Linear Algebra for AI/ML

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Topics

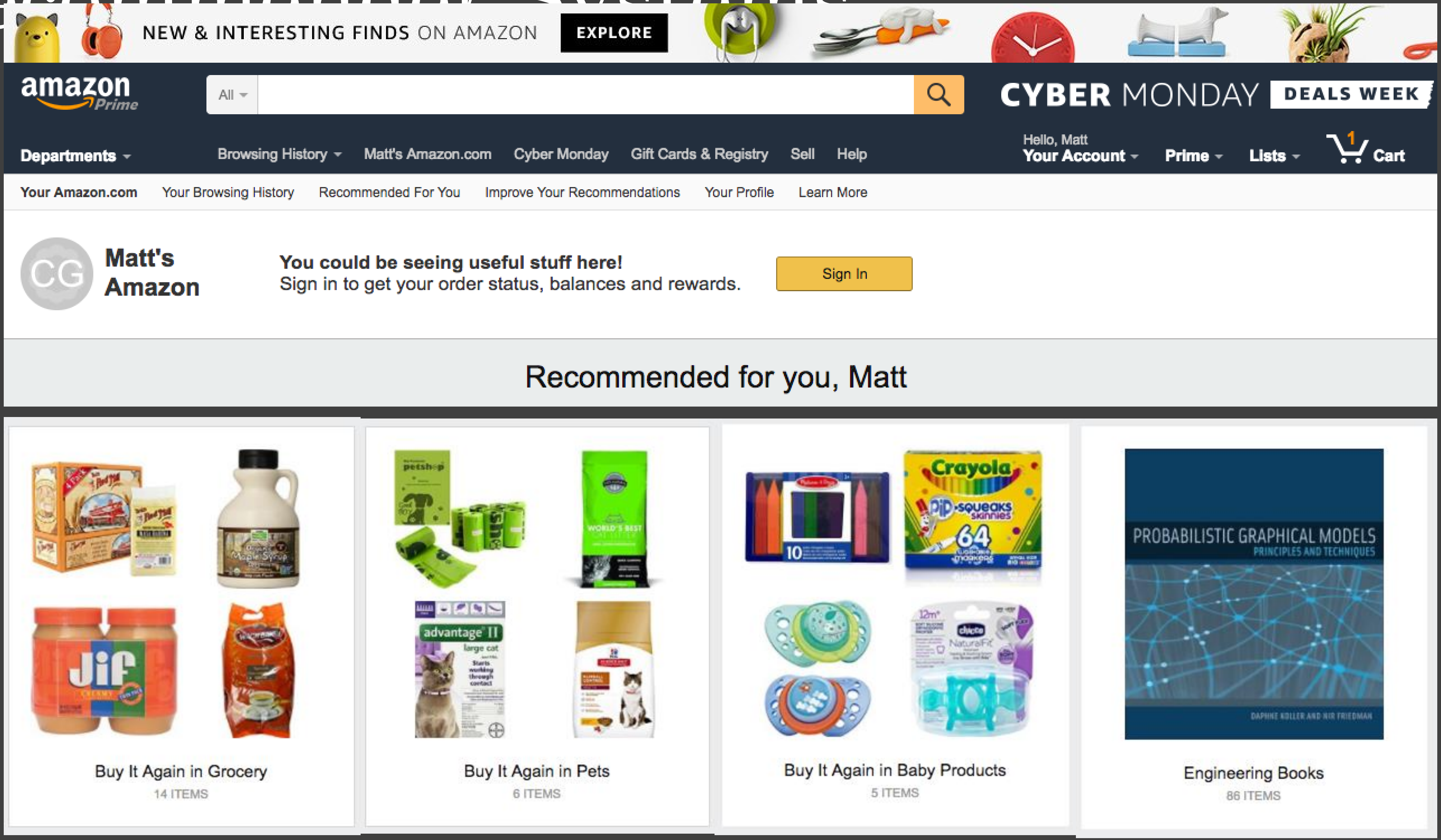
- **Recommender Systems**
  - Content Filtering
  - Collaborative Filtering
  - CF: Neighborhood Methods
  - CF: Latent Factor Methods
- **Matrix Factorization**
  - User / item vectors
  - Prediction model
  - Training by SGD

