Prob-Stat/QUIZ/2/B

Fill in the blanks (Numerical)

Date of Exam : 4th Oct, 2021

Time : 11:05 am to 11:55 am

Duration : 45min

No of questions: 10 out of 20 questions

Type: Random-sequential (navigation NOT allowed)

Each question carries 4 marks

NOTE: $\Phi(1.281552) = 0.9, \Phi(1) = 0.8413447, \Phi(2) = 0.9772499$

where $\Phi(\cdot)$ denotes the cdf of standard normal distribution

October 4, 2021

B:Q11. Let X be a continuous random variable with the density function.

$$f(x) = ke^{-|x-2|} \quad \text{for } -\infty < x < \infty,$$

where k is a suitable constant. Find $Var(X)$.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 2

ERROR RANGE: 0.005

B:Q12. A system functions if both of its parts function. The first part consists of 5 components in parallel with each component having reliability 0.4 after one month. The second part consists of 5 components in series each having a reliability of 0.97 after one month. What is the overall reliability of the system after one month?

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.792

ERROR RANGE: 0.005

Soln:$(1-(0.6)^5)*(0.97)^5 = 0.7919589$

B:Q13. The life span (in years) of a mobile battery follows a gamma distribution with mean 4 and variance 8. What is the probability that the battery will work for at least 4 years?

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.406

ERROR RANGE: 0.005

Solution: Here $r = 2, \lambda = 0.5$. So $f(x) = \frac{1}{4}xe^{(-\frac{x}{2})}, x > 0$. $P(X > 4) = 1 - pgamma(4, shape = 2, rate = 0.5) = 1 - 0.59399420 = 0.406$.

B: Q21. There are five multiple-choice questions in a test paper. Each question is with five answers as options, of which only one is correct. A student randomly ticks options in each question independently. Find the probability that he/she is correct on at most two questions?

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.942

ERROR RANGE: 0.005

Solution: $X \sim Bin(5, 0.2)$. So $P(X \leq 2) = (0.8)^5 + 5*0.2*(0.8)^4 + 10*(0.2)^2*(0.8)*3 = 0.94208$

B:Q22. The probability that a machine produces a defective item is 0.02. Each item is checked after its production. Assuming that these are independent trials, what is the probability that at least 100 items must be checked to find one that is defective?

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.1353

ERROR RANGE: 0.005

Solution: $P(X \geq 100) = P(X > 99) = (0.98)^9 9 \approxeq 0.1353$

B:Q23. Suppose there are 3 defective items in a lot of 50 items. A sample of size 10 is taken at random without replacement. Let X denote the number of defective items in the sample. What is the probability that the sample contains at most one defective item?

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.902

ERROR RANGE: 0.005

Soln: $(1 * choose(47, 10) + 3 * choose(47, 9))/choose(50, 10) = 0.9020408$

B:Q30. The percentage of impurities per batch in a certain chemical product is a random variable X that follows the beta distribution given by

$$f(x) = \begin{cases} 60x^3(1-x)^2 & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability that a randomly selected batch will have more than 25% impurities?
(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.9624

ERROR RANGE: 0.005

Soln: $1 - P(X \leq 0.25) = 0.9624023, X \sim Beta(4, 3)$

B:Q36. Let X be a Gamma distributed random variable with mean and variance equal to 2 and 4 respectively. Then Prob$(X > 6|X > 3)$ is ....
(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.2231

ERROR RANGE: 0.005

ANS: $X \sim exp(0.5)$ then $P(X > 6|X > 3) = P(X > 3) = 0.2231$

B:Q37. Let $X \sim \text{Normal}(2, 4)$. Then $P(X^2 + 2X \geqslant 0)$ is ....

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.864

ERROR RANGE: 0.005

Solution: The given probability $= P(X \leq -2) + P(X \geq 0) = P(Z \leq -2) + P(Z \geq -1) = \Phi(-2) + \Phi(1) = 0.0228 + 0.8413 = 0.8641$

B:Q39. The probability of a student successfully joining MS Teams for the class in one attempt is $\frac{3}{4}$. If the attempts are independent, what is the probability that the student will be able to join in at most 3 attempts.
(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.9844
ERROR RANGE: 0.005
Soln: $X \sim geo(0.75)$, $P(X \leq 2) = 0.984375$

B:Q41. Suppose that the number of admissions per day to a hospital has Poisson distribution with mean 2. At the beginning of a day there are 3 beds available for new admissions. Find the probability that the number of beds will be insufficient for new admissions for that day.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.1428765
ERROR RANGE: 0.005

Sol: Let X be the number of admissions. Then X is Poisson distributed with parameter $\lambda = 2$. Thus $P(X > 3) = 1 - P(X \leq 3) = 1 - e^{-2}(1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!}) = 1 - \frac{19}{3}e^{-2} \approx 0.1428765$

B:Q42. Suppose a stick of length 6 units is broken into two pieces (need not be of equal length). Assume that the choice of the breaking point follows uniform distribution over the length of the stick. Then determine the expected area of the right angled triangle formed by the two pieces as arms of the right angle.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 3
ERROR RANGE: 0.005

Sol: It is a right angled triangle. The legs are formed by the two pieces of the stick. The lengths are X and $6 - X$. The formula for the area of a right angle triangle is $A = \frac{1}{2}X(6 - X)$. Thus $E(A) = E(\frac{1}{2}x(6 - x)) = E(3X) - E(\frac{1}{2}X^2) = 3E(X) - \frac{1}{2}E(X^2)$. X is $U[0, 6]$. Now $f(x) = \frac{1}{6}$ for $x \in [0, 6]$. Thus $E(X) = 3$ and $E(X^2) = 12$.
Thus $E(A) = 9 - 6 = 3$.

