1. You are designing an oval-shaped race track. The race track consists of two straight stretches of length 500 meters and two semicircles of radius 100 meters. On a straight stretch, a car attains a maximum speed of 100 meters per second (near the end of the straight stretch). When traveling along a curved stretch, a car experiences a constant deceleration and leaves the curved stretch going 50 meters per second.

   Since you don't want cars to slide off the track as they travel along the curved part of the track, you plan on angling the curved part of the road (like a real race track). Assuming there is no friction (so the only forces a car is subjected to are the acceleration due to its movement and the acceleration due to gravity), compute the angle of elevation of the curved part of the race track so that a car won't slide off (Hint, this won't be constant!).

   You may assume that the acceleration due to gravity is g meters per second squared.

2. Given a curve $S \subset \mathbb{R}^n$, the *curvature* of S at the point $\vec{p} \in S$ is the magnitude of the acceleration when passing through $\vec{p}$ at unit speed (following the curve S). That is, if $\vec{r}(t)$ is an arc length parameterization of S and $\vec{r}(t_0) = \vec{p}$, then the curvature of S at the point $\vec{p}$ would be $\|\vec{a}(t_0)\|$.

   (a) Let $S_r \subset \mathbb{R}^2$ be a circle of radius r centered at the origin. Compute the curvature of S_r at the point $\vec{p} = (r, 0)$.

   (b) Given the following points that lie on the curve $C \subset \mathbb{R}^2$, estimate the curvature of C at $(1, 1)$.

x	y
0.7	0.49
0.8	0.64
0.9	0.81
1	1
1.1	1.21
1.2	1.44
1.3	1.69

3. (a) The level curves of the function $f : \mathbb{R}^2 \to [0, \infty)$ are given in polar coordinates by the function $L(h) = \{(\theta, h) : \theta \in [0, 2\pi)\}$. That is, $L(h)$ gives the level curve $f(x, y) = h$. Graph the level curves of f and then make a 3d sketch of the surface $(x, y, f(x, y))$.

   (b) The level curves of $g : \mathbb{R}^2 \to [1, \infty)$ are given in polar coordinates by the function $L(h) = \{(\theta, \sin(\theta)/h) : \theta \in (0, \pi)\}$. Graph the level curves of g and then make a 3d sketch of $(x, y, g(x, y))$.

4. Determine the following limits if they exist, otherwise show they don't exist.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} x + y$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$

   (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2}$

5. We've seen that for a function f and a direction $\vec{u}$, $\nabla f(\vec{a}) \cdot \vec{u}$ gives the directional derivative at $\vec{a}$ in the direction $\vec{u}$ *if f is differentiable*. This last part is key.

   Let $f(x, y) = \frac{x^2 y}{x^2 + y^2}$ if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Use the definition of the directional derivative to compute the directional derivative of f at $(0, 0)$ in the direction $\vec{u}$. Can the directional derivative be written as $\nabla f(0, 0) \cdot \vec{u}$ for all $\vec{u}$? How about for some $\vec{u}$?

   Show that f is not differentiable at $(0, 0)$.