1. (a) A valley can modeled by the equation $z = f(x, y) = 3e^{xy}$ where x is kilometers north and y is kilometers east. From a helicopter you observe a hiking trail that appears to be modeled by $x^3 + y^3 = 27$. Find the highest and lowest points along the hiking trail.
   (b) Write down an equation of the tangent plane (in vector form) at the minimum and maximum points along the hiking trail.

2. Let $A = \displaystyle\int_{x=0}^{x=6} \int_{y=x/3}^{y=2} x\sqrt{y^3+1}\,\mathrm{d}y\mathrm{d}x.$
   (a) Find a region R and a function f so that $A = \displaystyle\iint_R f(x, y)\,\mathrm{d}V$.
   (b) Find A.

3. Let C be a circular path in $\mathbb{R}^3$ that passes through the points $\vec{a} = (1, 2, 3)$, $\vec{b} = (1, 1, 3)$, and $\vec{c} = (-1, 1, 1)$. Let $f : \mathbb{R}^3 \to \mathbb{R}$ be distance from the origin. That is, $f(\vec{x}) = \|x\|$.
   (a) Find the radius of C. Then find two functions $g : \mathbb{R}^3 \to \mathbb{R}$ and $h : \mathbb{R}^3 \to \mathbb{R}$ so that $g(\vec{x}_0) = h(\vec{x}_0) = 0$ if and only if $\vec{x}_0 \in C$. (Hint, think about how to make two surfaces intersect at exactly C.)
   (b) Could you find a single function $r : \mathbb{R}^3 \to \mathbb{R}$ so that $r(\vec{x}_0) = 0$ if and only if $\vec{x}_0 \in C$? Why or why not?
   (c) Find a tangent vector to C at the point $\vec{c}$, and call this vector $T_{\vec{c}}$. Write down vector form of the plane of vectors orthogonal to $T_{\vec{c}}$.
   (d) Moving along C from the point $\vec{c}$, how fast is f changing *with respect to distance*?
   (e) Find the minimum and maximum values f attains along the curve C. (Hint, it's much easier to find the minimum and maximum values of f^2 and use those to minimize/maximize f.)

4. Consider the stretched polar coordinates system $\mathcal{SP}$. Coordinates in $\mathcal{SP}$ are given by pairs (ρ, ϕ) and relate to rectangular coordinates via the equations $x = \rho\cos\phi$ and $y = 2\rho\sin\phi$ where $\rho \in [0, \infty)$ and $\phi \in [0, 2\pi)$.
   (a) Draw the level curves $\rho = m$ for $m \in \{0, 1, 2, 3\}$ and $\phi = n$ for $n \in \{0, \pi/8, \pi/4, 3\pi/8, \pi/2\}$.
   (b) Compute the area enclosed by the curves $\rho = \rho_0$, $\rho = \rho_0 + \Delta\rho$, $\phi = \phi_0$, and $\phi = \phi_0 + \Delta\phi$ and call this area $\Delta V(\rho_0, \phi_0)$.
   (c) Give an isometric parameterization for... just kidding. Instead of forcing a parameterization to preserve area, let's measure how much it changes area. We'd like to compute the *density* of the coordinate system $\mathcal{SP}$. Consider the following: Let $G_n = \{(\frac{i}{n}, \frac{j}{n}) : i, j \in \mathbb{Z}\}$ be a grid of regularly spaced points. If we interpret G_n as points given in rectangular coordinates and we look at a square of size $\frac{1}{n} \times \frac{1}{n}$, we expect to find one point in that square. That is, if $S_n(x, y) = [x, x + \frac{1}{n}) \times [y, y + \frac{1}{n})$ is the square of width $\frac{1}{n}$ with lower left corner at the point (x, y), then the size of $S_n(x, y) \cap G_n = 1$ regardless of (x, y) and regardless of n. However, if we interpret G_n as specified in $\mathcal{SP}$ coordinates, suddenly the size of $S_n(x, y) \cap G_n$ depends on (x, y) and on n. The *density* of $\mathcal{SP}$ coordinates at the point (x, y) is defined as
   $$\mathrm{den}(x, y) = \lim_{n\to\infty} \text{expected size of } S_n(x, y) \cap G_n,$$
   when G_n is interpreted in $\mathcal{SP}$ coordinates. Alternatively (if the *expected size* language is too ambiguous), let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be the function defined by $(a, b) \mapsto (a\cos b, 2a\sin b)$ that takes $\mathcal{SP}$ coordinates to rectangular coordinates. Then,
   $$\mathrm{den}(x, y) = \lim_{n\to\infty} \frac{\text{area of } S_n(x, y)}{\text{area of } f(S_n(x, y))},$$
   where $f(S_n(x, y))$ is the image of $S_n(x, y)$ under f.

   Compute the density of $\mathcal{SP}$ coordinates.