Recommender Systems

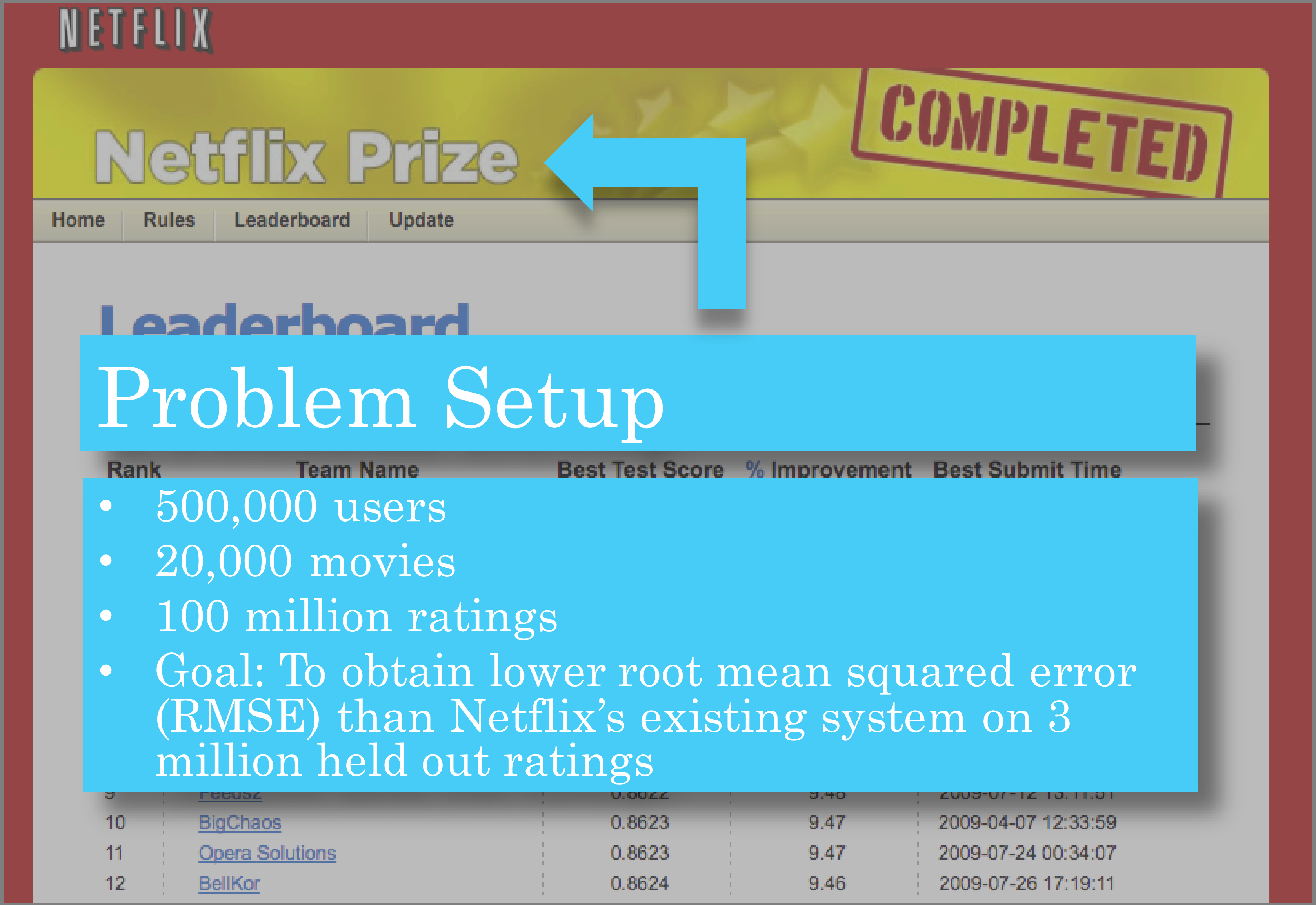
A Common Challenge:

- Assume you're a company selling **items** of some sort: movies, songs, products, etc.
- Company collects millions of **ratings** from **users** of their **items**
- To maximize profit / user happiness, you want to **recommend** items that users are likely to want

Recommender Systems



Recommender Systems



Problem Setup

- 500,000 users
- 20,000 movies
- 100 million ratings
- Goal: To obtain lower root mean squared error (RMSE) than Netflix's existing system on 3 million held out ratings

Recommender Systems

- **Setup:**
  - Items:
    movies, songs, products, etc.
    (often many thousands)
  - Users:
    watchers, listeners, purchasers, etc.
    (often many millions)
  - Feedback:
    5-star ratings, not-clicking 'next',
    purchases, etc.
- **Key Assumptions:**
  - Can represent ratings numerically as a user/item matrix
  - Users only rate a small number of items (the matrix is sparse)

	Doctor Strange	Star Trek: Beyond	Zootopia
Alice	1		5
Bob	3	4	
Charlie	3	5	2

Two Types of Recommender Systems

- *Example*: Pandora.com music recommendations (Music Genome Project)
- **Con:** Assumes access to side information about items (e.g. properties of a song)
- **Pro:** Got a new item to add? No problem, just be sure to include the side information

