

# Prob-Stat/QUIZ/1/SLOT-A

Fill in the blanks (Numerical)

Date : 7th Sep, 2021

Time : 12:05 pm to 12:55 am

Duration : 45min

No of questions: 10 out of 20 questions

Type: Random-sequential (navigation NOT allowed)

Each question carries 4 marks

September 7, 2021

**A:Q14.** To study the habit of students being absent in classes, a college survey revealed that sixty percent of the students have no interest in the studies, thirty percent wake up late in the morning and ten percent have some other problems. The probabilities of being absent in the class by three types of students are  $\beta$ ,  $\frac{\beta}{2}$ ,  $\frac{\beta}{4}$  respectively. If overall proportion of absentee students is  $93/160$ , then find the value of  $\beta$ .

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.75

ERROR RANGE: 0.005

**A:Q15.** In a colony each family has at least two children. The probability that a randomly chosen family from this society has exactly  $k$  children is  $\frac{4}{5} \cdot (\frac{1}{5})^{k-2}$ ,  $k = 2, 3, \dots$ , Each child can be a boy or girl with equal probability. Find the probability that a randomly selected family has exactly two boys.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.2743

ERROR RANGE: 0.005

A:Q16. Let  $E$  and  $F$  be two independent events with  $P(E) < P(F)$ . The probability that exactly two of them occur is  $\frac{11}{25}$  and the probability that none of them occurs is  $\frac{2}{25}$ . Find  $P(F) - P(E)$ .

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.2

ERROR RANGE: 0.005

A:Q17. Let  $X$  be a discrete random variable with with p.m.f.

$$p_X(k) = c k^2 \text{ for } k = -2, -1; \text{ and } p_X(k) = d k^{-2} \text{ for } k = 1, 2,$$

where  $c$  and  $d$  are suitable constants so that  $E(X) = \frac{9}{20}$ . Find  $E(X^2)$ .

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 2.05

ERROR RANGE: 0.005

A:Q18. According to Benford's law, in various types of data sets, the first digit  $X$  usually follows a discrete distribution with the p.m.f.

$$P(X = k) = \log_{10} \left( \frac{k+1}{k} \right), \quad k = 1, 2, \dots, 9.$$

Find the median of this distribution.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 3

ERROR RANGE: 0.005

A: Q23. Let  $A$  and  $B$  be events in a sample space  $S$  such that  $P(A) = \frac{1}{2} = P(B)$  and  $P(A^c \cap B^c) = \frac{1}{3}$ . Find  $P((A - B)|(A \cup B^c))$ .

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.2

ERROR RANGE: 0.005

A: Q25. Let  $X$  have the density function

$$f(x) = \begin{cases} \frac{2x}{k^2} & 0 \leq x \leq k \\ 0 & \text{otherwise} \end{cases}$$

For what value of  $k$  is the variance of  $X$  equal to 2?

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 6

ERROR RANGE:  $\mp 0.005$

A: Q29. Let  $X$  be a continuous random variable with p.d.f.

$$f(x) = \begin{cases} \frac{1}{3}e^x & x \leq 0 \\ \frac{2}{3}e^{-x} & x > 0 \end{cases}$$

Find the median of the distribution.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER :0.2876821

ERROR RANGE: 0.005

A: Q38. For some  $k > 0$ , let a function  $f(x)$  be defined as follows.

$$f(x) = \begin{cases} \frac{k}{x^2}e^{1/x} & \text{for } x \in (-1, 0), \\ 0 & \text{otherwise,} \end{cases}$$

The value of  $k$  for which  $f(x)$  becomes a probability density function is .....

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER: 2.718282

ERROR RANGE: 0.005

**A: Q52.** The police of a city plan to enforce speed limits during the morning rush hours on four different routes. The traps on routes A, B, C and D are operated 40%, 30%, 50% and 10% of the time respectively and independently. Ruby always speeds to work and she has probability 0.2, 0.1, 0.3 and 0.4 of using these routes respectively. Find the probability that she will be able to go to office without being caught on a given day.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.7

ERROR RANGE: 0.005

A: Q55. A circular target of unit radius is divided into four annular zones with radii  $\frac{1}{6}$ ,  $\frac{1}{2}$ ,  $\frac{2}{3}$  and 1 respectively. If 6 shots are fired and hit inside the disc of radius  $\frac{1}{2}$ , find the probability that at least one of these hits inside the disc of radius  $\frac{1}{6}$ .