B:Q43. Suppose a football team has 40% chance to win any one game and games are independent of one another. What is the probability that the team has fifth win on the 11-th game?

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.1003
ERROR RANGE: 0.005

Sol: Clearly, $X \sim NB(r = 5, p = 0.4)$ follows negative binomial distribution. Then $P(X = 11) = \binom{10}{4}(0.4)^5(1 - 0.4)^6 = 0.1003$.

B:Q44. Suppose a company buys batches of 20 components, and before dispatch, 5 of the components are selected at random without replacement from the batch and tested. The batch is rejected if at least two components are found to be below standard. Find the probability that a batch, which actually contains six components which are below-standard, will be rejected.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.4834881
ERROR RANGE: 0.005

Sol: X be the number of defective components and X follows hypergeometric distribution. Thus probability of acceptance is $P(X = 0) + P(X = 1) = \frac{\binom{14}{5}\binom{6}{0}}{\binom{20}{5}} + \frac{\binom{14}{4}\binom{6}{1}}{\binom{20}{5}} = 0.5165$. Thus probability of rejection $1 - 0.5165119 = 0.4834881$

B:Q45. Suppose the shoe size of Football players is normally distributed with mean 12 inches and variance 4 inches. It is also known that 10% of all Football players have a shoe size greater than c inches. Then the value of c is ———

(answer should be correct up to three decimal places, error range: 0.005)
NOTE: $\Phi(1.281552) = 0.9$

ANSWER : 14.5631
ERROR RANGE: 0.005

Sol: We have $\mu = 12, \sigma^2 = 4$. Then $P(X > c) = 0.1$. Take $Z = \frac{X-\mu}{\sigma}$. Then $P(X > c) = 0.1 \implies$

$P(Z > \frac{c-12}{2}) = 0.1 \implies P(Z \leq \frac{c-12}{2}) = 0.9$. Thus $\Phi(\frac{c-12}{2}) = 0.9 \implies \frac{c-12}{2} = 1.281552 \implies c = 14.5631$

B:Q46. If a random circle has a radius that is uniformly distributed on the interval $(0, 1)$ then the probability that the area exceeds 2 square units is ——

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.202
ERROR RANGE: 0.005

Sol:$P(A > 2) = P(\pi X^2 > 2) = P(X > \sqrt{\frac{2}{\pi}}) = \int_{\sqrt{\frac{2}{\pi}}}^{1} dx = 1 - \sqrt{\frac{2}{\pi}} = 1 - 0.7978845608 \approx 0.20211544$

B:Q48. Let the random variable X have a gamma distribution with mean 12 and variance 36. Find $E(X^{-3})$.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.006
ERROR RANGE: 0.005

Sol: We have $\frac{r}{\lambda} = 12$ and $\frac{r}{\lambda^2} = 36$. Then $r = 4, \lambda = \frac{1}{3}$. $E(X^{-3}) = \int_0^\infty \frac{1}{x^3} f(x)dx = \frac{1}{3!3^4} \int_0^\infty e^{-x/3} dx = 1/3!3^3 = 1/162 = 0.00617$

B:Q56. Patients arrive in a clinic according to a Poisson process with rate 10 patients per hour. The doctor starts to examine the patients only when the 4th patient arrives. The probability that in the first opening hour, the doctor does not start examining at all is

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.01033605
ERROR RANGE: ∓ 0.005
Soln: $X \sim Pois(10)$ then $P(X \leq 3) = 0.01033605$

B:Q69. Let the moments of a non-negative integer valued random variable X be given by $E(X^k) = 0.8$, where $k = 1, 2, \ldots$. What is the value of $P(X = 0)$?

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.200

ERROR RANGE: $\mp$0.005

Solution: It is easy to see that MGF= $M(t) = 0.2 + 0.8e^t$ which is equal to $E(e^{tX})$. Comparing we get $P(X = 0) = 0.2$.

B:Q72. Electronic items sold by a firm are known to contain 15% defectives. A buyer randomly picks four items from the lot available in the firm. Let X denote the number of defectives among these. The buyer will return the defectives for repair, and the cost of repair is $C = 2X^2 + X + 3$. Find the expected cost of repair.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 5.34

ERROR RANGE: 0.005

ANS:$X \sim Bin(4, 0.15)$ Then $E(C) = 2 * (Var(X) + E(X)^2) + E(X) + 3 = 2 * (4 * 0.15 * 0.85 + (4 * 0.15)^2) + (4 * 0.15) + 3 = 5.34$