

Contents

1 2-level optimisation by Quine-McCluskey method



Section outline

- 1 **2-level optimisation by Quine-McCluskey method**
 - QM method

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QM method

- Partition cubes into groups $\langle m : 1, n : D \rangle$ of m on-set and n DC-set minterms
- Combine adjacent terms between groups (matching in the position of 1's and don't cares) to get prime implicants
- Cover on-set minterms using prime implicants
 - reduce table by row dominance
 C_1 dominates C_2 if C_1 covers every on-set minterm covered by C_2
 - reduce table by column dominance
 m_2 is dominated m_1 if any P covering m_1 also covers m_2
however, if C_2 is a bigger cube (due to DCs) it may still be retained
 - reduce table by dropping essential PMs
 - finally apply branch and bound (recursive application of covering needed)
arbitrarily decide to keep or not to keep a cube
bound exploration if the cost matches or exceeds the cost of an explored solution
 - Petrick's method may be used to generate all possible covers (especially for the cyclic core)



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 0 : 1, 0 : D \rangle$				
m	a	b	c	d
0	0	0	0	0

$\langle 1 : 1, 0 : D \rangle$				
m	a	b	c	d
8	1	0	0	0



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 0 : 1, 0 : D \rangle$					
m	a	b	c	d	
0	0	0	0	0	

$\langle 1 : 1, 0 : D \rangle$					
m	a	b	c	d	
8	1	0	0	0	



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 0 : 1, 0 : D \rangle$				
m	a	b	c	d
0	0	0	0	0

$\langle 1 : 1, 0 : D \rangle$				
m	a	b	c	d
8	1	0	0	0

$\langle 0 : 1, 1 : D \rangle$				
cube	a	b	c	d
0, 8	-	0	0	0



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 0 : D \rangle$				
m	a	b	c	d
8	1	0	0	0

$\langle 2 : 1, 0 : D \rangle$				
m	a	b	c	d
5	0	1	0	1
9	1	0	0	1
10	1	0	1	0



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

 $\langle 1 : 1, 0 : D \rangle$

m	a	b	c	d
8	1	0	0	0

 $\langle 2 : 1, 0 : D \rangle$

m	a	b	c	d
5	0	1	0	1
9	1	0	0	1
10	1	0	1	0



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 0 : D \rangle$				
m	a	b	c	d
8	1	0	0	0

$\langle 2 : 1, 0 : D \rangle$				
m	a	b	c	d
5	0	1	0	1
9	1	0	0	1
10	1	0	1	0

$\langle 1 : 1, 1 : D \rangle$				
cube	a	b	c	d
8, 9	1	0	0	-



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 0 : D \rangle$				
m	a	b	c	d
8	1	0	0	0
$\langle 2 : 1, 0 : D \rangle$				
m	a	b	c	d
5	0	1	0	1
9	1	0	0	1
10	1	0	1	0

$\langle 1 : 1, 1 : D \rangle$				
cube	a	b	c	d
8, 9	1	0	0	-
8, 10	1	0	-	0



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 0 : D \rangle$				
m	a	b	c	d
5	0	1	0	1
9	1	0	0	1
10	1	0	1	0

$\langle 3 : 1, 0 : D \rangle$				
m	a	b	c	d
11	1	0	1	1
14	1	1	1	0



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 0 : D \rangle$					
	m	a	b	c	d
	5	0	1	0	1
	9	1	0	0	1
	10	1	0	1	0

$\langle 3 : 1, 0 : D \rangle$					
	m	a	b	c	d
	11	1	0	1	1
	14	1	1	1	0



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 0 : D \rangle$					
	m	a	b	c	d
	5	0	1	0	1
	9	1	0	0	1
	10	1	0	1	0

$\langle 3 : 1, 0 : D \rangle$					
	m	a	b	c	d
	11	1	0	1	1
	14	1	1	1	0

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	
5	0	1	0	1	
9	1	0	0	1	
10	1	0	1	0	
$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	
11	1	0	1	1	
14	1	1	1	0	

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1
10, 11	1	0	1	-



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	
5	0	1	0	1	
9	1	0	0	1	
10	1	0	1	0	
$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	
11	1	0	1	1	
14	1	1	1	0	

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

 $\langle 3 : 1, 0 : D \rangle$

m	a	b	c	d
11	1	0	1	1
14	1	1	1	0

 $\langle 4 : 1, 0 : D \rangle$

m	a	b	c	d
15	1	1	1	1



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 3 : 1, 0 : D \rangle$				
m	a	b	c	d
11	1	0	1	1
14	1	1	1	0

$\langle 4 : 1, 0 : D \rangle$				
m	a	b	c	d
15	1	1	1	1



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 3 : 1, 0 : D \rangle$				
m	a	b	c	d
11	1	0	1	1
14	1	1	1	0

$\langle 4 : 1, 0 : D \rangle$				
m	a	b	c	d
15	1	1	1	1

$\langle 3 : 1, 1 : D \rangle$				
cube	a	b	c	d
11, 15	1	-	1	1



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	
11	1	0	1	1	
14	1	1	1	0	

$\langle 4 : 1, 0 : D \rangle$					
m	a	b	c	d	
15	1	1	1	1	

$\langle 3 : 1, 1 : D \rangle$					
cube	a	b	c	d	
11, 15	1	-	1	1	
14, 15	1	1	1	-	



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 1 : D \rangle$				
cube	a	b	c	d
8, 9	1	0	0	-
8, 10	1	0	-	0

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 1 : D \rangle$				
cube	a	b	c	d
8, 9	1	0	0	-
8, 10	1	0	-	0

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 1 : D \rangle$				
cube	a	b	c	d
8, 9	1	0	0	-
8, 10	1	0	-	0

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0

$\langle 1 : 1, 2 : D \rangle$				
cube	a	b	c	d
8, 9, 10, 11	1	0	-	-



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 1 : D \rangle$				
cube	a	b	c	d
8, 9	1	0	0	-
8, 10	1	0	-	0

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0

$\langle 1 : 1, 2 : D \rangle$				
cube	a	b	c	d
8, 9, 10, 11	1	0	-	-
8, 10, 9, 11	1	0	-	-



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0

$\langle 3 : 1, 1 : D \rangle$				
cube	a	b	c	d
11, 15	1	-	1	1
14, 15	1	1	1	-



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0

$\langle 3 : 1, 1 : D \rangle$				
cube	a	b	c	d
11, 15	1	-	1	1
14, 15	1	1	1	-



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0

$\langle 3 : 1, 1 : D \rangle$				
cube	a	b	c	d
11, 15	1	-	1	1
14, 15	1	1	1	-

$\langle 2 : 1, 2 : D \rangle$				
cube	a	b	c	d
10, 11, 14, 15	1	-	1	-



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0

$\langle 3 : 1, 1 : D \rangle$				
cube	a	b	c	d
11, 15	1	-	1	1
14, 15	1	1	1	-

$\langle 2 : 1, 2 : D \rangle$				
cube	a	b	c	d
10, 11, 14, 15	1	-	1	-
10, 14, 11, 15	1	-	1	-



$$f(a, b, c, d) = \sum_m (0, 5, 8, 9, 10, 11, 14, 15)$$

PIs	0	5	8	9	10	11	14	15
8, 9, 10, 11			X	X	X	X		
10, 11, 14, 15					X	X	X	X
0, 8	X		X					
5		X						

$$f = a\bar{b} + ac + \bar{b}\bar{c}\bar{d} + \bar{a}b\bar{c}d$$



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

 $\langle 1 : 1, 0 : D \rangle$

m	a	b	c	d
1	0	0	0	1
2	0	0	1	0
8	1	0	0	0

 $\langle 2 : 1, 0 : D \rangle$

m	a	b	c	d
3	0	0	1	1
9	1	0	0	1
10	1	0	1	0



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

 $\langle 1 : 1, 0 : D \rangle$

m	a	b	c	d
1	0	0	0	1
2	0	0	1	0
8	1	0	0	0

 $\langle 2 : 1, 0 : D \rangle$

m	a	b	c	d
3	0	0	1	1
9	1	0	0	1
10	1	0	1	0



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 0 : D \rangle$				
m	a	b	c	d
1	0	0	0	1
2	0	0	1	0
8	1	0	0	0

