CS21201 Discrete Structures, Autumn 2021–2022

First Test

Date: Sep 07, 2021 | Time: 10:15am–11:30am | Maximum marks: 40

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1. Prove the following identity for all positive integers n.

$$\binom{2n}{n} = C(n) + \sum_{k=0}^{n-1} \binom{2n-2k-1}{n-k} C(k).$$

Here, $C(r)$ is the r-th Catalan number, and $\binom{s}{t} = \dfrac{s!}{t!(s-t)!}$ is the binomial coefficient s-choose-t. **(10)**

2. From the semester examination results, the head of the CSE Department makes the following observations.

F_1 : *If Andy is meritorious, then Bob is not studious or Paul is not attentive.*

F_2 : *If Paul is attentive, then Liya is exceptional.*

F_3 : *If Liya is exceptional and Bob is studious, then Andy is meritorious.*

F_4 : *Bob is studious.*

You task is to answer the question: "*Is Paul attentive?*"

Frame the arguments logically, and formally deduce (applying logical inferencing rules) the answer being asked here. Present your solution as indicated in the following parts.

(a) Write all the propositions (that you use) with English statements (meaning). **(2)**

(b) Build a suitable propositional logic formula to encode each of the *four* statements F_1–F_4 given above. **(4)**

(c) Use logical inferencing rules to derive the answer, and conclude whether Paul is attentive or not. **(4)**

3. Consider the following statements.

F_1 : *Tony and Mike are fans of Sachin Tendulkar.*

F_2 : *Every fan of Sachin Tendulkar is either a sports-lover, or a commentator, or both.*

F_3 : *No commentator likes rain during matches.*

F_4 : *All sports-lover likes high-scoring games.*

F_5 : *Mike dislikes whatever Tony likes and likes whatever Tony dislikes.*

F_6 : *Tony likes rain during matches and high-scoring games.*

Your tasks are to do the following.

(a) Write all the predicates (that you use) with English statements (meaning). **(2)**

(b) Encode the above statements in predicate (first-order) logic. **(3)**

(c) Use logical inferencing techniques (deduction rules) to prove that

G : "*There is a fan of Sachin Tendulkar who is a commentator, but not a sports-lover.*" **(5)**

4. All the integers in the sequence 2021, 20821, 208821, 2088821, 20888821, ... (with any non-negative number of occurrences of the digit 8 between 20 and 21) are divisible by a common prime p. Find p, and prove the assertion. No credits without a valid proof. You may use $45^2 = 2025$ for simplifying your calculations. **(10)**