

Prob-Stat/QUIZ/2/A

Fill in the blanks (Numerical)

Date of Exam : 5th Oct, 2021

Time : 12:05 pm to 12:55 pm

Duration : 45min

No of questions: 10 out of 20 questions

Type: Random-sequential (navigation NOT allowed)

Each question carries 4 marks

NOTE: $\Phi(2.5) = 0.9937903$, $\Phi(0.625) = 0.7340145$

$\Phi(1.174987) = 0.88$, $\Phi(2) = 0.9772499$, $\Phi(1.33) = 0.9082409$

where $\Phi(\cdot)$ denotes the cdf of standard normal distribution

October 4, 2021

A:Q14. Let X be a Poisson random variable with $P(X = 2) = P(X = 3)$. Find $P(X \leq 3)$

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.6472

ERROR RANGE: 0.005

Solution: Given $\lambda^2/2 = \lambda^3/6$ gives $\lambda = 3$. Now $P(X \leq 3) = 13e^{-3} = 0.6472$.

A:Q15. Let a random variable X have a binomial distribution with mean 4 and variance 3. Find $P(3 < X < 6)$.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.4053583

ERROR RANGE: 0.005

Soln: Note $X \sim \text{bin}(n = 16, p = 0.25)$. Hence $P(3 < X < 6) = P(X = 4) + P(X = 5) = 0.4053583$

A:Q16. In a software company there are 20 engineers and 5 of them suffer from diabetes. If 4 of engineers are randomly selected for a bonus, find the probability that the number of diabetes patients among them is at most 2.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.968

ERROR RANGE: 0.005

Soln: $(1 * \text{choose}(15, 4) + 5 * \text{choose}(15, 3) + \text{choose}(5, 2) * \text{choose}(15, 2)) / \text{choose}(20, 4) = 0.9680083$

A:Q17. Let a random variable X have moment generating function

$$M_X(t) = \frac{1}{7^{10}} (4 + 3e^t)^{10}$$

Find the variance of X .

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 2.44898

ERROR RANGE: 0.005

Soln: $10 * 3/7 * 4/7 = 2.44898$

A:Q18. There are three posts for which interviews are being conducted. The probability of each candidate being selected is $\frac{1}{4}$ independently of others. Candidates are being interviewed in succession until three are selected. Find the probability that at least 6 candidates will be interviewed.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.8964844

ERROR RANGE: 0.005

Soln: Probability of success $p = 0.25$

Required probability $1 - (p^3 + 3 * p^3 * (1 - p) + \text{choose}(4, 2) * p^3 * (1 - p)^2) = 0.8964844$

A:Q19. The life (in months) of an electronic item is seen to follow a distribution with the density function $f(x) = 9x^2e^{-3x^3}$, $x > 0$. What is the probability that the item is working after one month given that it was working after 15 days?

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.072439757

ERROR RANGE: 0.005

Solution: $P(X > 1|X > 0.5) = \frac{P(X>1)}{P(X>0.5)} = \frac{e^{-3}}{e^{-3/8}} = e^{-21/8} = 0.072439757$

A:Q20. Let a point R be chosen at random with uniform distribution on the line segment PQ of length 10. Let X denote the length of line segment PR . Find $E(|PR - QR|)$.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 5

ERROR RANGE: 0.005

Solution: Here $X \sim U(0, 10)$. $E(|PR - QR|) = E(|2X - 10|) = 5$.

A:Q47. Suppose X is a normal random variable with mean 5. If $P(X > 0) = 0.88$ then $Var(X)$ is — .

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 18.10814

ERROR RANGE: 0.005

Sol: $P(X > 0) = 0.88 \implies P(Z > \frac{0-5}{\sigma}) = .88$. Thus $1 - \Phi(\frac{-5}{\sigma}) = 0.88 \implies 1 - (1 - \Phi(\frac{5}{\sigma})) = 0.88$. Thus $\Phi(\frac{5}{\sigma}) = 0.88$, Thus $\frac{5}{\sigma} = 1.174987$. Hence $Var(X) = \sigma^2 = 18.10814$

A:Q49. Let X be a random variable has uniform distribution on the interval 0 to 10. Find $P(X + \frac{10}{X} \geq 7)$.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.7

ERROR RANGE: 0.005

Sol: $P(X + \frac{10}{X} \geq 7) = P(X^2 + 10 \geq 7X) = P((X - 5)(X - 2) \geq 0) = P(X \leq 2 \text{ or } X \geq 5) = 1 - P(2 < X < 5) = 1 - \int_2^5 f(x)dx = 1 - \frac{3}{10} = \frac{7}{10}$.

A:Q61. Scores on a certain standardized test, IQ scores, are normally distributed with mean $\mu = 100$ and standard deviation $\sigma = 15$. An individual is selected at random. What is the probability his/her score X satisfies $120 < X < 130$.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.069

ERROR RANGE: ∓ 0.005

Solution: $P(120 < X < 130) = P(\frac{120-100}{15} < \frac{X-100}{15} < \frac{130-100}{15}) = P(1.33 < Z < 2)$, where Z follows standard normal distribution.

So $P(1.33 < Z < 2) = \Phi(2) - \Phi(1.33) = 0.9772 - 0.9082 = 0.069$

NOTE: PROVIDE VALUES

A:Q62. An analog signal received at a detector (measured in microvolts) may be modeled as a normal distribution $N(200, 256)$ at a fixed point in time. What is the probability that the signal is larger than 240 microvolts, given that it is larger than 210 microvolts?