$\langle 2 : 1, 0 : D \rangle$				
m	a	b	c	d
3	0	0	1	1
9	1	0	0	1
10	1	0	1	0

$\langle 1 : 1, 1 : D \rangle$				
cube	a	b	c	d
1, 3	0	0	-	1



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 0 : D \rangle$				
m	a	b	c	d
1	0	0	0	1
2	0	0	1	0
8	1	0	0	0

$\langle 2 : 1, 0 : D \rangle$				
m	a	b	c	d
3	0	0	1	1
9	1	0	0	1
10	1	0	1	0

$\langle 1 : 1, 1 : D \rangle$				
cube	a	b	c	d
1, 3	0	0	-	1
1, 9	-	0	0	1



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 0 : D \rangle$				
m	a	b	c	d
1	0	0	0	1
2	0	0	1	0
8	1	0	0	0

$\langle 2 : 1, 0 : D \rangle$				
m	a	b	c	d
3	0	0	1	1
9	1	0	0	1
10	1	0	1	0

$\langle 1 : 1, 1 : D \rangle$				
cube	a	b	c	d
1, 3	0	0	-	1
1, 9	-	0	0	1
2, 3	0	0	1	-



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 0 : D \rangle$				
m	a	b	c	d
1	0	0	0	1
2	0	0	1	0
8	1	0	0	0

$\langle 2 : 1, 0 : D \rangle$				
m	a	b	c	d
3	0	0	1	1
9	1	0	0	1
10	1	0	1	0

$\langle 1 : 1, 1 : D \rangle$				
cube	a	b	c	d
1, 3	0	0	-	1
1, 9	-	0	0	1
2, 3	0	0	1	-
2, 10	-	0	1	0



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 0 : D \rangle$					
m	a	b	c	d	
1	0	0	0	1	
2	0	0	1	0	
8	1	0	0	0	

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	
3	0	0	1	1	
9	1	0	0	1	
10	1	0	1	0	

$\langle 1 : 1, 1 : D \rangle$					
cube	a	b	c	d	
1, 3	0	0	-	1	
1, 9	-	0	0	1	
2, 3	0	0	1	-	
2, 10	-	0	1	0	
8, 9	1	0	0	-	



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 0 : D \rangle$				
m	a	b	c	d
1	0	0	0	1
2	0	0	1	0
8	1	0	0	0

$\langle 2 : 1, 0 : D \rangle$				
m	a	b	c	d
3	0	0	1	1
9	1	0	0	1
10	1	0	1	0

$\langle 1 : 1, 1 : D \rangle$				
cube	a	b	c	d
1, 3	0	0	-	1
1, 9	-	0	0	1
2, 3	0	0	1	-
2, 10	-	0	1	0
8, 9	1	0	0	-
8, 10	1	0	-	0



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

 $\langle 2 : 1, 0 : D \rangle$

m	a	b	c	d
3	0	0	1	1
9	1	0	0	1
10	1	0	1	0

 $\langle 3 : 1, 0 : D \rangle$

m	a	b	c	d
7	0	1	1	1
11	1	0	1	1
14	1	1	1	0



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

 $\langle 2 : 1, 0 : D \rangle$

m	a	b	c	d
3	0	0	1	1
9	1	0	0	1
10	1	0	1	0

 $\langle 3 : 1, 0 : D \rangle$

m	a	b	c	d
7	0	1	1	1
11	1	0	1	1
14	1	1	1	0



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 0 : D \rangle$				
m	a	b	c	d
3	0	0	1	1
9	1	0	0	1
10	1	0	1	0

$\langle 3 : 1, 0 : D \rangle$				
m	a	b	c	d
7	0	1	1	1
11	1	0	1	1
14	1	1	1	0

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
3, 7	0	-	1	1



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 0 : D \rangle$				
m	a	b	c	d
3	0	0	1	1
9	1	0	0	1
10	1	0	1	0

$\langle 3 : 1, 0 : D \rangle$				
m	a	b	c	d
7	0	1	1	1
11	1	0	1	1
14	1	1	1	0

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
3, 7	0	-	1	1
3, 11	-	0	1	1



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 0 : D \rangle$				
m	a	b	c	d
3	0	0	1	1
9	1	0	0	1
10	1	0	1	0

$\langle 3 : 1, 0 : D \rangle$				
m	a	b	c	d
7	0	1	1	1
11	1	0	1	1
14	1	1	1	0

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
3, 7	0	-	1	1
3, 11	-	0	1	1
9, 11	1	0	-	1



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	
3	0	0	1	1	
9	1	0	0	1	
10	1	0	1	0	

$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	
7	0	1	1	1	
11	1	0	1	1	
14	1	1	1	0	

$\langle 2 : 1, 1 : D \rangle$					
cube	a	b	c	d	
3, 7	0	-	1	1	
3, 11	-	0	1	1	
9, 11	1	0	-	1	
10, 11	1	0	1	-	



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	
3	0	0	1	1	
9	1	0	0	1	
10	1	0	1	0	

$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	
7	0	1	1	1	
11	1	0	1	1	
14	1	1	1	0	

$\langle 2 : 1, 1 : D \rangle$					
cube	a	b	c	d	
3, 7	0	-	1	1	
3, 11	-	0	1	1	
9, 11	1	0	-	1	
10, 11	1	0	1	-	
10, 14	1	-	1	0	



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 3 : 1, 0 : D \rangle$				
m	a	b	c	d
7	0	1	1	1
11	1	0	1	1
14	1	1	1	0

$\langle 4 : 1, 0 : D \rangle$				
m	a	b	c	d
15	1	1	1	1



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 3 : 1, 0 : D \rangle$				
m	a	b	c	d
7	0	1	1	1
11	1	0	1	1
14	1	1	1	0

$\langle 4 : 1, 0 : D \rangle$				
m	a	b	c	d
15	1	1	1	1



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 3 : 1, 0 : D \rangle$				
m	a	b	c	d
7	0	1	1	1
11	1	0	1	1
14	1	1	1	0

$\langle 4 : 1, 0 : D \rangle$				
m	a	b	c	d
15	1	1	1	1

$\langle 3 : 1, 1 : D \rangle$				
cube	a	b	c	d
7, 15	-	1	1	1



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 3 : 1, 0 : D \rangle$				
m	a	b	c	d
7	0	1	1	1
11	1	0	1	1
14	1	1	1	0
$\langle 4 : 1, 0 : D \rangle$				
m	a	b	c	d
15	1	1	1	1

$\langle 3 : 1, 1 : D \rangle$				
cube	a	b	c	d
7, 15	-	1	1	1
11, 15	1	-	1	1



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 3 : 1, 0 : D \rangle$				
m	a	b	c	d
7	0	1	1	1
11	1	0	1	1
14	1	1	1	0

