MATH 281-1 **Due: Thursday, October 8**

HOMEWORK 2 (*October 2, 2015*)

1. A rocket follows a straight path. Its position along the path is t^2 meters from the origin at time t. The radius of the exhaust pipe is governed by the function $r(t) = 8 - t^{1/3}$.
   (a) The EPA wants an estimate of the total volume of exhaust from $t = 0$ to $t = 8$ seconds. They request you estimate this volume by sampling the radius of the exhaust pipe and the length of the exhaust column at 8 regularly timed intervals.
   Write and evaluate a summation representing the EPA's requested estimate.
   (b) Use an integral to produce a more accurate estimate of the total amount of exhaust (you may assume the exhaust in an exhaust column of height h and radius r is $\pi r^2 h$).

2. You travel, starting from the origin and heading in the positive direction, along the x-axis with a speed given by $s(x) = \sqrt{x}$ units per second, where x is your position along the x-axis. You sample the height of a function as you travel and discover $h(t) = (2 - t)^2$ where t is time in seconds.
   (a) How long does it take you to get to $x = 10$?
   (b) Give a relationship between x and t.
   (c) Reparameterize h in terms of x.
   (d) Write an integral formula for the area under h from $x = 0$ to $x = 10$.

Recall that Work $= \vec{F} \cdot \vec{d}$ where $\vec{F}$ is a force vector and $\vec{d}$ is a displacement vector.

3. A turbulent river pushes a particle at the point (x, y) with a force
$$\vec{F}(x, y) = (-yx, x).$$
   You are pushing a box through the river from $(0, 0)$ to $(1, 1)$ along a straight path.
   (a) Does $\vec{F}(0, 0) \cdot (1, 1)$ give you the total work done? How about $\vec{F}(1, 1) \cdot (1, 1)$? Why or why not?
   (b) Suppose you take tiny steps of size $\sqrt{2}/n$ on your way from $(0, 0)$ to $(1, 1)$. Write a summation to estimate the total work done by using $\vec{F}(x, y) \cdot \vec{d}$ as an approximation of the work done when moving from the origin of $\vec{d}$ to the tip of $\vec{d}$.
   (c) Write down the integral that results from your summation in (b) when you let $n \to \infty$.
   (d) What is the total work done?

4. A vector moves along the path $\vec{r}(t) = \begin{bmatrix} \sin t \\ 2\cos t \\ t \end{bmatrix}$ through a force field given by $\vec{F}(x, y, z) = \begin{bmatrix} yz \\ xz \\ xy \end{bmatrix}$.
   Find the total work done between $t = 0$ and $t = \pi$.

5. An *isometric parameterization* of a 2D surface S is a parameterization $p(t, s)$ that is area and length preserving. That is, the speed with respect to the first variable is 1, the speed with respect to the second variable is 1, and the area of the image of the square with corners $(\alpha, \beta), (\alpha + 1, \beta), (\alpha, \beta + 1), (\alpha + 1, \beta + 1)$ is 1.
   Consider the surface $S \subset \mathbb{R}^3$ parameterized by
$$s(x, y) = (x^2, y, |y|^{3/2}).$$
   Produce an isometric parameterization $p : \mathbb{R}^2 \to \mathbb{R}^3$ and verify each property of the parameterization. (Hint, use a computer to visualize the surface and imagine it as a piece of paper. How choose easy direction vectors to parameterize with. Also, don't forget that $\|\vec{a} \times \vec{b}\|$ gives you the area of the parallelogram with sides $\vec{a}$ and $\vec{b}$.)