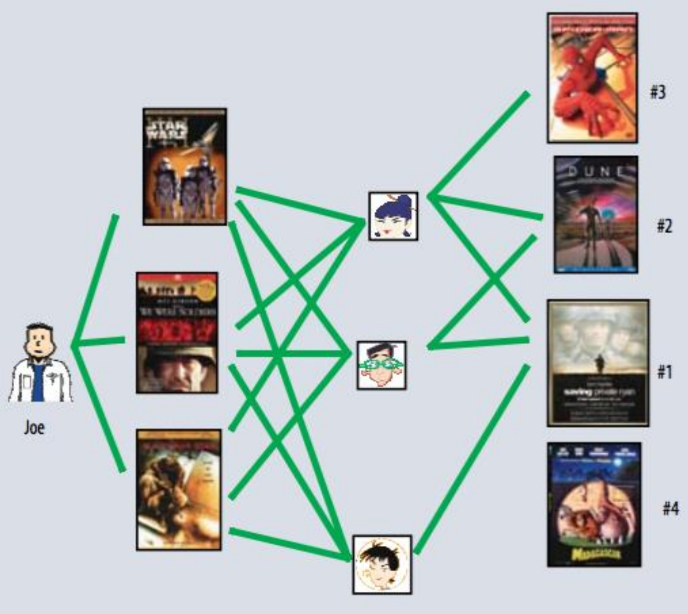
- *Example*: Netflix movie recommendations
- **Pro:** Does not assume access to side information about items (e.g. does not need to know about movie genres)
- **Con:** Does not work on new items that have no ratings

Collaborative Filtering

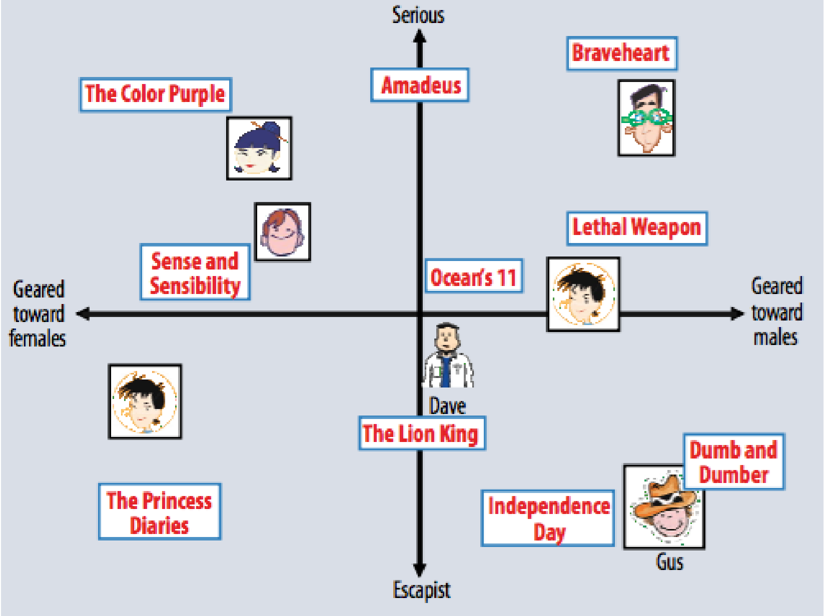
- **Everyday Examples of Collaborative Filtering...**
  - Bestseller lists
  - Top 40 music lists
  - The "recent returns" shelf at the library
  - Unmarked but well-used paths thru the woods
  - The printer room at work
  - "Read any good books lately?"
  - …
- **Common insight: personal tastes are correlated**
  - If Alice and Bob both like X and Alice likes Y then Bob is more likely to like Y
  - especially (perhaps) if Bob knows Alice