$\langle 4 : 1, 0 : D \rangle$				
m	a	b	c	d
15	1	1	1	1

$\langle 3 : 1, 1 : D \rangle$				
cube	a	b	c	d
7, 15	-	1	1	1
11, 15	1	-	1	1
14, 15	1	1	1	-



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 1 : D \rangle$				
cube	a	b	c	d
1, 3	0	0	-	1
1, 9	-	0	0	1
2, 3	0	0	1	-
2, 10	-	0	1	0
8, 9	1	0	0	-
8, 10	1	0	-	0

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
3, 7	0	-	1	1
3, 11	-	0	1	1
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0

$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 1 : D \rangle$				
cube	a	b	c	d
1, 3	0	0	-	1
1, 9	-	0	0	1
2, 3	0	0	1	-
2, 10	-	0	1	0
8, 9	1	0	0	-
8, 10	1	0	-	0

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
3, 7	0	-	1	1
3, 11	-	0	1	1
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0

$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 1 : D \rangle$				
cube	a	b	c	d
1, 3	0	0	-	1
1, 9	-	0	0	1
2, 3	0	0	1	-
2, 10	-	0	1	0
8, 9	1	0	0	-
8, 10	1	0	-	0
$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
3, 7	0	-	1	1
3, 11	-	0	1	1
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0

$\langle 1 : 1, 2 : D \rangle$				
cube	a	b	c	d
1, 3, 9, 11	-	0	-	1

$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 1 : D \rangle$				
cube	a	b	c	d
1, 3	0	0	-	1
1, 9	-	0	0	1
2, 3	0	0	1	-
2, 10	-	0	1	0
8, 9	1	0	0	-
8, 10	1	0	-	0
$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
3, 7	0	-	1	1
3, 11	-	0	1	1
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0

$\langle 1 : 1, 2 : D \rangle$				
cube	a	b	c	d
1, 3, 9, 11	-	0	-	1
2, 3, 10, 11	-	0	1	-

$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 1 : 1, 1 : D \rangle$				
cube	a	b	c	d
1, 3	0	0	-	1
1, 9	-	0	0	1
2, 3	0	0	1	-
2, 10	-	0	1	0
8, 9	1	0	0	-
8, 10	1	0	-	0

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
3, 7	0	-	1	1
3, 11	-	0	1	1
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0

$\langle 1 : 1, 2 : D \rangle$				
cube	a	b	c	d
1, 3, 9, 11	-	0	-	1
2, 3, 10, 11	-	0	1	-
8, 9, 10, 11	1	0	-	-

$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
3, 7	0	-	1	1
3, 11	-	0	1	1
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0

$\langle 3 : 1, 1 : D \rangle$				
cube	a	b	c	d
7, 15	-	1	1	1
11, 15	1	-	1	1
14, 15	1	1	1	-



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
3, 7	0	-	1	1
3, 11	-	0	1	1
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0

$\langle 3 : 1, 1 : D \rangle$				
cube	a	b	c	d
7, 15	-	1	1	1
11, 15	1	-	1	1
14, 15	1	1	1	-



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
3, 7	0	-	1	1
3, 11	-	0	1	1
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0

$\langle 3 : 1, 1 : D \rangle$				
cube	a	b	c	d
7, 15	-	1	1	1
11, 15	1	-	1	1
14, 15	1	1	1	-

$\langle 2 : 1, 2 : D \rangle$				
cube	a	b	c	d
3, 7, 11, 15	-	-	1	1



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
3, 7	0	-	1	1
3, 11	-	0	1	1
9, 11	1	0	-	1
10, 11	1	0	1	-
10, 14	1	-	1	0

$\langle 3 : 1, 1 : D \rangle$				
cube	a	b	c	d
7, 15	-	1	1	1
11, 15	1	-	1	1
14, 15	1	1	1	-

$\langle 2 : 1, 2 : D \rangle$				
cube	a	b	c	d
3, 7, 11, 15	-	-	1	1
10, 11, 14, 15	1	-	1	-



$$f(a, b, c, d) = \sum_m (1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

PIs	1	2	3	7	8	9	10	11	14	15
1, 3, 9, 11	(X)		X			X		X		
2, 3, 10, 11		(X)	X				X	X		
8, 9, 10, 11					(X)	X	X	X		
3, 7, 11, 15			X	(X)				X		X
10, 11, 14, 15							X	X	(X)	X

$$f = \bar{B}D + \bar{B}R + A\bar{B} + RD + AR$$



$$f(a, b, c, d) = \sum_m(4, 6, 9, 10, 11, 13) + \sum_d(2, 12, 15)$$

 $\langle 1 : 1, 0 : D \rangle$

m	a	b	c	d
2	0	0	1	0
4	0	1	0	0

 $\langle 2 : 1, 0 : D \rangle$

m	a	b	c	d
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0



$$f(a, b, c, d) = \sum_m(4, 6, 9, 10, 11, 13) + \sum_d(2, 12, 15)$$

 $\langle 1 : 1, 0 : D \rangle$

m	a	b	c	d
2	0	0	1	0
4	0	1	0	0

 $\langle 2 : 1, 0 : D \rangle$

m	a	b	c	d
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0



$$f(a, b, c, d) = \sum_m(4, 6, 9, 10, 11, 13) + \sum_d(2, 12, 15)$$

$\langle 1 : 1, 0 : D \rangle$					
m	a	b	c	d	
2	0	0	1	0	
4	0	1	0	0	
$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	
6	0	1	1	0	
9	1	0	0	1	
10	1	0	1	0	
12	1	1	0	0	

$\langle 1 : 1, 1 : D \rangle$				
cube	a	b	c	d
2, 6	0	-	1	0



$$f(a, b, c, d) = \sum_m(4, 6, 9, 10, 11, 13) + \sum_d(2, 12, 15)$$

$\langle 1 : 1, 0 : D \rangle$					
	m	a	b	c	d
	2	0	0	1	0
	4	0	1	0	0
$\langle 2 : 1, 0 : D \rangle$					
	m	a	b	c	d
	6	0	1	1	0
	9	1	0	0	1
	10	1	0	1	0
	12	1	1	0	0

$\langle 1 : 1, 1 : D \rangle$					
	cube	a	b	c	d
	2, 6	0	-	1	0
	2, 10	-	0	1	0



$$f(a, b, c, d) = \sum_m(4, 6, 9, 10, 11, 13) + \sum_d(2, 12, 15)$$

$\langle 1 : 1, 0 : D \rangle$					
	m	a	b	c	d
	2	0	0	1	0
	4	0	1	0	0
$\langle 2 : 1, 0 : D \rangle$					
	m	a	b	c	d
	6	0	1	1	0
	9	1	0	0	1
	10	1	0	1	0
	12	1	1	0	0

$\langle 1 : 1, 1 : D \rangle$					
	cube	a	b	c	d
	2, 6	0	-	1	0
	2, 10	-	0	1	0
	4, 6	0	1	-	0



$$f(a, b, c, d) = \sum_m(4, 6, 9, 10, 11, 13) + \sum_d(2, 12, 15)$$

$\langle 1 : 1, 0 : D \rangle$					
	m	a	b	c	d
	2	0	0	1	0
	4	0	1	0	0
$\langle 2 : 1, 0 : D \rangle$					
	m	a	b	c	d
	6	0	1	1	0
	9	1	0	0	1
	10	1	0	1	0
	12	1	1	0	0

