MATH 281-1 **Due: Thursday, October 1**

HOMEWORK 1 (*September 23, 2015*)

1. Find unit vectors in the directions $\vec{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $\vec{v} = \hat{x} + 3\hat{y}$, $\vec{w} = -\hat{x} - \hat{y} - \hat{z}$, and $\vec{r} = \frac{\vec{v}}{\|\vec{v}\|} + \frac{\vec{w}}{\|\vec{w}\|}$.

2. Use the algebraic definition of the dot product to show for any $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^n$ and $\alpha \in \mathbb{R}$
   (i) $\vec{a} \cdot (\alpha\vec{b}) = \alpha(\vec{a} \cdot \vec{b})$
   (ii) $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

   (See page 7 of the Evans text for the definition of $\alpha\vec{v}$ and $\vec{u} + \vec{v}$.)

3. For vectors $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^n$, is $\vec{a} \cdot \vec{b} \cdot \vec{c}$ defined? Explain your answer in terms of the definition of the dot product.

4. Consider the plane $\mathcal{P}$ defined as the set of solutions to the equation
   $$3x - 2y + z = 4.$$
   Let $\vec{p} \in \mathcal{P}$ be a point in $\mathcal{P}$ interpreted as a vector.
   (a) What is $\vec{p} \cdot \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$. Explain.
   (b) The plane $\mathcal{Q}$ is the plane $\mathcal{P}$ translated in the $\hat{y}$ direction by one unit. Let $\vec{q} \in \mathcal{Q}$. What is $\vec{q} \cdot \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$?

5. Let $\vec{r} = \begin{bmatrix} 1.5 \\ 1 \end{bmatrix}$ and consider the sets
   $$A = \{\vec{x} \in \mathbb{R}^2 : \vec{x} = 2t\vec{r} \text{ for some } t \in \mathbb{Z}\} \qquad B = \{\vec{x} \in \mathbb{R}^2 : \vec{x} = 3t\vec{r} \text{ for some } t \in \mathbb{Z}\}$$
   $$C = \{\vec{x} \in \mathbb{R}^2 : \vec{x} = \vec{a} + \vec{b} \text{ for some } \vec{a} \in A \text{ and } \vec{b} \in B\}.$$
   (a) On separate axes, draw A, B, and C.
   (b) Prove or disprove the following statements:
      (i) $A = \{\vec{x} \in \mathbb{R}^2 : \vec{x} = t\vec{r} \text{ for some } t \in \mathbb{Z}\}$
      (ii) $C = \{\vec{x} \in \mathbb{R}^2 : \vec{x} = t\vec{r} \text{ for some } t \in \mathbb{Z}\}$
      (iii) $C = \{\vec{x} \in \mathbb{R}^2 : \vec{x} = t\vec{r} \text{ for some } t \in \mathbb{R}\}$

      Make sure to include any relevant definitions. (*Hint:* you may take it as a fact that if $a, b \in \mathbb{Z}$ then $a \pm b \in \mathbb{Z}$ and $ab \in \mathbb{Z}$.)