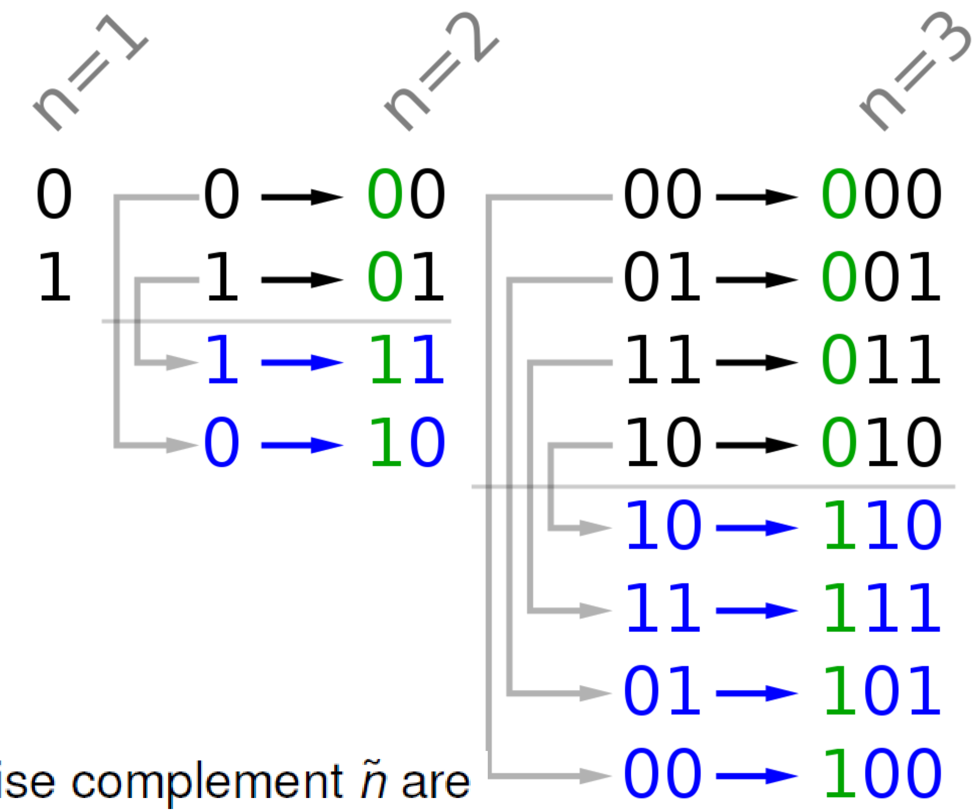
Gray Code

Reflected Binary Code

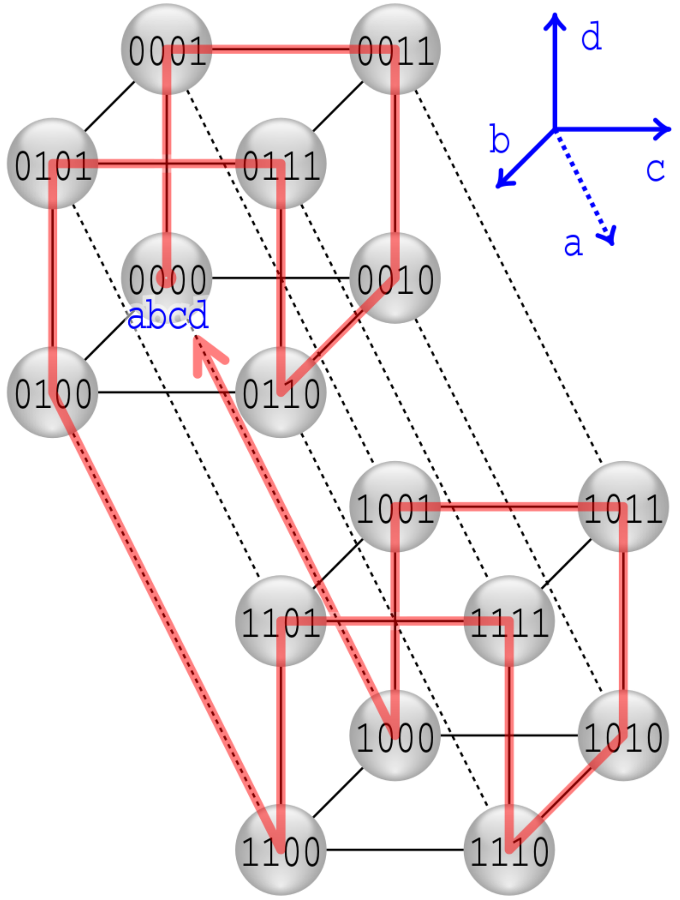
N	Binary				Gray			
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
2	0	0	1	0	0	0	1	1
3	0	0	1	1	0	0	1	0
4	0	1	0	0	0	1	1	0
5	0	1	0	1	0	1	1	1
6	0	1	1	0	0	1	0	1
7	0	1	1	1	0	1	0	0
8	1	0	0	0	1	1	0	0
9	1	0	0	1	1	1	0	1
10	1	0	1	0	1	1	1	1
11	1	0	1	1	1	1	1	0
12	1	1	0	0	1	0	1	0
13	1	1	0	1	1	0	1	1
14	1	1	1	0	1	0	0	1
15	1	1	1	1	1	0	0	0