$\langle 1 : 1, 1 : D \rangle$					
	cube	a	b	c	d
	2, 6	0	-	1	0
	2, 10	-	0	1	0
	4, 6	0	1	-	0
	4, 12	-	1	0	0



$$f(a, b, c, d) = \sum_m(4, 6, 9, 10, 11, 13) + \sum_d(2, 12, 15)$$

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	
6	0	1	1	0	
9	1	0	0	1	
10	1	0	1	0	
12	1	1	0	0	
$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	
11	1	0	1	1	
13	1	1	0	1	



$$f(a, b, c, d) = \sum_m (4, 6, 9, 10, 11, 13) + \sum_d (2, 12, 15)$$

 $\langle 2 : 1, 0 : D \rangle$

m	a	b	c	d
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0

 $\langle 3 : 1, 0 : D \rangle$

m	a	b	c	d
11	1	0	1	1
13	1	1	0	1



$$f(a, b, c, d) = \sum_m(4, 6, 9, 10, 11, 13) + \sum_d(2, 12, 15)$$

$\langle 2 : 1, 0 : D \rangle$					
	m	a	b	c	d
	6	0	1	1	0
	9	1	0	0	1
	10	1	0	1	0
	12	1	1	0	0
$\langle 3 : 1, 0 : D \rangle$					
	m	a	b	c	d
	11	1	0	1	1
	13	1	1	0	1

$\langle 2 : 1, 1 : D \rangle$					
	cube	a	b	c	d
	9, 11	1	0	-	1



$$f(a, b, c, d) = \sum_m(4, 6, 9, 10, 11, 13) + \sum_d(2, 12, 15)$$

$\langle 2 : 1, 0 : D \rangle$					
	m	a	b	c	d
	6	0	1	1	0
	9	1	0	0	1
	10	1	0	1	0
	12	1	1	0	0
$\langle 3 : 1, 0 : D \rangle$					
	m	a	b	c	d
	11	1	0	1	1
	13	1	1	0	1

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1
9, 13	1	-	0	1



$$f(a, b, c, d) = \sum_m(4, 6, 9, 10, 11, 13) + \sum_d(2, 12, 15)$$

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	
6	0	1	1	0	
9	1	0	0	1	
10	1	0	1	0	
12	1	1	0	0	
$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	
11	1	0	1	1	
13	1	1	0	1	

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1
9, 13	1	-	0	1
10, 11	1	0	1	-



$$f(a, b, c, d) = \sum_m(4, 6, 9, 10, 11, 13) + \sum_d(2, 12, 15)$$

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	
6	0	1	1	0	
9	1	0	0	1	
10	1	0	1	0	
12	1	1	0	0	
$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	
11	1	0	1	1	
13	1	1	0	1	

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1
9, 13	1	-	0	1
10, 11	1	0	1	-
12, 13	1	1	0	-



$$f(a, b, c, d) = \sum_m (4, 6, 9, 10, 11, 13) + \sum_d (2, 12, 15)$$

 $\langle 3 : 1, 0 : D \rangle$

m	a	b	c	d
11	1	0	1	1
13	1	1	0	1

 $\langle 4 : 1, 1 : D \rangle$

m	a	b	c	d
15	1	1	1	1



$$f(a, b, c, d) = \sum_m (4, 6, 9, 10, 11, 13) + \sum_d (2, 12, 15)$$

 $\langle 3 : 1, 0 : D \rangle$

m	a	b	c	d
11	1	0	1	1
13	1	1	0	1

 $\langle 4 : 1, 1 : D \rangle$

m	a	b	c	d
15	1	1	1	1



$$f(a, b, c, d) = \sum_m (4, 6, 9, 10, 11, 13) + \sum_d (2, 12, 15)$$

$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	
11	1	0	1	1	
13	1	1	0	1	

$\langle 4 : 1, 1 : D \rangle$					
m	a	b	c	d	
15	1	1	1	1	

$\langle 3 : 1, 1 : D \rangle$					
cube	a	b	c	d	
11, 15	1	-	1	1	



$$f(a, b, c, d) = \sum_m (4, 6, 9, 10, 11, 13) + \sum_d (2, 12, 15)$$

$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	
11	1	0	1	1	
13	1	1	0	1	

$\langle 4 : 1, 1 : D \rangle$					
m	a	b	c	d	
15	1	1	1	1	

$\langle 3 : 1, 1 : D \rangle$					
cube	a	b	c	d	
11, 15	1	-	1	1	
13, 15	1	1	-	1	



$$f(a, b, c, d) = \sum_m(4, 6, 9, 10, 11, 13) + \sum_d(2, 12, 15)$$

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1
9, 13	1	-	0	1
10, 11	1	0	1	-
12, 13	1	1	0	-
$\langle 3 : 1, 1 : D \rangle$				
cube	a	b	c	d
11, 15	1	-	1	1
13, 15	1	1	-	1



$$f(a, b, c, d) = \sum_m(4, 6, 9, 10, 11, 13) + \sum_d(2, 12, 15)$$

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1
9, 13	1	-	0	1
10, 11	1	0	1	-
12, 13	1	1	0	-
$\langle 3 : 1, 1 : D \rangle$				
cube	a	b	c	d
11, 15	1	-	1	1
13, 15	1	1	-	1



$$f(a, b, c, d) = \sum_m(4, 6, 9, 10, 11, 13) + \sum_d(2, 12, 15)$$

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1
9, 13	1	-	0	1
10, 11	1	0	1	-
12, 13	1	1	0	-
$\langle 3 : 1, 1 : D \rangle$				
cube	a	b	c	d
11, 15	1	-	1	1
13, 15	1	1	-	1

$\langle 2 : 1, 2 : D \rangle$				
cube	a	b	c	d
9, 11, 13, 15	1	-	-	1



$$f(a, b, c, d) = \sum_m(4, 6, 9, 10, 11, 13) + \sum_d(2, 12, 15)$$

$\langle 2 : 1, 1 : D \rangle$				
cube	a	b	c	d
9, 11	1	0	-	1
9, 13	1	-	0	1
10, 11	1	0	1	-
12, 13	1	1	0	-
$\langle 3 : 1, 1 : D \rangle$				
cube	a	b	c	d
11, 15	1	-	1	1
13, 15	1	1	-	1

$\langle 2 : 1, 2 : D \rangle$				
cube	a	b	c	d
9,11,13,15	1	-	-	1
9,13,11,15	1	-	-	1



$$f(a, b, c, d) = \sum_m (4, 6, 9, 10, 11, 13) + \sum_d (2, 12, 15)$$

PIs	4	6	9	10	11	13
2, 6		X				
2, 10				X		
4, 6	X	X				
4, 12	X					
9, 11, 13, 15			(X)		X	X
10, 11				X	X	
12, 13						X

- $\langle 9, 11, 13, 15 \rangle$ dominates $\langle 12, 13 \rangle$
- $\langle 10, 11 \rangle$ dominates $\langle 2, 10 \rangle$
- $\langle 4, 6 \rangle$ dominates $\langle 4, 12 \rangle$ and $\langle 2, 6 \rangle$

$$f = ad + \bar{a}b\bar{d}$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 0 : 1, 0 : D \rangle$					
m	a	b	c	d	e
0	0	0	0	0	0

$\langle 1 : 1, 0 : D \rangle$					
m	a	b	c	d	e
1	0	0	0	0	1
2	0	0	0	1	0
8	0	1	0	0	0



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 0 : 1, 0 : D \rangle$					
m	a	b	c	d	e
0	0	0	0	0	0

$\langle 1 : 1, 0 : D \rangle$					
m	a	b	c	d	e
1	0	0	0	0	1
2	0	0	0	1	0
8	0	1	0	0	0