Two Types of Collaborative Filtering

1. Neighborhood Methods



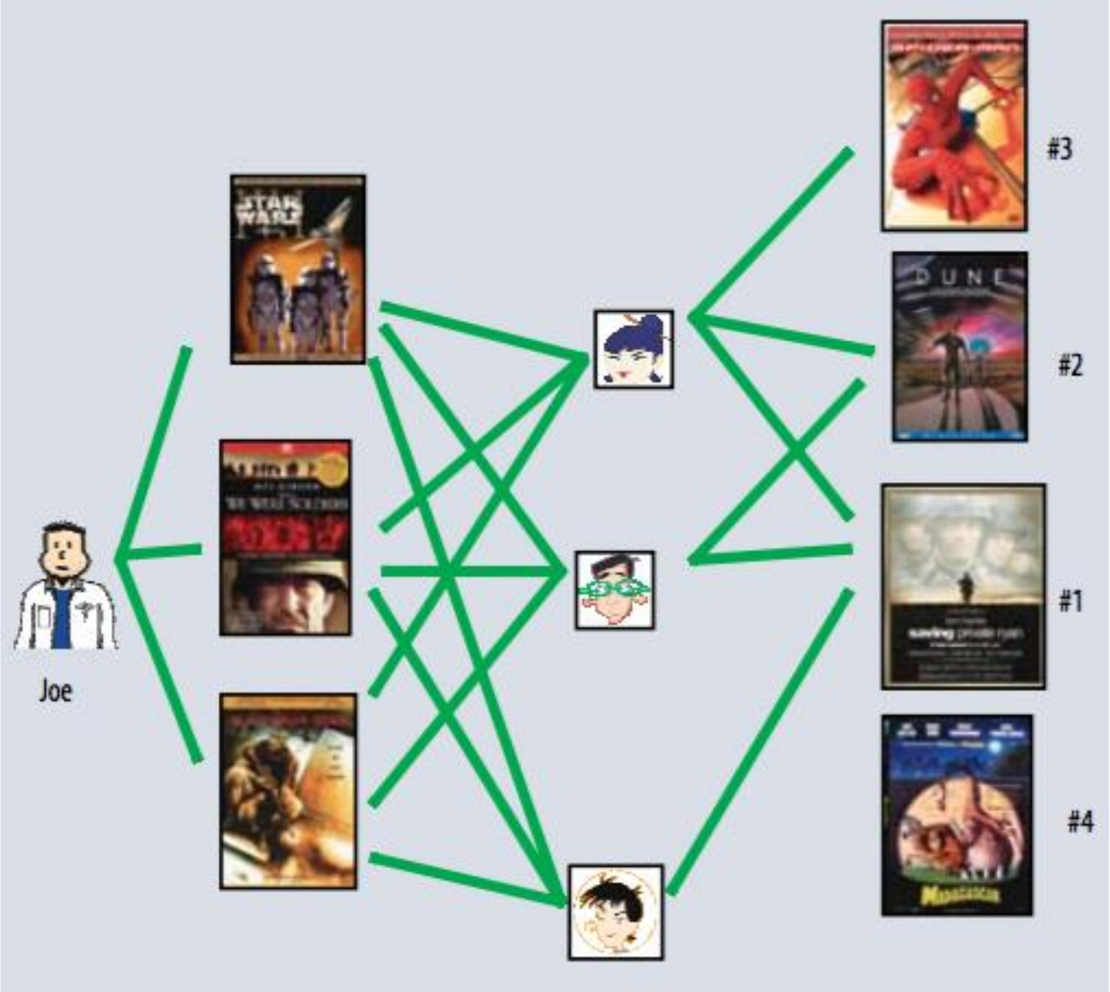
2. Latent Factor Methods



Koren et al. (2009)

Two Types of Collaborative Filtering

1. Neighborhood Methods



In the figure, assume that a green line indicates the movie was **watched**

Algorithm:

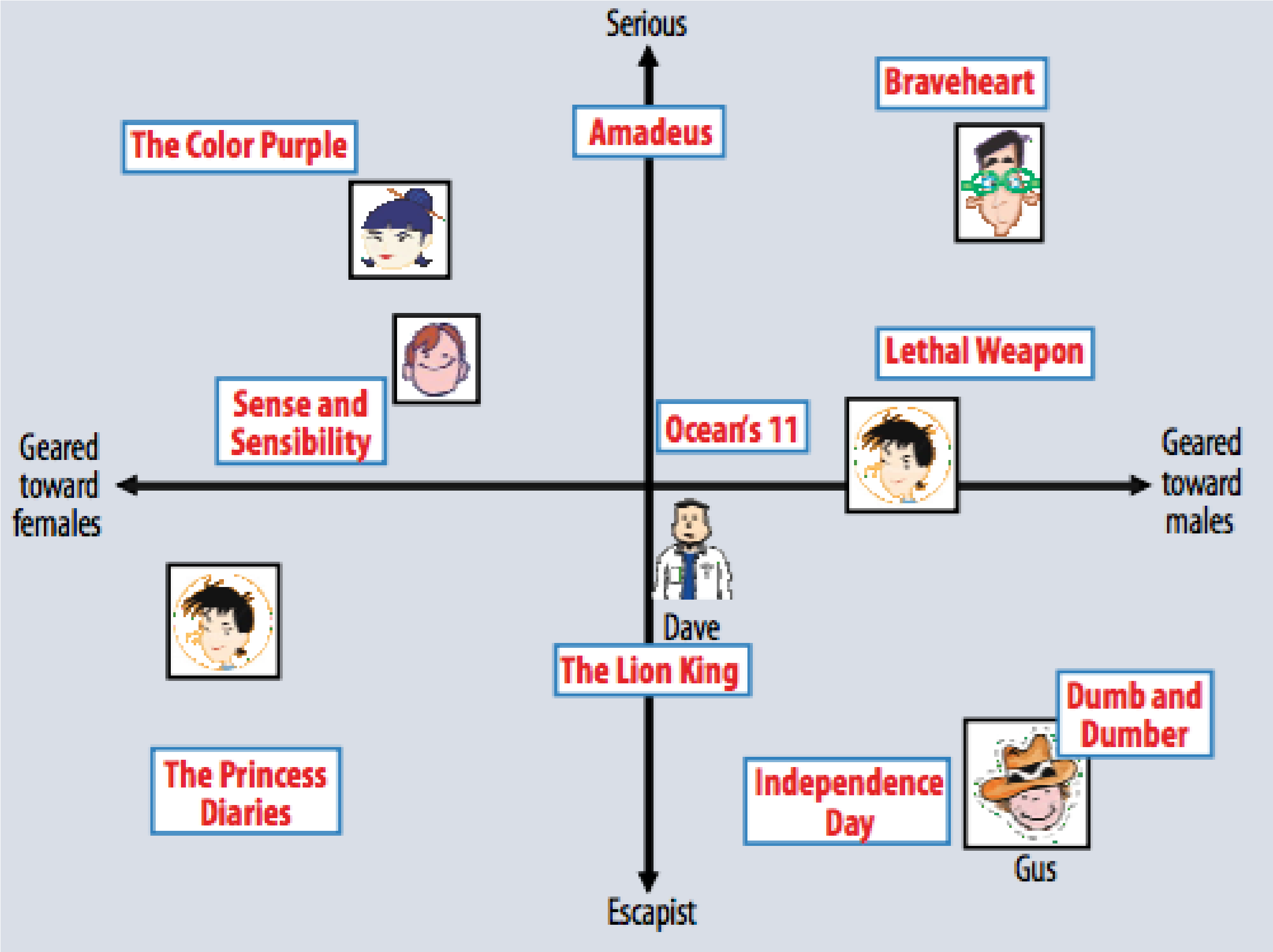
1. **Find neighbors** based on similarity of movie preferences
2. **Recommend** movies that those neighbors watched

Koren et al. (2009)

Two Types of Collaborative Filtering

2. Latent Factor Methods

- Assume that both movies and users live in some **low-dimensional space** describing their properties
- **Recommend** a movie based on its **proximity** to the user in the latent space



Koren et al. (2009)

Matrix Factorization

Matrix Factorization

(with matrices)

- **User vectors:**

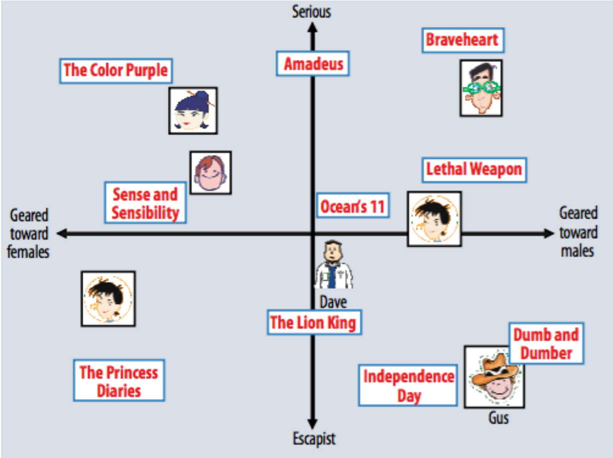
$$(W_{u*})^T \in \mathbb{R}^r$$

- **Item vectors:**

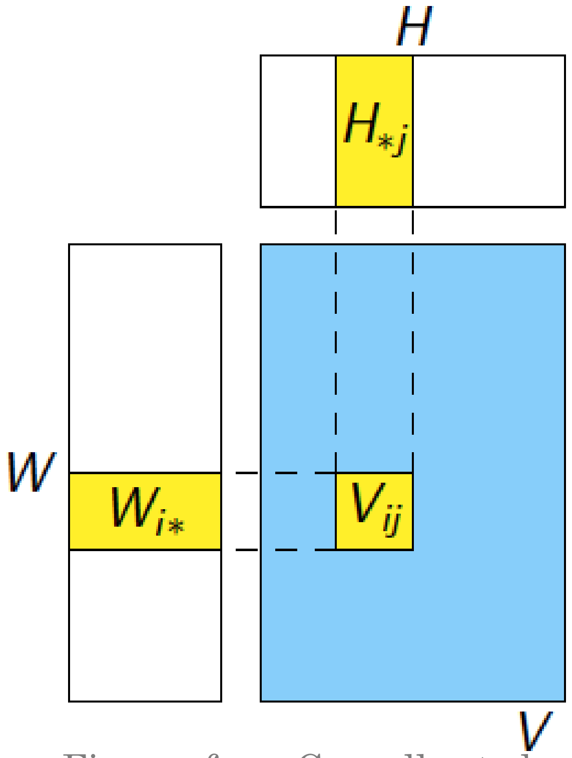
$$H_{*i} \in \mathbb{R}^r$$

- **Rating prediction:**

$$V_{ui} = W_{u*}H_{*i}$$
$$= [WH]_{ui}$$



Figures from Koren et al. (2009)