Recursive Reflect and Prefix



- *n* and it's bitwise complement *ñ* are placed symmetrically about the middle of the table
- Their Gray codes should differ only in the MSB

Visualization on a Tesseract



Some Properties…

- Gray code is cyclic (in the range 0..15, 0 and 15 being adjacent for a 4-bit code) and also a reflected code – not cyclic in 0..9
    - Hamming distance between successive Gray code is 1
    - Hamiltonian cycle on the vertices of the Hypercube
- Balanced gray codes
    - Number of transitions for each bit position is same through a complete cycle

Is the Gray code weighted?

- Can we find weights such that $\sum_i w_i x_{i,j} = j$?
- Suppose it's weighted
- Utilise the property that adjacent codes differ in one place only
- $\forall i \exists j | (j+1) - j = \sum_i w_i \left(x_{i,j+1} - x_{i,j} \right) = \pm w_i = 1$ (why?)
- This precludes representation of 2^n values for a n-bit Gray code

Applications

- Switch Based Communication
- Analog to Digital Conversion
- Memory Address Bus (power efficient for array access)
- Multi-clock Systems

Binary to Gray Code Conversion

- The most significant bit (MSB) of the Gray code is always equal to the MSB of the given Binary code.
- Other bits of the output Gray code can be obtained by XORing binary code bit at the index and previous index.

Binary $b_2\, b_1\, b_0$	Gray Code $g_2\, g_1\, g_0$
000	000
001	001
010	011
011	010
100	110
101	111
110	101
111	100

$$g_i = b_i \oplus b_{i+1},\ g_{n-1} = b_{n-1}$$
$$g_i \oplus b_{i+1} = b_i \oplus b_{i+1} \oplus b_{i+1} = b_i \oplus 0 = b_i$$

Gray to Binary Code Conversion

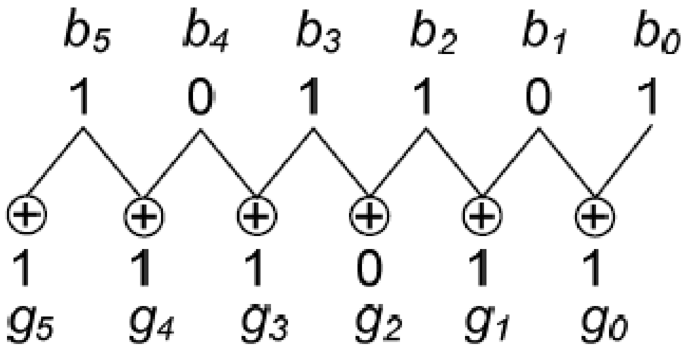
- The Most Significant Bit (MSB) of the binary code is always equal to the MSB of the given binary number.
- Other bits of the output binary code can be obtained by checking gray code bit at that index. If current gray code bit is 0, then copy previous binary code bit, else copy invert of previous binary code bit.

Gray Code $g_2g_1g_0$	**Binary** $b_2b_1b_0$
000	000
001	001
011	010
010	011
110	100
111	101
101	110
100	111

Binary – Gray Conversion

Example:

Binary:



Gray:

Gray-to-binary:

- $b_i = g_i$ if no. of 1's preceding g_i is even
- $b_i = g_i'$ if no. of 1's preceding g_i is odd

Error Detecting Codes

N		Even Parity BCD						2-out-of-5, $\binom{5}{2} = 10$						63210 BCD				
		8	4	2	1	p		0	1	2	4	7		6	3	2	1	0
0		0	0	0	0	0		0	0	0	1	1		0	0	1	1	0
1		0	0	0	1	1		1	1	0	0	0		0	0	0	1	1
2		0	0	1	0	1		1	0	1	0	0		0	0	1	0	1
3		0	0	1	1	0		0	1	1	0	0		0	1	0	0	1
4		0	1	0	0	1		1	0	0	1	0		0	1	0	1	0
5		0	1	0	1	0		0	1	0	1	0		0	1	1	0	0
6		0	1	1	0	0		0	0	1	1	0		1	0	0	0	1
7		0	1	1	1	1		1	0	0	0	1		1	0	0	1	0
8		1	0	0	0	1		0	1	0	0	1		1	0	1	0	0
9		1	0	0	1	0		0	0	1	0	1		1	1	0	0	0

- Hamming distance: number of bits differing between two codes
- If minimum Hamming distance between any two code words is d then $d - 1$ single bit errors can be detected