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 0 : 1, 0 : D \rangle$					
m	a	b	c	d	e
0	0	0	0	0	0

$\langle 1 : 1, 0 : D \rangle$					
m	a	b	c	d	e
1	0	0	0	0	1
2	0	0	0	1	0
8	0	1	0	0	0

$\langle 0 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
0, 1	0	0	0	0	-



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 0 : 1, 0 : D \rangle$					
m	a	b	c	d	e
0	0	0	0	0	0

$\langle 1 : 1, 0 : D \rangle$					
m	a	b	c	d	e
1	0	0	0	0	1
2	0	0	0	1	0
8	0	1	0	0	0

$\langle 0 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
0, 1	0	0	0	0	-
0, 2	0	0	0	-	0



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 0 : 1, 0 : D \rangle$					
m	a	b	c	d	e
0	0	0	0	0	0

$\langle 1 : 1, 0 : D \rangle$					
m	a	b	c	d	e
1	0	0	0	0	1
2	0	0	0	1	0
8	0	1	0	0	0

$\langle 0 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
0, 1	0	0	0	0	-
0, 2	0	0	0	-	0
0, 8	0	-	0	0	0



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

 $\langle 1 : 1, 0 : D \rangle$

m	a	b	c	d	e
1	0	0	0	0	1
2	0	0	0	1	0
8	0	1	0	0	0

 $\langle 2 : 1, 0 : D \rangle$

m	a	b	c	d	e
3	0	0	0	1	1
6	0	0	1	1	0
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1
20	1	0	1	0	0



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 1 : 1, 0 : D \rangle$					
m	a	b	c	d	e
1	0	0	0	0	1
2	0	0	0	1	0
8	0	1	0	0	0

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	e
3	0	0	0	1	1
6	0	0	1	1	0
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1
20	1	0	1	0	0

$\langle 1 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
1, 3	0	0	0	-	1



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 1 : 1, 0 : D \rangle$						
m	a	b	c	d	e	
1	0	0	0	0	1	
2	0	0	0	1	0	
8	0	1	0	0	0	

$\langle 2 : 1, 0 : D \rangle$						
m	a	b	c	d	e	
3	0	0	0	1	1	
6	0	0	1	1	0	
9	0	1	0	0	1	
10	0	1	0	1	0	
17	1	0	0	0	1	
20	1	0	1	0	0	

$\langle 1 : 1, 1 : D \rangle$						
cube	a	b	c	d	e	
1, 3	0	0	0	-	1	
1, 9	0	-	0	0	1	



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 1 : 1, 0 : D \rangle$						
m	a	b	c	d	e	
1	0	0	0	0	1	
2	0	0	0	1	0	
8	0	1	0	0	0	

$\langle 2 : 1, 0 : D \rangle$						
m	a	b	c	d	e	
3	0	0	0	1	1	
6	0	0	1	1	0	
9	0	1	0	0	1	
10	0	1	0	1	0	
17	1	0	0	0	1	
20	1	0	1	0	0	

$\langle 1 : 1, 1 : D \rangle$						
cube	a	b	c	d	e	
1, 3	0	0	0	-	1	
1, 9	0	-	0	0	1	
1, 17	-	0	0	0	1	



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 1 : 1, 0 : D \rangle$					
m	a	b	c	d	e
1	0	0	0	0	1
2	0	0	0	1	0
8	0	1	0	0	0

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	e
3	0	0	0	1	1
6	0	0	1	1	0
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1
20	1	0	1	0	0

$\langle 1 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
1, 3	0	0	0	-	1
1, 9	0	-	0	0	1
1, 17	-	0	0	0	1
2, 3	0	0	0	1	-



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 1 : 1, 0 : D \rangle$					
m	a	b	c	d	e
1	0	0	0	0	1
2	0	0	0	1	0
8	0	1	0	0	0

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	e
3	0	0	0	1	1
6	0	0	1	1	0
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1
20	1	0	1	0	0

$\langle 1 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
1, 3	0	0	0	-	1
1, 9	0	-	0	0	1
1, 17	-	0	0	0	1
2, 3	0	0	0	1	-
2, 6	0	0	-	1	0



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 1 : 1, 0 : D \rangle$					
m	a	b	c	d	e
1	0	0	0	0	1
2	0	0	0	1	0
8	0	1	0	0	0

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	e
3	0	0	0	1	1
6	0	0	1	1	0
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1
20	1	0	1	0	0

$\langle 1 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
1, 3	0	0	0	-	1
1, 9	0	-	0	0	1
1, 17	-	0	0	0	1
2, 3	0	0	0	1	-
2, 6	0	0	-	1	0
2, 10	0	-	0	1	0



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 1 : 1, 0 : D \rangle$					
m	a	b	c	d	e
1	0	0	0	0	1
2	0	0	0	1	0
8	0	1	0	0	0

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	e
3	0	0	0	1	1
6	0	0	1	1	0
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1
20	1	0	1	0	0

$\langle 1 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
1, 3	0	0	0	-	1
1, 9	0	-	0	0	1
1, 17	-	0	0	0	1
2, 3	0	0	0	1	-
2, 6	0	0	-	1	0
2, 10	0	-	0	1	0
8, 9	0	1	0	0	-



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 1 : 1, 0 : D \rangle$					
m	a	b	c	d	e
1	0	0	0	0	1
2	0	0	0	1	0
8	0	1	0	0	0

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	e
3	0	0	0	1	1
6	0	0	1	1	0
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1
20	1	0	1	0	0

$\langle 1 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
1, 3	0	0	0	-	1
1, 9	0	-	0	0	1
1, 17	-	0	0	0	1
2, 3	0	0	0	1	-
2, 6	0	0	-	1	0
2, 10	0	-	0	1	0
8, 9	0	1	0	0	-
8, 10	0	1	0	-	0



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

 $\langle 2 : 1, 0 : D \rangle$

m	a	b	c	d	e
3	0	0	0	1	1
6	0	0	1	1	0
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1
20	1	0	1	0	0

 $\langle 3 : 1, 0 : D \rangle$

m	a	b	c	d	e
11	0	1	0	1	1
21	1	0	1	0	1
25	1	1	0	0	1
28	1	1	1	0	0

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

 $\langle 2 : 1, 0 : D \rangle$

m	a	b	c	d	e
3	0	0	0	1	1
6	0	0	1	1	0
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1
20	1	0	1	0	0

 $\langle 3 : 1, 0 : D \rangle$

m	a	b	c	d	e
11	0	1	0	1	1
21	1	0	1	0	1
25	1	1	0	0	1
28	1	1	1	0	0

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	e
3	0	0	0	1	1
6	0	0	1	1	0
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1
20	1	0	1	0	0

$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	e
11	0	1	0	1	1
21	1	0	1	0	1
25	1	1	0	0	1
28	1	1	1	0	0

$\langle 2 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
3, 11	0	-	0	1	1

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	e
3	0	0	0	1	1
6	0	0	1	1	0
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1
20	1	0	1	0	0
$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	e
11	0	1	0	1	1
21	1	0	1	0	1
25	1	1	0	0	1
28	1	1	1	0	0

$\langle 2 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
3, 11	0	-	0	1	1
9, 11	0	1	0	-	1

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	e
3	0	0	0	1	1
6	0	0	1	1	0
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1
20	1	0	1	0	0
$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	e
11	0	1	0	1	1
21	1	0	1	0	1
25	1	1	0	0	1
28	1	1	1	0	0