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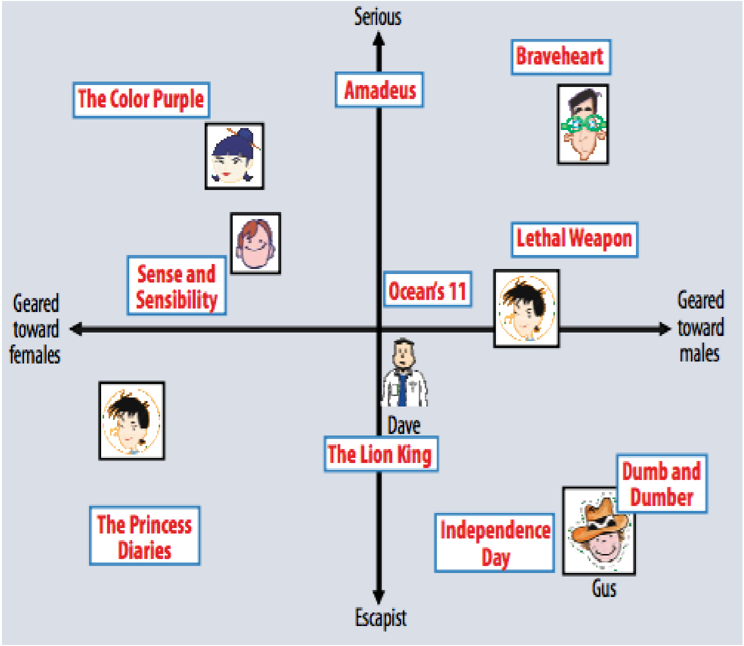

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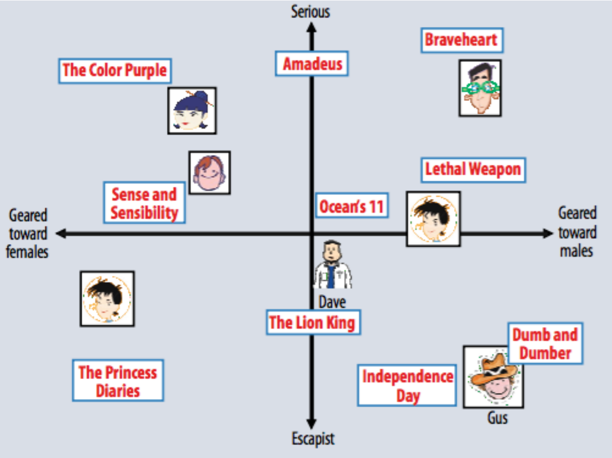


Figures from Koren et al. (2009)

- **Set of non-zero entries:**

$$\mathcal{Z} = \{(u, i) : v_{ui} \neq 0\}$$

- **Objective:**

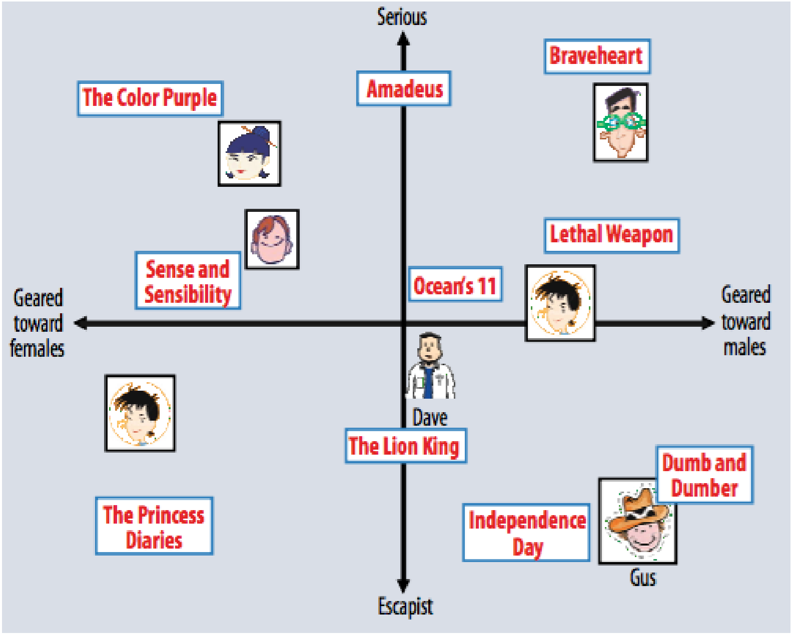
$$\underset{\mathbf{w},\mathbf{h}}{\text{argmin}} \sum_{(u,i)\in\mathcal{Z}} (v_{ui} - \mathbf{w}_u^T \mathbf{h}_i)^2$$

