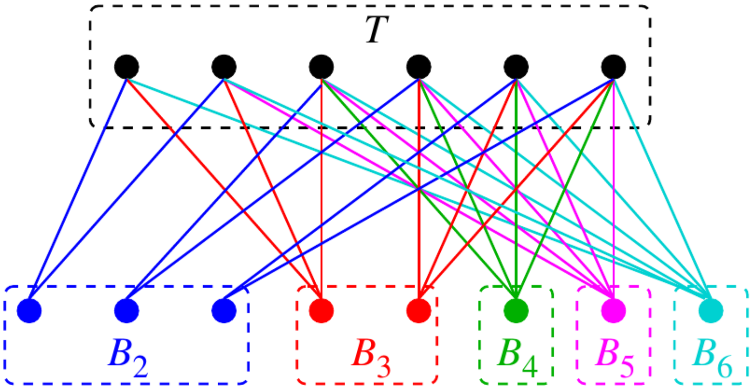
A logarithmic approximation algorithm for MIN-VERTEX-COVER

```
Initialize U = ∅.
while (E is not empty) {
    Find a vertex u ∈ V of largest (remaining) degree.
    Add u to U.
    Delete from E all the (remaining) edges with u as one endpoint.
}
Return U.
```

Tightness of the approximation ratio

The logarithmic approximation factor for the greedy vertex cover algorithm is optimal



A 2-approximation algorithm for MIN-VERTEX-COVER

Initialize $U = \emptyset$.

while (E is not empty) {

  Pick any edge $e = (u, v)$ from E.

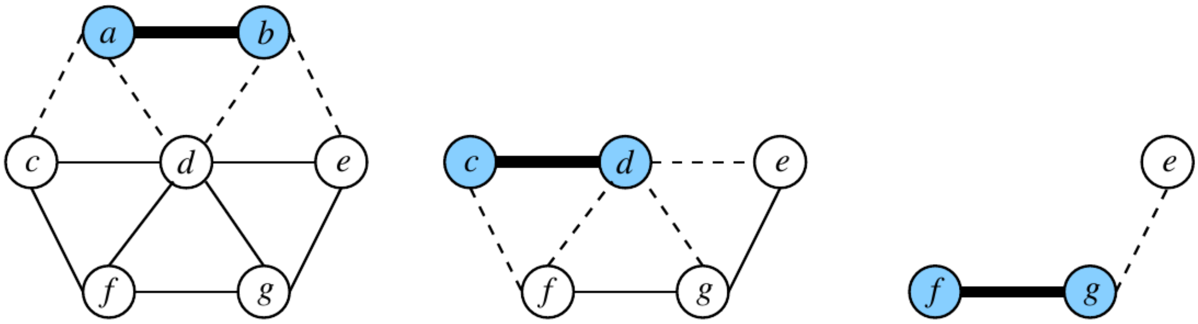
  Add u and v to U.

  Remove u and v from V.

  Remove from E all edges incident on u or v.

}

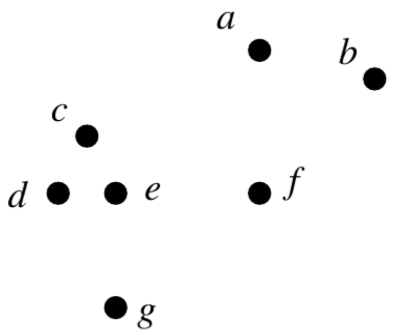
Return U.



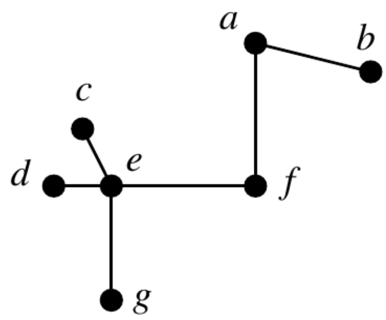
$U = \{a,b\}$ $\quad$ $U = \{a,b,c,d\}$ $\quad$ $U = \{a,b,c,d,f,g\}$

A 2-approximation algorithm for ETSP

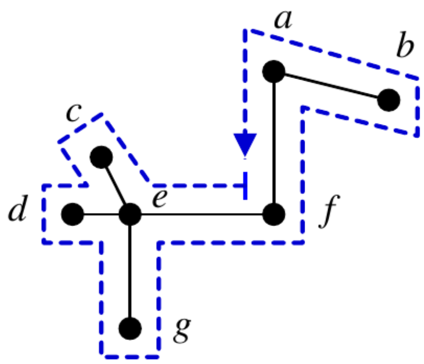
```
Compute a minimum spanning tree T of G under the given cost function.
Choose an arbitrary vertex u_0 of T.
Treat T as a tree rooted at u_0.
Impose an arbitrary ordering on the children of each node.
Make a pre-order traversal of T (starting at the root u_0).
Suppose that the traversal returns the list u_0, u_1, u_2, ..., u_{n-1} of visited nodes.
Return the Hamiltonian cycle Z = (u_0, u_1, u_2, ..., u_{n-1}, u_0).
```



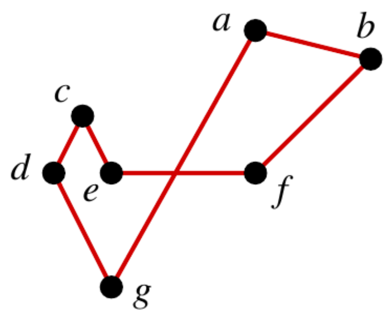
(a) Location of the cities



(b) Computation of an MST



(c) Preorder traversal of MST



(d) The TSP cycle

$p_i' = \lfloor p_i / \sigma \rfloor$

SOPT′ = optimal scaled profit
SOPT = corresponding original profit
OPT = optimal original profit
OPT′ = corresponding scaled profit

$\text{OPT}' \le \text{SOPT}'$
$\text{SOPT} \le \text{OPT}$

$p_i' = \lfloor p_i / \sigma \rfloor \ge p_i / \sigma - 1$ gives $p_i - \sigma p'_i \le \sigma$.
OPT corresponds to m objects, so $\text{OPT} - \sigma\, \text{OPT}' \le m\sigma \le n\sigma$.

$p_i' = \lfloor p_i / \sigma \rfloor \le p_i / \sigma$, so $\sigma\, \text{SOPT}' \le \text{SOPT}$.

Combining

$\text{SOPT} \ge \sigma\, \text{SOPT}' \ge \sigma\, \text{OPT}' \ge \text{OPT} - n\sigma$.

We require $\text{SOPT} \ge (1 - \varepsilon)\text{OPT}$. This is guaranteed by all

$\sigma \le (\varepsilon \times \text{OPT}) / n$.

$\text{OPT} \ge p_{max}$ so we take

$\sigma = (\varepsilon \times p_{max}) / n$.