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.50673

ERROR RANGE: 0.005

A: Q57. Suppose the random variable  $X$  has probability density function

$$f(x) = \begin{cases} cx^{-4}, & x \geq c \\ 0, & \text{otherwise} \end{cases}$$

for a constant  $c > 0$ . Find the probability that  $X$  is at least  $2\sqrt{3}$  given that  $X$  is at least  $\sqrt{3}$ .

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.125

ERROR RANGE: 0.005

A: Q58. Let a random variable  $X$  have the probability density function

$$f(x) = \begin{cases} cx^2, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

for some constant  $c > 0$ . Consider a random rectangle whose sides are  $X$  and  $2 - X$ . The expected value of the area of the rectangle is

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.6

ERROR RANGE:  $\mp 0.005$

**A: Q59.** The distribution function of a random variable  $X$  is given by

$$F(x) = \begin{cases} cx^3, & 0 \leq x < 3 \\ 1, & x \geq 3 \\ 0, & \text{otherwise} \end{cases}$$

for a constant  $c > 0$ . If  $P(X = 3) = 0$ , the median of the distribution is

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 2.38110 ERROR RANGE: 0.005

**A: Q61.** Consider an experiment where the letters of the word SUCCESS is rearranged randomly. Let  $A$  be the event that the word is rearranged so that the three S's come consecutively. Then the probability of the event  $A$  is

(answer should be correct up to three decimal places, error range: 0.005)

Answer: 0.14286

ERROR RANGE: 0.005

**A: Q62.** Urn-1 contains one white and two black marbles, Urn-2 contains one black and two white marbles, Urn-3 contains three black and three white marbles. A fair dice is rolled. If 1 or 2 or 3 show up, then Urn-1 is selected; if 4 is shows up, then Urn-2

is selected and if 5 or 6 show up, then Urn-3 is selected. A marble is then drawn at random from the urn selected. Let  $A$  be the event that the marble drawn is white. If  $U, V, W$  denote the events that the Urn-1, Urn-2, Urn-3 is selected respectively. Then the conditional probability  $P(V|A)$  is

(answer should be correct up to three decimal places, error range: 0.005)

Answer: 0.25

ERROR RANGE: 0.005

A: Q67. Consider a random variable  $X$  with  $P(X = x) = \frac{x^2}{a}$  for  $x \in \{-3, -2, -1, 1, 2, 3\}$ ,  $a > 0$  and  $P(X = x) = 0$  for  $x \notin \{-3, -2, -1, 1, 2, 3\}$ . Then  $P(X < -1)$  is

(answer should be correct up to three decimal places, error range: 0.005)

Answer: 0.464

ERROR RANGE: 0.005

A: Q68. Consider a continuous random variable  $X$  with the probability density function  $f(x) = ae^{-2|x|}$ ,  $-\infty < x < \infty$ , where  $a$  is a positive constant. Find the variance of  $X$ .

(answer should be correct up to three decimal places, error range: 0.005)

Answer: 0.5

ERROR RANGE: 0.005

A: Q69. Four buses carrying 148 job seeking IIT KGP students arrive at a job convention. The buses carry 40, 33, 25, and 50 students respectively. One of the students is randomly selected from the set of all the students with equal probability. Let  $X$  denote the

number of students that were on the bus carrying this randomly selected student. One of the four bus drivers is randomly selected with equal probability. Let  $Y$  denote the number of students on his bus. Find  $E(X) - E(Y)$ .

(answer should be correct up to three decimal places, error range: 0.005)

Answer: 2.284

ERROR RANGE: 0.005

A: Q76. Six distinct balls are placed randomly into three boxes A, B, C. For each ball the probability of going into a specific box is  $1/3$ . Find the probability that box A will contain at least two balls.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.648834

ERROR RANGE: 0.005