \ldots$$

for some constant k. Find the probability that X is either a multiple of 2 or a multiple of 3.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER: 0.4603175
ERROR RANGE: 0.005

B:Q39. Each of the slopes and y-intercepts of two lines can be chosen from the set $\{-1, 1\}$ independently with probability of choosing 1 being same as that of choosing -1. Then the probability that the lines will intersect in a unique point is .....

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER: 0.5
ERROR RANGE: 0.005

B:Q40. A discrete random variable with the range $\{1, 2, 3, 4\}$ is such that $P(X = 1) = 4P(X = 3)$, $9P(X = 2) = 2P(X = 4)$ and $P(X = 2) = 2P(X = 3)$. Then $P(X \leq 2)$ is .....

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER: 0.375
ERROR RANGE: 0.005

B:Q41. Suppose 60% of new taxi drivers had driving lesson. In the first year of driving, suppose new taxi drivers without driving lesson have probability 0.08 of having an accident, whereas, new taxi drivers with driving lesson have 0.05 probability of having an accident. Determine the probability that a new taxi driver had driving lesson, given that the driver does not have any accident in the first year of driving.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.6077
ERROR RANGE: ∓ 0.005

B:Q45. Let the density function of a random variable X is

$$f(x) = \begin{cases} 3x^2, & \text{for } 0 < x < 1, \\ 0, & \text{otherwise} \end{cases}$$

Then the expected area of a random isosceles right angled triangle with hypotenuse X is —-

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.15
ERROR RANGE: 0.005

B:Q47. Let X be a continuous random variable with the probability density function

$$f_X(x) = \begin{cases} x, & \text{for } 0 < x \leq 1 \\ 2 - x, & \text{for } 1 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Find $P(0.3 < X \leq 1.5)$.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.83
ERROR RANGE: 0.005

B:Q49. Let X be a random variable with the p.d.f.

$$f(x) = \begin{cases} 6x(1-x), & \text{for } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the variance of X .

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.05
ERROR RANGE: 0.005

B:Q71. Suppose 10 students appear in an online test. Each of them will get 5 questions randomly chosen without replacement out of a set of 10 questions. If the probability that exactly 3 students will get same set of questions in same (sequential) order is $N \times 10^{-7}$, find the value of N.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 1.311039
ERROR RANGE: 0.005

B:Q72. Suppose for some damage in a keyboard of a computer only the vowel keys got defected. If a vowel key is pressed it may produce the same with probability 0.5 or any of the rest vowels with equal probability. If the probability that you will see "WEEKLY DEPENDENT" when you have typed "WEAKLY DEPENDENT" is $1/N$. Then find the value of positive integer N.

ANSWER : 128
ERROR RANGE: 0

B:Q73. A fair dice is rolled once. If the observed face value is X then it is again rolled independently X times more. Find the probability that the first roll will have face value greater than or equal to the face values observed in subsequent roll(s).

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.3336977
ERROR RANGE: ∓ 0.005

B: Q74. If the random variable X has p.d.f.

$$f(x) = \begin{cases} cx^3 & \text{if } 0 \leq x \leq 2, \\ 0 & \text{otherwise} \end{cases}$$

with a suitable constant c, then find the median of the distribution.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 1.681793
ERROR RANGE: 0.005

B:Q75. A random point is uniformly chosen from the interior of a unit disk. Find the maximum probability that the point will lie in a isosceles triangle which has a vertex at the centre of the disk and and the other two are on the circumference of the disk.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.1591549
ERROR RANGE: 0.005

B:Q78. Let X be a random variable with $E(X) = \mu$, $Var(X) = \sigma^2$ with $\sigma > 0$ and $k = 1.5$. If X has the p.d.f.

$$f(x) = \begin{cases} \frac{1}{2\sqrt{3}} & \text{if } -\sqrt{3} \leq x \leq \sqrt{3}, \\ 0 & \text{otherwise} \end{cases}$$

find $P(|X - \mu| \geq k\sigma)$.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.1339746
ERROR RANGE: 0.005

B:Q79. The lifetime of an electric bulb is denoted by a random variable X with p.d.f. $f(x)$. Find the probability that the bulb will be in function even after mean life time i.e. $E(X)$, where

$$f(x) = \begin{cases} \frac{1}{4}e^{-x/4} & \text{if } 0 < x < \infty, \\ 0 & \text{otherwise} \end{cases}$$

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.3678794

ERROR RANGE: ∓ 0.005

B:Q80. Consider the c.d.f. $F(x)$ of a random variable X .

$$F(x) = \begin{cases} 1 - e^{-x} - xe^{-x} & \text{if } 0 < x < \infty, \\ 0 & \text{otherwise} \end{cases}$$

Find the value of $Var(X)$.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 2.00

ERROR RANGE: 0.005