$\langle 2 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
3, 11	0	-	0	1	1
9, 11	0	1	0	-	1
9, 25	-	1	0	0	1

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	e
3	0	0	0	1	1
6	0	0	1	1	0
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1
20	1	0	1	0	0

$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	e
11	0	1	0	1	1
21	1	0	1	0	1
25	1	1	0	0	1
28	1	1	1	0	0

$\langle 2 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
3, 11	0	-	0	1	1
9, 11	0	1	0	-	1
9, 25	-	1	0	0	1
10, 11	0	1	0	1	-

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	e
3	0	0	0	1	1
6	0	0	1	1	0
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1
20	1	0	1	0	0
$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	e
11	0	1	0	1	1
21	1	0	1	0	1
25	1	1	0	0	1
28	1	1	1	0	0

$\langle 2 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
3, 11	0	-	0	1	1
9, 11	0	1	0	-	1
9, 25	-	1	0	0	1
10, 11	0	1	0	1	-
17, 21	1	0	-	0	1

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	e
3	0	0	0	1	1
6	0	0	1	1	0
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1
20	1	0	1	0	0

$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	e
11	0	1	0	1	1
21	1	0	1	0	1
25	1	1	0	0	1
28	1	1	1	0	0

$\langle 2 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
3, 11	0	-	0	1	1
9, 11	0	1	0	-	1
9, 25	-	1	0	0	1
10, 11	0	1	0	1	-
17, 21	1	0	-	0	1
17, 25	1	-	0	0	1

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	e
3	0	0	0	1	1
6	0	0	1	1	0
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1
20	1	0	1	0	0

$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	e
11	0	1	0	1	1
21	1	0	1	0	1
25	1	1	0	0	1
28	1	1	1	0	0

$\langle 2 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
3, 11	0	-	0	1	1
9, 11	0	1	0	-	1
9, 25	-	1	0	0	1
10, 11	0	1	0	1	-
17, 21	1	0	-	0	1
17, 25	1	-	0	0	1
20, 21	1	0	1	0	-

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 2 : 1, 0 : D \rangle$					
m	a	b	c	d	e
3	0	0	0	1	1
6	0	0	1	1	0
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1
20	1	0	1	0	0

$\langle 3 : 1, 0 : D \rangle$					
m	a	b	c	d	e
11	0	1	0	1	1
21	1	0	1	0	1
25	1	1	0	0	1
28	1	1	1	0	0

$\langle 2 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
3, 11	0	-	0	1	1
9, 11	0	1	0	-	1
9, 25	-	1	0	0	1
10, 11	0	1	0	1	-
17, 21	1	0	-	0	1
17, 25	1	-	0	0	1
20, 21	1	0	1	0	-
20, 28	1	-	1	0	0

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 3 : 1, 0 : D \rangle$						
m	a	b	c	d	e	
11	0	1	0	1	1	
21	1	0	1	0	1	
25	1	1	0	0	1	
28	1	1	1	0	0	

$\langle 4 : 1, 0 : D \rangle$						
m	a	b	c	d	e	
23	1	0	1	1	1	
30	1	1	1	1	0	



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 3 : 1, 0 : D \rangle$						
m	a	b	c	d	e	
11	0	1	0	1	1	
21	1	0	1	0	1	
25	1	1	0	0	1	
28	1	1	1	0	0	

$\langle 4 : 1, 0 : D \rangle$						
m	a	b	c	d	e	
23	1	0	1	1	1	
30	1	1	1	1	0	



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 3 : 1, 0 : D \rangle$						
m	a	b	c	d	e	
11	0	1	0	1	1	
21	1	0	1	0	1	
25	1	1	0	0	1	
28	1	1	1	0	0	
$\langle 4 : 1, 0 : D \rangle$						
m	a	b	c	d	e	
23	1	0	1	1	1	
30	1	1	1	1	0	

$\langle 3 : 1, 1 : D \rangle$						
cube	a	b	c	d	e	
21, 23	1	0	1	-	1	



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 3 : 1, 0 : D \rangle$						
m	a	b	c	d	e	
11	0	1	0	1	1	
21	1	0	1	0	1	
25	1	1	0	0	1	
28	1	1	1	0	0	
$\langle 4 : 1, 0 : D \rangle$						
m	a	b	c	d	e	
23	1	0	1	1	1	
30	1	1	1	1	0	

$\langle 3 : 1, 1 : D \rangle$						
cube	a	b	c	d	e	
21, 23	1	0	1	-	1	
28, 30	1	1	1	-	0	



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

 $\langle 4 : 1, 0 : D \rangle$

m	a	b	c	d	e
23	1	0	1	1	1
30	1	1	1	1	0

 $\langle 5 : 1, 0 : D \rangle$

m	a	b	c	d	e
31	1	1	1	1	1



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

 $\langle 4 : 1, 0 : D \rangle$

m	a	b	c	d	e
23	1	0	1	1	1
30	1	1	1	1	0

 $\langle 5 : 1, 0 : D \rangle$

m	a	b	c	d	e
31	1	1	1	1	1



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 4 : 1, 0 : D \rangle$					
m	a	b	c	d	e
23	1	0	1	1	1
30	1	1	1	1	0
$\langle 5 : 1, 0 : D \rangle$					
m	a	b	c	d	e
31	1	1	1	1	1

$\langle 4 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
23, 31	1	-	1	1	1



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 4 : 1, 0 : D \rangle$					
m	a	b	c	d	e
23	1	0	1	1	1
30	1	1	1	1	0
$\langle 5 : 1, 0 : D \rangle$					
m	a	b	c	d	e
31	1	1	1	1	1

$\langle 4 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
23, 31	1	-	1	1	1
30, 31	1	1	1	1	-



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 0 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
0, 1	0	0	0	0	-
0, 2	0	0	0	-	0
0, 8	0	-	0	0	0

$\langle 1 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
1, 3	0	0	0	-	1
1, 9	0	-	0	0	1
1, 17	-	0	0	0	1
2, 6	0	0	-	1	0
2, 10	0	-	0	1	0
8, 9	0	1	0	0	-
2, 3	0	0	0	1	-
8, 10	0	1	0	-	0

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 0 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
0, 1	0	0	0	0	-
0, 2	0	0	0	-	0
0, 8	0	-	0	0	0

$\langle 1 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
1, 3	0	0	0	-	1
1, 9	0	-	0	0	1
1, 17	-	0	0	0	1
2, 6	0	0	-	1	0
2, 10	0	-	0	1	0
8, 9	0	1	0	0	-
2, 3	0	0	0	1	-
8, 10	0	1	0	-	0

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 0 : 1, 1 : D \rangle$						
cube	a	b	c	d	e	
0, 1	0	0	0	0	-	
0, 2	0	0	0	-	0	
0, 8	0	-	0	0	0	

$\langle 1 : 1, 1 : D \rangle$						
cube	a	b	c	d	e	
1, 3	0	0	0	-	1	
1, 9	0	-	0	0	1	
1, 17	-	0	0	0	1	
2, 6	0	0	-	1	0	
2, 10	0	-	0	1	0	
8, 9	0	1	0	0	-	
2, 3	0	0	0	1	-	
8, 10	0	1	0	-	0	

$\langle 0 : 1, 2 : D \rangle$						
cube	a	b	c	d	e	
0, 1, 8, 9	0	-	0	0	-	

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 0 : 1, 1 : D \rangle$						
cube	a	b	c	d	e	
0, 1	0	0	0	0	-	
0, 2	0	0	0	-	0	
0, 8	0	-	0	0	0	

