Algorithms-1 - CS21003
(Class Test III)
Date: 01 – November – 2021

Maximum marks: 30 Duration: 1 hour

File naming convention: e.g., 18CS3004_G3_CT3.pdf (or any other extension for images). In case of multiple files, use _1, _2 etc at the end.

Submission is via Google form only. Email submissions will NOT be accepted. Please manage your time well keeping in mind that Internet and power disruptions are a new normal!

No clarifications from the TAs today. You can make any assumption as long as it is rational and you clearly state the same while solving the problem.

Plagiarism, in any form (including Internet source) will be severely penalized.

Whenever pseudocodes are asked, you can write C/C++ style code/pseudocode

Question 1

Give an algorithm that given two sets X and Y each consisting of n real numbers finds the minimum distance between a point of X and a point of Y. In particular, it computes the minimum possible value of $|x - y|$ over all pairs $x \in X$ and $y \in Y$. For full credit, your algorithm should run in time $O(n \log(n))$.

For example, if $X = \{0, 1, 8\}$ and $Y = \{11, 3, 5\}$, then the closest pair would be 1 and 3 with distance 2. Write a pseudocode. Demonstrate the correctness of your algorithm. **[7 marks]**

Question 2

Mac and Peter are stranded on opposite sides of Jaipur. The map of Jaipur is given by a directed graph where each edge in the graph takes only one minute to traverse. Each minute each of the two travellers can either traverse through an edge or stay where they are. Give an algorithm that given the map of Jaipur along with the initial locations of Mac and Peter computes the minimum amount of time required for them to meet at the same location. Write a pseudocode. What is the best complexity that you can achieve? **[5 marks]**

Question 3

Consider the 0-1 knapsack problem where you are allowed to take items more than once, have a total capacity of 10 and have the following items available (see Table 1). What is the best combination of items to take? Illustrate the steps by which you arrive at this combination. **[3 marks]**

Item	Weight	Value
A	2	4
B	3	5
C	4	9
D	5	10
E	7	16

Table 1: Item-weight table for the 0-1 knapsack.

Question 4

Answer the following questions:

(a). For each of the following operations, mention if the operation is asymptotically more efficient in a height-balanced binary search tree compared to a max heap or vice versa? If both the data structures have

the same asymptotic complexity, just mention that it is same. Also mention the individual complexities. i) searching a key element ii) insertion iii) deletion of maximum element iv) deletion of minimum element

(b). Suppose you have a max-heap of n nodes. Let h_i be the height of the sub-tree, rooted at a node at index i. What would be an asymptotic bound on $\sum_{i=1}^{n} h_i$? Briefly explain your answer.

(c). You are given an array $A[0...n-1]$ storing exactly n of the $n+1$ integers in the range $[a, a+n]$. This means that exactly one integer x in the range $[a, a+n]$ is missing in A. Mention an efficient algorithm to find the missing element with $O(1)$ extra space. Pseudocode is not required. What is the complexity?

(d). You are given a max-heap of n elements. You want to store all elements larger than a value x in a min-heap. What would be the algorithm? You can provide a pseudocode. Feel free to reuse the standard functions for a heap without giving the pseudocode. Assume there are m ($<< n$) such elements. What is the complexity? **[3+3+3+6 = 15 marks]**