Matrix Factorization

(with vectors)

- **Regularized Objective:**

$$\underset{\mathbf{w},\mathbf{h}}{\operatorname{argmin}} \sum_{(u,i)\in\mathcal{Z}} (v_{ui} - \mathbf{w}_u^T \mathbf{h}_i)^2$$
$$+ \lambda(\sum_i ||\mathbf{w}_i||^2 + \sum_u ||\mathbf{h}_u||^2)$$



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Matrix Factorization

(with vectors)

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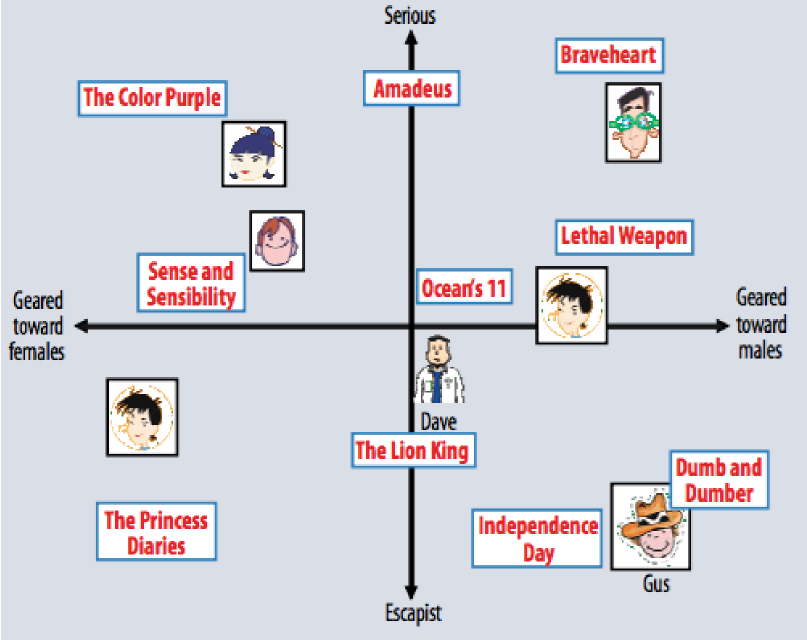
$$+ \lambda(\sum_i ||\mathbf{w}_i||^2 + \sum_u ||\mathbf{h}_u||^2)$$

- **SGD update for random (u,i):**

$$e_{ui} \leftarrow v_{ui} - \mathbf{w}_u^T \mathbf{h}_i$$

$$\mathbf{w}_u \leftarrow \mathbf{w}_u + \gamma(e_{ui}\mathbf{h}_i - \lambda \mathbf{w}_u)$$

$$\mathbf{h}_i \leftarrow \mathbf{h}_i + \gamma(e_{ui}\mathbf{w}_u - \lambda \mathbf{h}_i)$$



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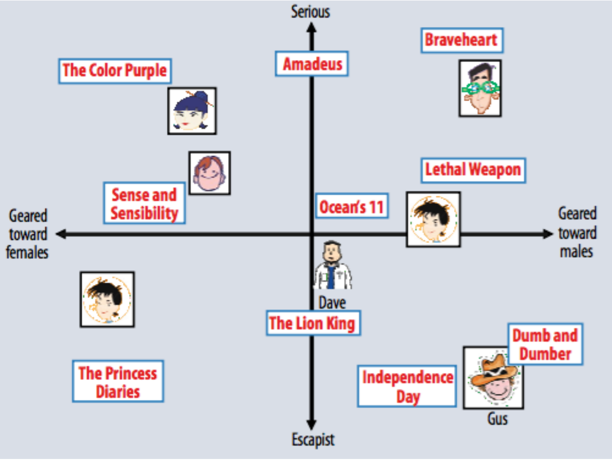