$\langle 1 : 1, 1 : D \rangle$						
cube	a	b	c	d	e	
1, 3	0	0	0	-	1	
1, 9	0	-	0	0	1	
1, 17	-	0	0	0	1	
2, 6	0	0	-	1	0	
2, 10	0	-	0	1	0	
8, 9	0	1	0	0	-	
2, 3	0	0	0	1	-	
8, 10	0	1	0	-	0	

$\langle 0 : 1, 2 : D \rangle$						
cube	a	b	c	d	e	
0, 1, 8, 9	0	-	0	0	-	
0, 1, 2, 3	0	0	0	-	-	

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 0 : 1, 1 : D \rangle$						
cube	a	b	c	d	e	
0, 1	0	0	0	0	-	
0, 2	0	0	0	-	0	
0, 8	0	-	0	0	0	

$\langle 1 : 1, 1 : D \rangle$						
cube	a	b	c	d	e	
1, 3	0	0	0	-	1	
1, 9	0	-	0	0	1	
1, 17	-	0	0	0	1	
2, 6	0	0	-	1	0	
2, 10	0	-	0	1	0	
8, 9	0	1	0	0	-	
2, 3	0	0	0	1	-	
8, 10	0	1	0	-	0	

$\langle 0 : 1, 2 : D \rangle$						
cube	a	b	c	d	e	
0, 1, 8, 9	0	-	0	0	-	
0, 1, 2, 3	0	0	0	-	-	
0, 2, 8, 10	0	-	0	-	0	

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 1 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
1, 3	0	0	0	-	1
1, 9	0	-	0	0	1
1, 17	-	0	0	0	1
2, 6	0	0	-	1	0
2, 10	0	-	0	1	0
8, 9	0	1	0	0	-
2, 3	0	0	0	1	-
8, 10	0	1	0	-	0

$\langle 2 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
3, 11	0	-	0	1	1
9, 11	0	1	0	-	1
9, 25	-	1	0	0	1
10, 11	0	1	0	1	-
17, 21	1	0	-	0	1
17, 25	1	-	0	0	1
20, 21	1	0	1	0	-
20, 28	1	-	1	0	0



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 1 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
1, 3	0	0	0	-	1
1, 9	0	-	0	0	1
1, 17	-	0	0	0	1
2, 6	0	0	-	1	0
2, 10	0	-	0	1	0
8, 9	0	1	0	0	-
2, 3	0	0	0	1	-
8, 10	0	1	0	-	0

$\langle 2 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
3, 11	0	-	0	1	1
9, 11	0	1	0	-	1
9, 25	-	1	0	0	1
10, 11	0	1	0	1	-
17, 21	1	0	-	0	1
17, 25	1	-	0	0	1
20, 21	1	0	1	0	-
20, 28	1	-	1	0	0

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 1 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
1, 3	0	0	0	-	1
1, 9	0	-	0	0	1
1, 17	-	0	0	0	1
2, 6	0	0	-	1	0
2, 10	0	-	0	1	0
8, 9	0	1	0	0	-
2, 3	0	0	0	1	-
8, 10	0	1	0	-	0

$\langle 1 : 1, 2 : D \rangle$					
cube	a	b	c	d	e
1, 3, 9, 11	0	-	0	-	1

$\langle 2 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
3, 11	0	-	0	1	1
9, 11	0	1	0	-	1
9, 25	-	1	0	0	1
10, 11	0	1	0	1	-
17, 21	1	0	-	0	1
17, 25	1	-	0	0	1
20, 21	1	0	1	0	-
20, 28	1	-	1	0	0

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 1 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
1, 3	0	0	0	-	1
1, 9	0	-	0	0	1
1, 17	-	0	0	0	1
2, 6	0	0	-	1	0
2, 10	0	-	0	1	0
8, 9	0	1	0	0	-
2, 3	0	0	0	1	-
8, 10	0	1	0	-	0

$\langle 1 : 1, 2 : D \rangle$					
cube	a	b	c	d	e
1, 3, 9, 11	0	-	0	-	1
1, 9, 17, 25	-	-	0	0	1

$\langle 2 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
3, 11	0	-	0	1	1
9, 11	0	1	0	-	1
9, 25	-	1	0	0	1
10, 11	0	1	0	1	-
17, 21	1	0	-	0	1
17, 25	1	-	0	0	1
20, 21	1	0	1	0	-
20, 28	1	-	1	0	0

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 1 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
1, 3	0	0	0	-	1
1, 9	0	-	0	0	1
1, 17	-	0	0	0	1
2, 6	0	0	-	1	0
2, 10	0	-	0	1	0
8, 9	0	1	0	0	-
2, 3	0	0	0	1	-
8, 10	0	1	0	-	0

$\langle 1 : 1, 2 : D \rangle$					
cube	a	b	c	d	e
1, 3, 9, 11	0	-	0	-	1
1, 9, 17, 25	-	-	0	0	1
2, 10, 3, 11	0	-	0	1	-

$\langle 2 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
3, 11	0	-	0	1	1
9, 11	0	1	0	-	1
9, 25	-	1	0	0	1
10, 11	0	1	0	1	-
17, 21	1	0	-	0	1
17, 25	1	-	0	0	1
20, 21	1	0	1	0	-
20, 28	1	-	1	0	0

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 1 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
1, 3	0	0	0	-	1
1, 9	0	-	0	0	1
1, 17	-	0	0	0	1
2, 6	0	0	-	1	0
2, 10	0	-	0	1	0
8, 9	0	1	0	0	-
2, 3	0	0	0	1	-
8, 10	0	1	0	-	0

$\langle 1 : 1, 2 : D \rangle$					
cube	a	b	c	d	e
1, 3, 9, 11	0	-	0	-	1
1, 9, 17, 25	-	-	0	0	1
2, 10, 3, 11	0	-	0	1	-
8, 9, 10, 11	0	1	0	-	-

$\langle 2 : 1, 1 : D \rangle$					
cube	a	b	c	d	e
3, 11	0	-	0	1	1
9, 11	0	1	0	-	1
9, 25	-	1	0	0	1
10, 11	0	1	0	1	-
17, 21	1	0	-	0	1
17, 25	1	-	0	0	1
20, 21	1	0	1	0	-
20, 28	1	-	1	0	0

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 0 : 1, 2 : D \rangle$					
cube	a	b	c	d	e
0, 1, 8, 9	0	-	0	0	-
0, 1, 2, 3	0	0	0	-	-
0, 2, 8, 10	0	-	0	-	0

$\langle 1 : 1, 2 : D \rangle$					
cube	a	b	c	d	e
1, 3, 9, 11	0	-	0	-	1
1, 9, 17, 25	-	-	0	0	1
2, 10, 3, 11	0	-	0	1	-
8, 9, 10, 11	0	1	0	-	-



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

$\langle 0 : 1, 2 : D \rangle$					
cube	a	b	c	d	e
0, 1, 8, 9	0	-	0	0	-
0, 1, 2, 3	0	0	0	-	-
0, 2, 8, 10	0	-	0	-	0
$\langle 1 : 1, 2 : D \rangle$					
cube	a	b	c	d	e
1, 3, 9, 11	0	-	0	-	1
1, 9, 17, 25	-	-	0	0	1
2, 10, 3, 11	0	-	0	1	-
8, 9, 10, 11	0	1	0	-	-

$\langle 0 : 1, 3 : D \rangle$					
cube	a	b	c	d	e
0, 1, 8, 9, 2, 10, 3, 11	0	-	0	-	-