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.023

ERROR RANGE: ∓ 0.005

Solution: $P(X > 240 | X > 210) = \frac{P(X > 240)}{P(X > 210)} = \frac{1 - P(X \leq 240)}{1 - P(X \leq 210)} = \frac{0.00621}{0.20599} = 0.023$

NOTE: $\Phi(2.5) = 0.9937903$, $\Phi(0.625) = 0.7340145$

A:Q64. The continuous random variable X has pdf

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

If $Y = 3X + 2$, then find $P(Y > 3)$.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.972

ERROR RANGE: ∓ 0.005

Solution: $P(Y > 3) = P(X > \frac{1}{3}) = \frac{35}{36} = 0.9722$.

A:Q66. In a biological experiment students are observing the eye colour for a large number of flies sequentially. It is known that 25% of flies have white eyes and 75% of flies have red eyes. The eye colour of individual flies are observed independently. What is the probability that the first fly with white eyes is fourth fly inspected?

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.105

ERROR RANGE: ∓ 0.005

Solution: The answer is q^3p , where $q = 3/4$ and $p = 1/4$. So $q^3p = \frac{27}{256} = 0.10546875$

A:Q67. Assume that the length of a phone call in minutes is an exponential random variable X with mean 10 minutes. If someone is already on a call at booth before you arrive, find the probability that you will have to wait between 5 and 10 minutes?

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.2386512

ERROR RANGE: ∓ 0.005

Solution: Recall pdf =

$$f(x) = \begin{cases} \frac{1}{10} e^{-\frac{x}{10}} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

We need to find $P(5 < X < 10) = \int_5^{10} \frac{1}{10} e^{-\frac{x}{10}} = e^{-0.5} - e^{-1} = 0.2386512$

A:Q73. A recruiting firm finds that 25% of the applicants for a particular sales position are fluent in Spanish. Applicants are selected at random from the pool and interviewed sequentially. Suppose that 10 applicants have been interviewed and no person fluent in Spanish has been identified. What is the conditional probability that exactly 15 unqualified applicants will be interviewed before finding the first applicant who is fluent Spanish?

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.05932617

ERROR RANGE: 0.005

ANS: X follows $\text{geo}(0.25)$.

$P(X=15 | X \geq 10) = 0.25 \cdot (0.75)^5 = 0.05932617$

A:Q75. Let X be a random variable having normal distribution with p.d.f. $f(x)$ satisfying $f(3.5) = f(6.5)$ with a finite variance. Find the probability that $X \in A$, where $A = [3, 4] \cup [5, 6] \cup [7, \infty)$.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.5

ERROR RANGE: 0.005

ANS: $E(X) = 5$ hence $P(X \in [3, 4]) = P(X \in (6, 7))$
 $P(X \in [3, 4] \cup [5, 6] \cup [7, \infty)) = P(X \in [5, 6] \cup (6, 7) \cup [7, \infty)) = 0.5$

A:Q76. Let X follows $N(0, 3)$. Find the value of $\left(\frac{Var(X^5)}{Var(X^3)}\right)$

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 567

ERROR RANGE: 0.005

ANS: $\left(\frac{Var(X^5)}{Var(X^3)}\right) = \left(\frac{E(X^{10})}{E(X^6)}\right) = \left(\frac{10!\sigma^{10}}{2^5 5!}\right) \left(\frac{6!\sigma^6}{2^3 3!}\right)^{-1} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{5 \cdot 4 \cdot 2^2} \sigma^4 = 9 \cdot 7 \cdot \sigma^4 = 9 \cdot 7 \cdot 9 = 567$

A:Q78. Coefficient of variation of a random variable X is defined as $C_v(X) = \frac{\sigma_X}{\mu_X}$ where $\mu_X = E(X) \neq 0$ and $\sigma_X = \sqrt{Var(X)} < \infty$. If X has the c.d.f.

$$F_X(x) = \begin{cases} 1 - \left(\frac{10}{x}\right)^{10} & \text{if } x \geq 10 \\ 0 & \text{if } x < 10 \end{cases}$$

Find the coefficient of variation of X .

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.1118034

ERROR RANGE: 0.005

ANS: If X follows $\text{Pareto}(m, \alpha)$ then $E(X) = \frac{m\alpha}{\alpha-1}$ and $Var(X) = \frac{m^2\alpha}{(\alpha-1)^2(\alpha-2)}$.
Hence $C_v(X) = (\alpha(\alpha-2))^{-1/2} = 1/\sqrt{10 \cdot 8} = 0.1118034$

A:Q79. At a certain location on highway, the number of cars exceeding the speed limit by more than 100 km per hour in half an hour is a random variable having a Poisson distribution with rate 8.4. What is the

probability that the the waiting time is less than 5 minutes for the first car exceeding the speed limit by more than 100 km per hour?

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.753403

ERROR RANGE: 0.005

ANS: Here X follows $Exp(8.4)$.

Hence $P(X < 5/30) = 1 - exp(-1.4) = 0.753403$

A:Q80. WhatsApp messages arrive in your phone following a Poisson process with a rate 6 per hour. Find the probability that 3rd message will not be received in half an hour.

(answer should be correct up to three decimal places, error range: 0.005)

ANSWER : 0.4231901

ERROR RANGE: 0.005

ANS: Number of messages entering in half an hour $X \sim Pois(6/2)$

Hence $P(X < 3) = 0.4231901$