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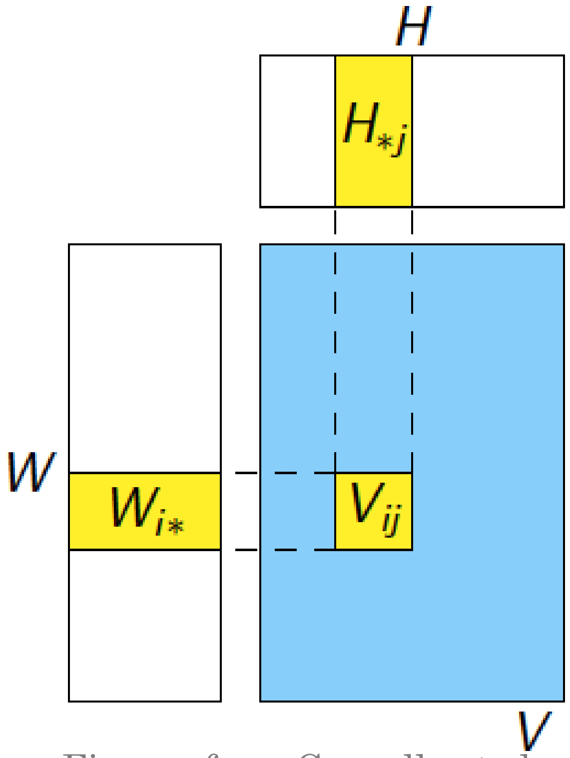

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Matrix Factorization

- **SGD (with matrices)**

require that the loss can be written as

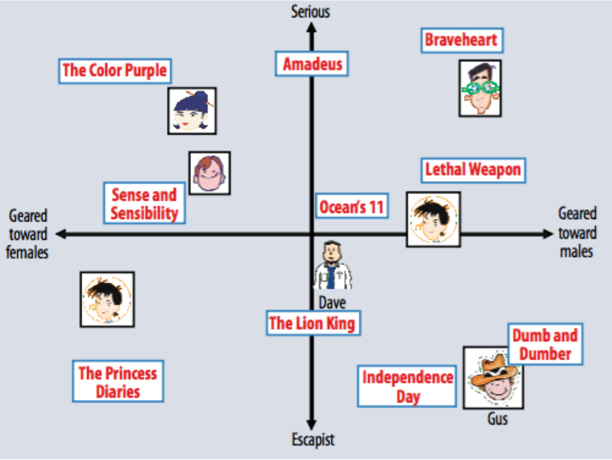
$$L = \sum_{(i,j) \in Z} l(\boldsymbol{V}_{ij}, \boldsymbol{W}_{i*}, \boldsymbol{H}_{*j})$$

Algorithm 1 SGD for Matrix Factorization

Require: A training set Z, initial values $\boldsymbol{W}_0$ and $\boldsymbol{H}_0$
  while not converged **do** {step}
    Select a training point $(i, j) \in Z$ uniformly at random.
    $\boldsymbol{W}'_{i*} \leftarrow \boldsymbol{W}_{i*} - \epsilon_n N \frac{\partial}{\partial \boldsymbol{W}_{i*}} l(\boldsymbol{V}_{ij}, \boldsymbol{W}_{i*}, \boldsymbol{H}_{*j})$
    $\boldsymbol{H}_{*j} \leftarrow \boldsymbol{H}_{*j} - \epsilon_n N \frac{\partial}{\partial \boldsymbol{H}_{*j}} l(\boldsymbol{V}_{ij}, \boldsymbol{W}_{i*}, \boldsymbol{H}_{*j})$
    $\boldsymbol{W}_{i*} \leftarrow \boldsymbol{W}'_{i*}$
  end while

step size

Figure from Gemulla et al. (2011)



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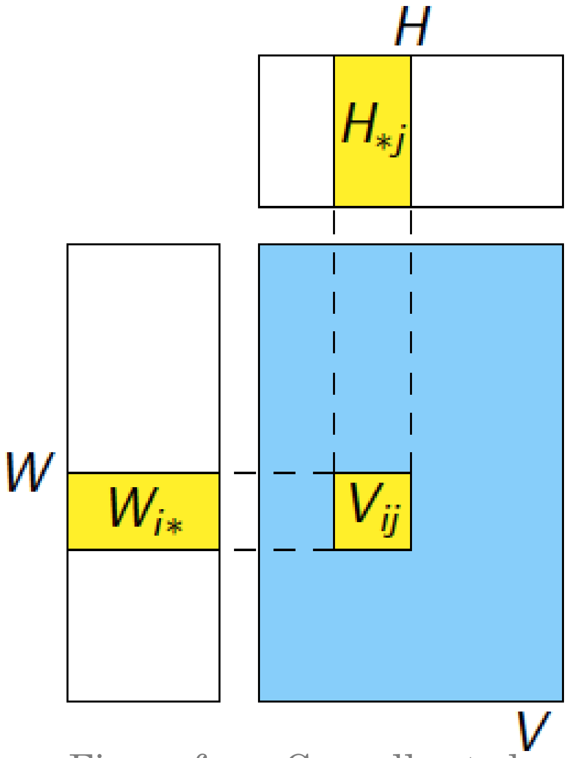

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Thank you!