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

2, 6	P ₁
17, 21	P ₂
20, 21	P ₃
20, 28	P ₄
21, 23	P ₅
28, 30	P ₆
23, 31	P ₇
30, 31	P ₈
1, 9, 17, 25	P ₉
0, 1, 8, 9, 2, 10, 3, 11	P ₁₀



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	0	1	2	3	6	8	9	10	11	17	20	21	23	25	28	30	31
P ₁			X		X												
P ₂										X		X					
P ₃											X	X					
P ₄											X				X		
P ₅												X	X				
P ₆															X	X	
P ₇													X				X
P ₈																X	X
P ₉		X					X			X				X			
P ₁₀	X	X	X	X		X	X	X	X								



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	0	1	2	3	6	8	9	10	11	17	20	21	23	25	28	30	31
P ₁			X		X												
P ₂										X		X					
P ₃											X	X					
P ₄											X				X		
P ₅												X	X				
P ₆															X	X	
P ₇													X				X
P ₈																X	X
P ₉		X					X			X				X			
P ₁₀	X	X	X	X		X	X	X	X								



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	0	1	2	3	6	8	9	10	11	17	20	21	23	25	28	30	31
P ₁			X		X												
P ₂										X		X					
P ₃											X	X					
P ₄											X				X		
P ₅												X	X				
P ₆															X	X	
P ₇													X				X
P ₈																X	X
P ₉		X					X			X				X			
P ₁₀	X	X	X	X		X	X	X	X								

P₁₀ is an essential cube

$$f = \bar{a}\bar{c} +$$

$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	6	17	20	21	23	25	28	30	31
P ₁	X								
P ₂		X		X					
P ₃			X	X					
P ₄			X				X		
P ₅				X	X				
P ₆							X	X	
P ₇					X				X
P ₈								X	X
P ₉		X				X			

$$f = \bar{a}\bar{c} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	6	17	20	21	23	25	28	30	31
P ₁	X								
P ₂		X		X					
P ₃			X	X					
P ₄			X				X		
P ₅				X	X				
P ₆							X	X	
P ₇					X				X
P ₈								X	X
P ₉		X				X			

$$f = \bar{a}\bar{c} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	6	17	20	21	23	25	28	30	31
P ₁	(X)								
P ₂		X		X					
P ₃			X	X					
P ₄			X				X		
P ₅				X	X				
P ₆							X	X	
P ₇					X				X
P ₈								X	X
P ₉		X				X			

P₁ is an essential cube

$$f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	17	20	21	23	25	28	30	31
P ₂	X		X					
P ₃		X	X					
P ₄		X				X		
P ₅			X	X				
P ₆						X	X	
P ₇				X				X
P ₈							X	X
P ₉	X				X			

$$f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	17	20	21	23	25	28	30	31
P ₂	X		X					
P ₃		X	X					
P ₄		X				X		
P ₅			X	X				
P ₆						X	X	
P ₇				X				X
P ₈							X	X
P ₉	X				X			

$$f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	17	20	21	23	25	28	30	31
P ₂	X		X					
P ₃		X	X					
P ₄		X				X		
P ₅			X	X				
P ₆						X	X	
P ₇				X				X
P ₈							X	X
P ₉	X				(X)			

P₉ is an essential cube

$$f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	20	21	23	28	30	31
P ₂		X				
P ₃	X	X				
P ₄	X			X		
P ₅		X	X			
P ₆				X	X	
P ₇			X			X
P ₈					X	X

$$f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	20	21	23	28	30	31
P ₂		X				
P ₃	X	X				
P ₄	X			X		
P ₅		X	X			
P ₆				X	X	
P ₇			X			X
P ₈					X	X

P₃ dominates P₂

$$f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	20	21	23	28	30	31
P ₃	X	X				
P ₄	X			X		
P ₅		X	X			
P ₆				X	X	
P ₇			X			X
P ₈					X	X

$$f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	20	21	23	28	30	31
P ₃	X	X				
P ₄	X			X		
P ₅		X	X			
P ₆				X	X	
P ₇			X			X
P ₈					X	X

$$P_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	23	28	30	31
P ₄		X		
P ₅	X			
P ₆		X	X	
P ₇	X			X
P ₈			X	X

$$P_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	23	28	30	31
P ₄		X		
P ₅	X			
P ₆		X	X	
P ₇	X			X
P ₈			X	X

P₆ dominates P₄

$$P_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	23	28	30	31
P ₅	X			
P ₆		X	X	
P ₇	X			X
P ₈			X	X

$$P_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	23	28	30	31
P ₅	X			
P ₆		X	X	
P ₇	X			X
P ₈			X	X

P₇ dominates P₅

$$P_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	23	28	30	31
P ₆		X	X	
P ₇	X			X
P ₈			X	X

$$P_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	23	28	30	31
P ₆		X	X	
P ₇	X			X
P ₈			X	X

23 dominates 31

$$P_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	23	28	30
P ₆		X	X
P ₇	X		
P ₈			X

$$P_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	23	28	30
P ₆		X	X
P ₇	X		
P ₈			X

P₆ dominates P₈

$$P_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	23	28	30
P ₆		X	X
P ₇	X		

$$P_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	23	28	30
P ₆		(X)	(X)
P ₇	(X)		

P₆ and P₇ are essential cubes

$$P_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} + abc\bar{e} + acde$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	20	21	23	28	30	31
P ₄	X			X		
P ₅		X	X			
P ₆				X	X	
P ₇			X			X
P ₈					X	X

$$\bar{P}_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	20	21	23	28	30	31
P ₄	(X)			X		
P ₅		X	X			
P ₆				X	X	
P ₇			X			X
P ₈					X	X

$$\bar{P}_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	20	21	23	28	30	31
P ₄	(X)			X		
P ₅		X	X			
P ₆				X	X	
P ₇			X			X
P ₈					X	X

P₄ is an essential cube

$$\bar{P}_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + ac\bar{d}\bar{e} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	21	23	30	31
P ₅	X	X		
P ₆			X	
P ₇		X		X
P ₈			X	X

$$\bar{P}_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + ac\bar{d}\bar{e} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	21	23	30	31
P ₅	(X)	X		
P ₆			X	
P ₇		X		X
P ₈			X	X

$$\bar{P}_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + ac\bar{d}\bar{e} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	21	23	30	31
P ₅	(X)	X		
P ₆			X	
P ₇		X		X
P ₈			X	X

P₅ is an essential cube

$$\bar{P}_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + ac\bar{d}\bar{e} + a\bar{b}ce +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	30	31
P ₆	X	
P ₇		X
P ₈	X	X

$$\bar{P}_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + ac\bar{d}\bar{e} + a\bar{b}ce +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

PIs	30	31
P ₆	X	
P ₇		X
P ₈	X	X

P₈ dominates P₆ and P₇

$$\bar{P}_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + ac\bar{d}\bar{e} +$$



$$f(a, b, c, d, e) = \sum_m (0, 1, 2, 3, 6, 8, 9, 10, 11, 17, 20, 21, 23, 25, 28, 30, 31)$$

Pls	30	31
P ₈	(X)	(X)

P₈ is an essential cube

$$\bar{P}_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + ac\bar{d}\bar{e} + a\bar{b}ce + abcd$$

$$P_3 : f = \bar{a}\bar{c} + \bar{a}\bar{b}d\bar{e} + \bar{c}\bar{d}e + a\bar{b}c\bar{d} + abc\bar{e} + acde$$

Both decisions lead to (distinct) solutions of the same cost

